Analog IC Design : 2022-23

Lecture 9

Frequency Response of Single Stage Amplifiers

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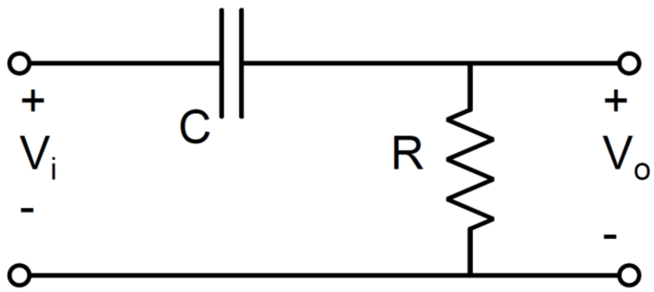
High Pass Filters



$$V_{out} = V_{in}\frac{R}{R + \frac{1}{j\omega C}}$$

$$Gain = \frac{V_{out}}{V_{in}} = \frac{R}{R+\frac{1}{j\omega C}} = \frac{1}{1+\frac{1}{j\omega RC}}$$

$$Critical\ (or)\ Cutoff\ frequency\ \omega_0 = \frac{1}{RC}$$

$$Gain = \frac{1}{1+\frac{\omega_0}{s}} = \frac{s}{s+\omega_0}$$

One pole at -1/(*RC*) and one zero at 0

$$Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega_0}{\omega})^2}}\angle\tan^{-1}(\frac{\omega_0}{\omega})$$

High Pass Filters

$$\omega_0 = \frac{1}{RC}$$

$$A_v = Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega_0}{\omega})^2}} \angle \tan^{-1}(\frac{\omega_0}{\omega})$$

$$Voltage\ Gain\ in\ dB = 10\log(Av)$$

$$Power\ Gain\ in\ dB = 20\log(Av)$$

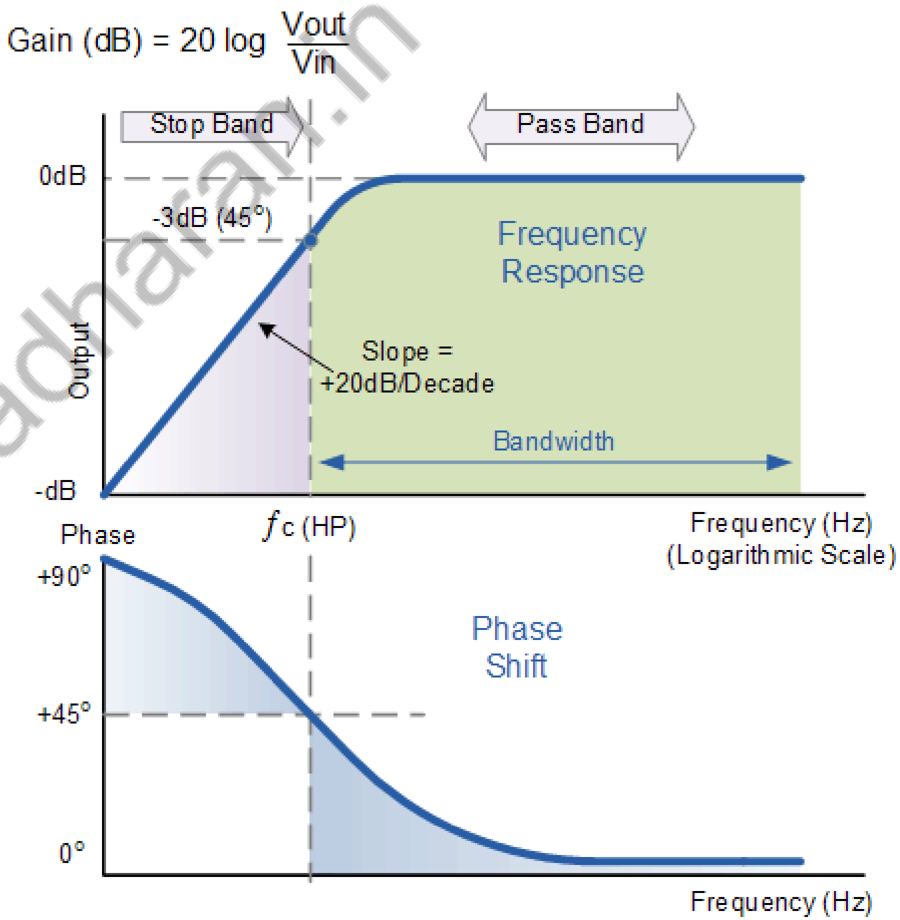
$$Half\ Power\ Gain\ in\ dB = 20\log(0.707) = -3\ dB$$

$$Half\ Power\ A_v = \frac{1}{\sqrt{2}}$$

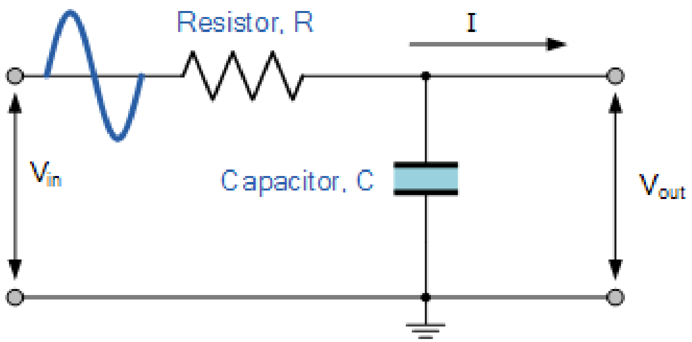
$For\ \omega \gg \omega_0\ \ A_v = 1\ Power\ Gain = 0\ dB, \phi = 0$

$For\ \omega = \omega_0\ A_v = \frac{1}{\sqrt{2}}\ \ Power\ Gain = -3\ dB, \phi = 45^0$

$For\ \omega < \omega_0\ Slope\ 20\ dB/\text{decade}\ \ \ for \omega << \omega_0\ \phi = 90^0$



Low Pass Filters



$$V_{out} = V_{in} \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

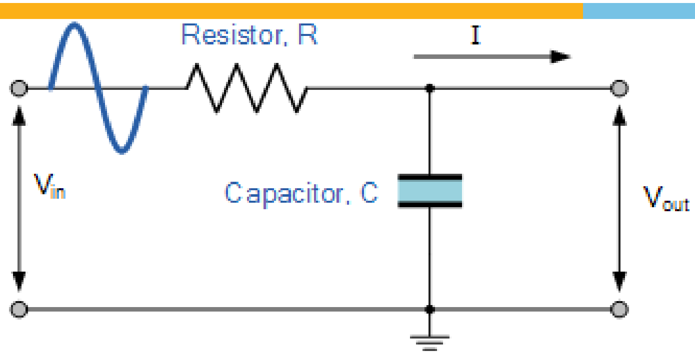
$$\omega_0 = \frac{1}{RC}$$

$$Gain = \frac{V_{out}}{V_{in}} = \frac{1}{1+jRC\omega} = \frac{1}{1+\frac{j\omega}{\omega_0}} = \frac{\omega_0}{s+\omega_0} = \frac{1}{1+\frac{s}{\omega_0}}$$

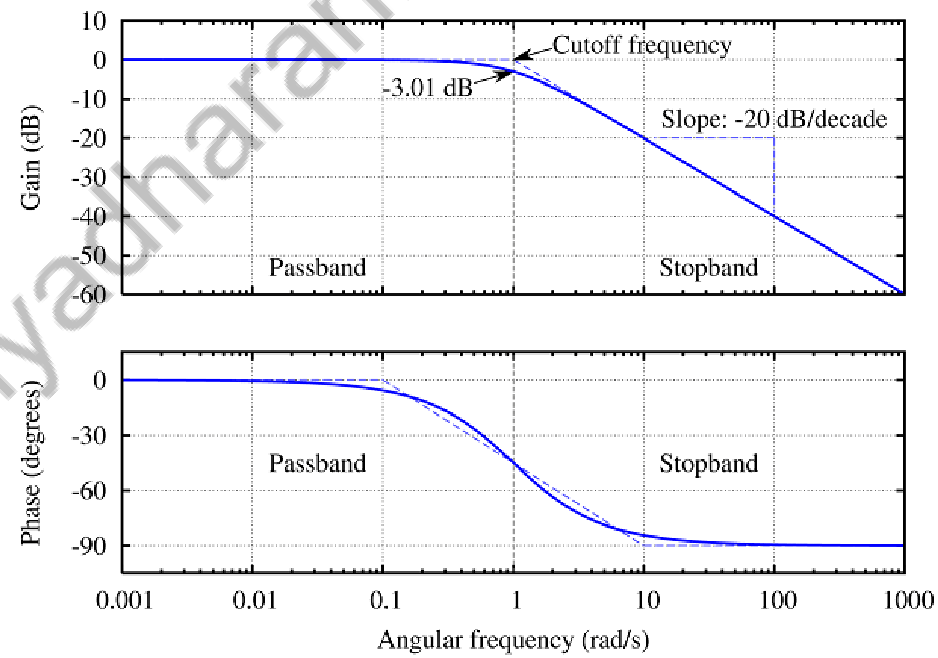
$$Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_0})^2}} \angle -\tan^{-1}(\frac{\omega}{\omega_0})$$

One pole at -1/(*RC*)

Low Pass Filters



$$Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_0})^2}} \angle -\tan^{-1}(\frac{\omega}{\omega_0}) \qquad \omega_0 = \frac{1}{RC}$$



$$Power\ Gain\ in\ dB = 20\log(Av)$$

$$Half\ Power\ Gain\ in\ dB = 20\log(0.5) = -3\ dB$$

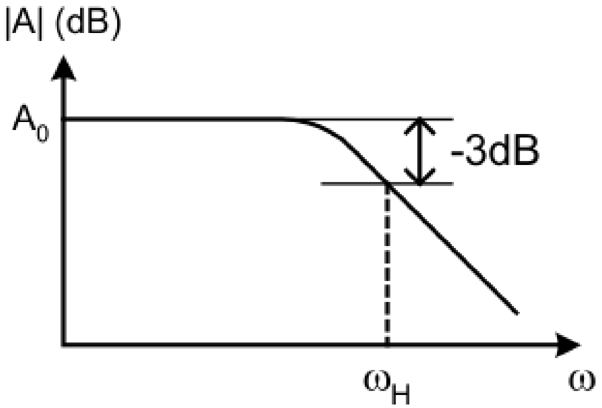
$$Half\ Power Av = \frac{1}{\sqrt{2}}$$

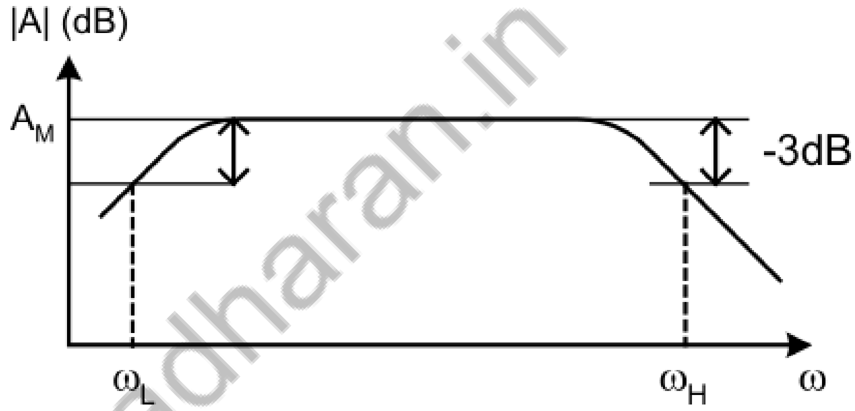
$$For\ \omega \ll \omega_0 \quad A_v = 1\ Power\ Gain = 0\ dB, \phi = 0$$

$$For\ \omega = \omega_0\, A_v = \frac{1}{\sqrt{2}} \quad Power\ Gain = -3\ dB, \phi = -45^0$$

$$For\ \omega > \omega_0\ Slope\ 20\ dB/\text{decade} \quad for \omega \gg \omega_0\ \phi = -90^0$$

Amplifier Transfer Function





Direct-Coupled (DC) amplifiers exhibit low-pass characteristics – flat gain from DC to ω_H

Capacitively coupled amplifiers exhibit band-pass characteristics , attenuation at low frequency due to impedance from coupling capacitances increasing for low frequencies Gain drops due to effects of internal capacitances of the device

Bandwidth is the frequency range over which gain is flat BW = $\omega_H - \omega_L \approx \omega_H$ ($\omega_H >> \omega_L$)

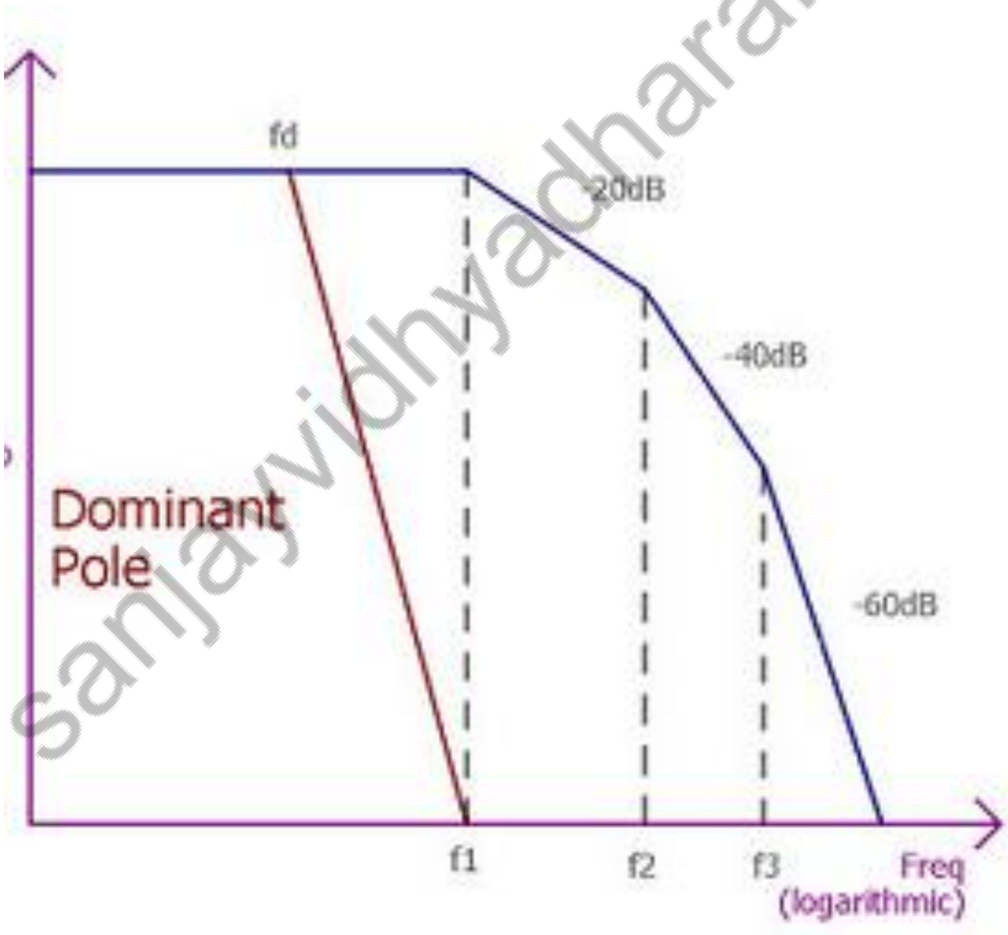
Gain-Bandwidth Product (GB) – Amplifier figure of merit GB $\equiv A_M\omega_H$

where A_M is the midband gain

It is possible to trade off gain for bandwidth

High-Frequency Response

$$H(s) = \frac{N(s)}{D(s)} = K\frac{(s-z_1)(s-z_2)\ldots(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)\ldots(s-p_{n-1})(s-p_n)},$$



High-Frequency Response

We can express function FH(s) with the general form:

$$F_H(s) = \frac{(1+s/\omega_{Z1})(1+s/\omega_{Z2})\cdots(1+s/\omega_{ZnH})}{(1+s/\omega_{P1})(1+s/\omega_{P2})\cdots(1+s/\omega_{PnH})}$$

Where ω_P and ω_Z represent the frequencies of high-frequency poles and zeros

The zeros are usually at infinity or sufficiently high frequency such that the numerator → 1 and assuming there is one dominant pole (other poles at much higher frequencies), we can approximate the function as…

$$F_H(s) = \frac{1}{(1+s/\omega_{P1})}$$

$$\omega_H \cong \omega_{P1}$$

– This simplifies the determination of the BW or ω_H

If a dominant pole does not exist, the upper 3-dB frequency ω_H can be found from a plot of $|F_H(j\omega)|$. Alternatively, we can approximate with following formula (see S&S p593 for derivation).

$$\omega_H = 1\Big/\sqrt{\frac{1}{\omega_{P1}^2} + \frac{1}{\omega_{P2}^2} + \cdots - \frac{2}{\omega_{Z1}^2} - \frac{2}{\omega_{Z2}^2}\cdots}$$

– Note: if ω_{P1} is a dominant pole, then reduces to $\omega_H=\omega_{P1}$

Open-Circuit Time Constant Method

The approach:

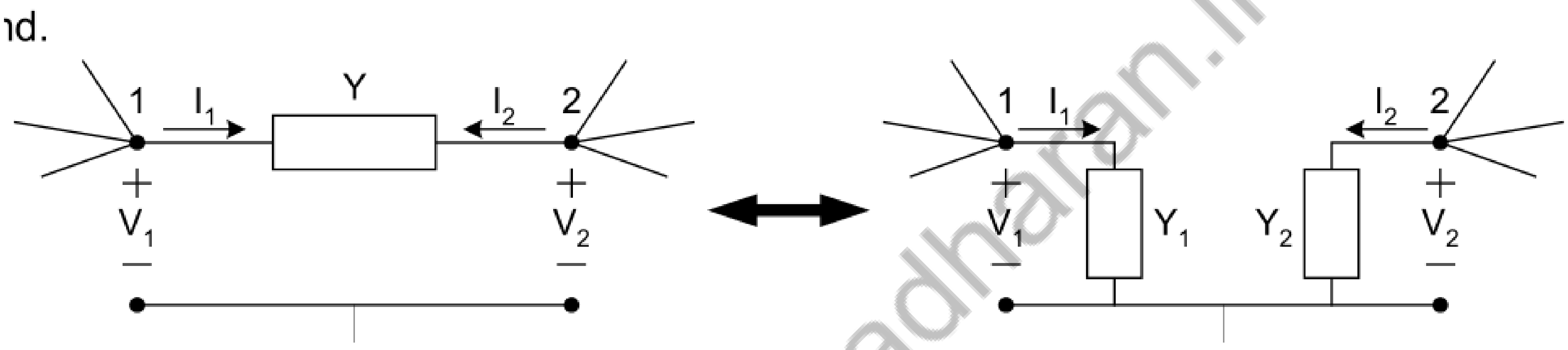
- For each capacitor:
  - set input signal to zero
  - replace all other capacitors with open circuits
  - find the effective resistance (R_{io}) seen by the capacitor C_i
- Sum the individual time constants (RCs or also called the open-circuit time constants)

$$b_1 = \sum_{i=1}^{nH} C_i R_{io}$$

- This method for determining b_1 is exact. The approximation comes from using this result to determine ω_H.

$$\omega_H \cong \frac{1}{\sum_{i=1}^{nH} C_i R_{io}}$$

Miller's Theorem



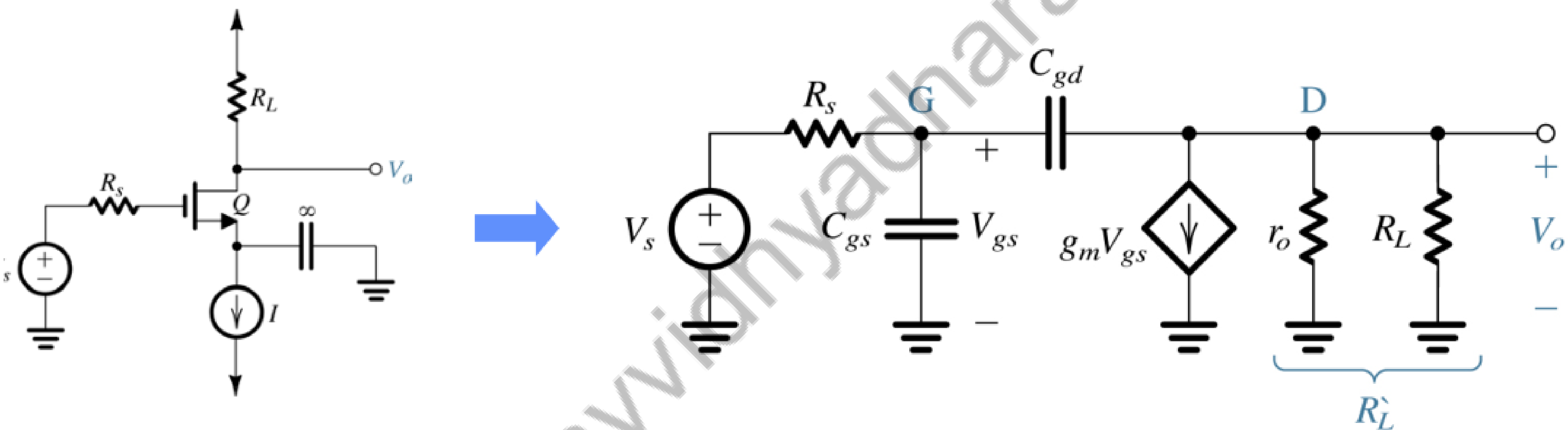
$$I_1 = Y(V_1 - V_2) = YV_1(1 - V_2/V_1) \qquad I_2 = Y(V_2 - V_1) = YV_2(1 - V_1/V_2)$$

$$I_1 = YV_1(1 - K) \qquad I_2 = YV_2(1 - 1/K)$$

$$I_1 = Y_1V_1 \qquad I_2 = Y_2V_2$$

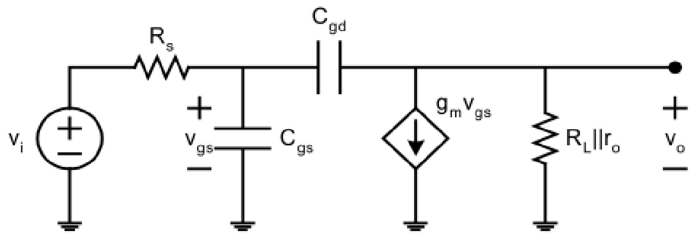
$$Y_1 = Y(1 - K) \qquad Y_2 = Y(1 - 1/K)$$

High-Frequency Response of CS Amp



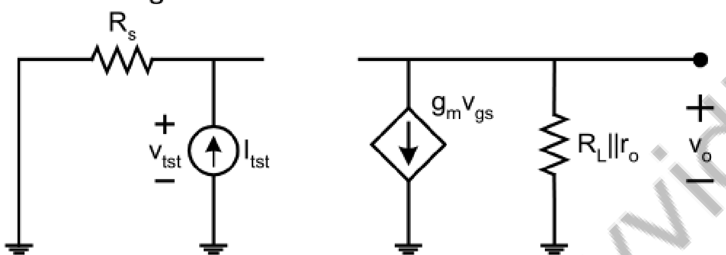
High-Frequency Response of CS Amp

Find the RC time constants associated with C_{gd} and C_{gs} in the following circuit



$$A_M = \frac{v_o}{v_i} = -g_m(R_L \parallel r_o)$$

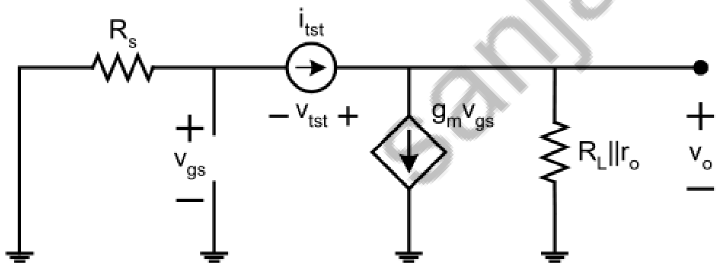
Replace C_{gd} with an open-ckt and find the resistance seen by C_{gs}



$$R_{gs} = \frac{v_{tst}}{i_{tst}} = R_s$$

$$\tau_{gs} = R_{gs}C_{gs} = R_s C_{gs}$$

Replace C_{gs} with an open-ckt and find the resistance seen by C_{gd}



$$i_{tst} = -v_{gs}/R_s \qquad i_{tst} = g_m v_{gs} + \frac{v_{gs} + v_{tst}}{R_L \parallel r_o}$$

$$R_{gd} = v_{tst}/i_{tst} = (R_L \parallel r_o) + g_m R_s (R_L \parallel r_o) + R_s$$

$$\tau_{gd} = R_{gd}C_{gd} = [(R_L \parallel r_o) + g_m R_s (R_L \parallel r_o) + R_s]C_{gd}$$

High-Frequency Response of CS Amp

- Summing to two time constants yields ω_H

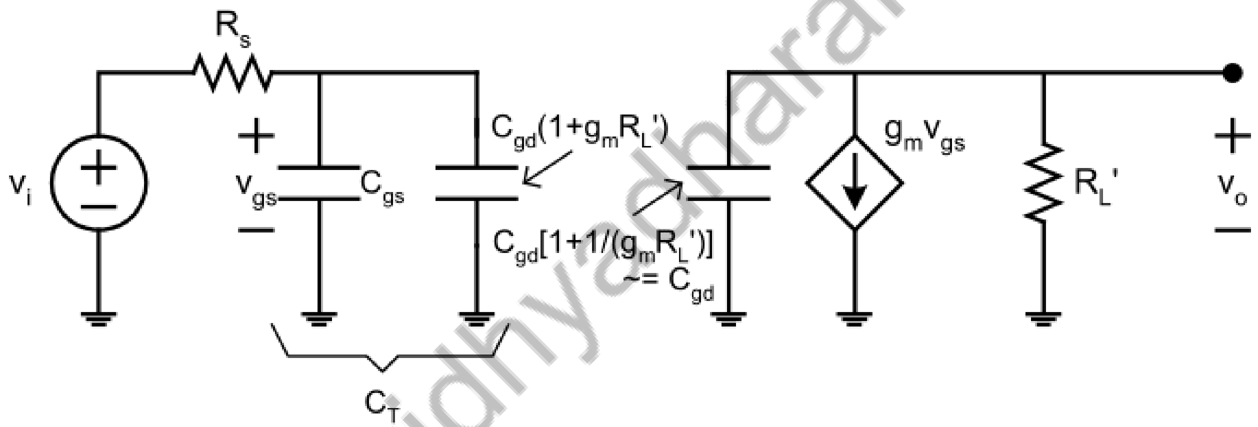
$$\omega_H \cong \frac{1}{\tau_{gs} + \tau_{gd}}$$

$$\omega_H \cong \frac{1}{R_s C_{gs} + \left[(R_L \| r_o) + g_m R_s (R_L \| r_o) + R_s \right] C_{gd}}$$

$$\omega_H \approx \frac{1}{R_s C_{gs} + g_m (R_L || r_0) C_{gd} R_s}$$

High-Frequency Response of CS Amp

Redraw the high-frequency small-signal model using Miller's theorem



– Assuming a dominant pole introduced by C_{gd} in parallel with C_{gs}

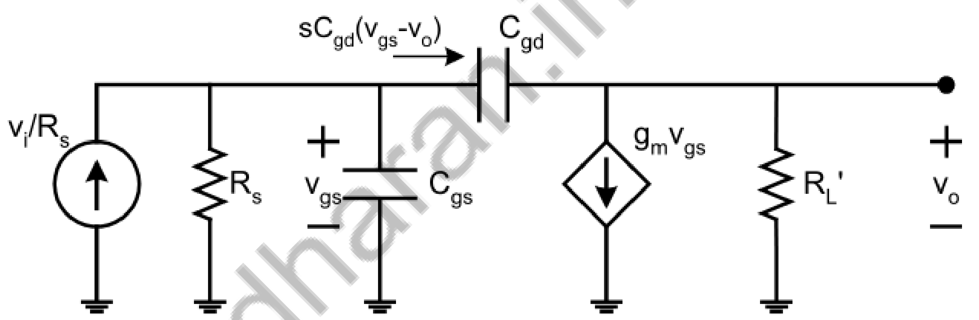
$$\omega_H \cong \frac{1}{\left[C_{gs} + C_{gd}\left(1 + g_m R_L'\right)\right] R_s} = \frac{1}{C_T R_s}$$

– Miller multiplication of C_{gd} results in a large input capacitance

High-Frequency Response of CS Amp

$$\frac{v_i(s)}{R_s} = \frac{v_{gs}}{R_s} + sC_{gs}v_{gs} + sC_{gd}\left(v_{gs} - v_o\right)$$

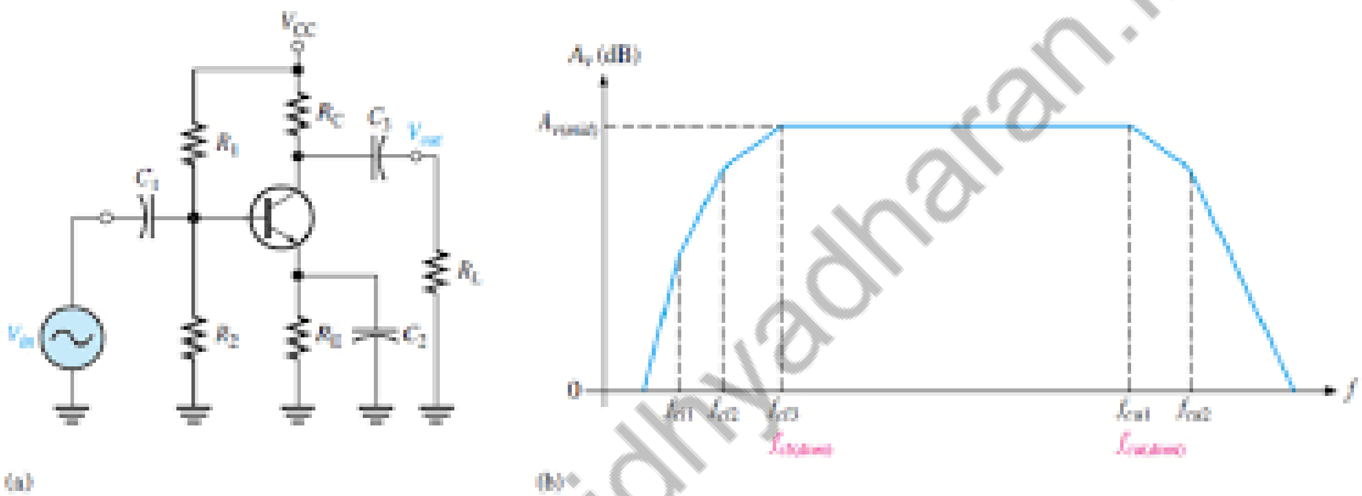
$$sC_{gd}\left(v_{gs} - v_o\right) = g_m v_{gs} + \frac{v_o(s)}{R_L'}$$

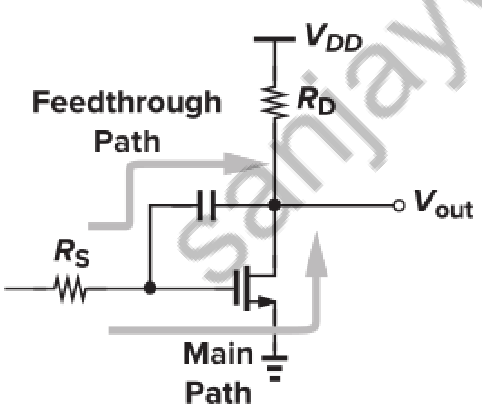


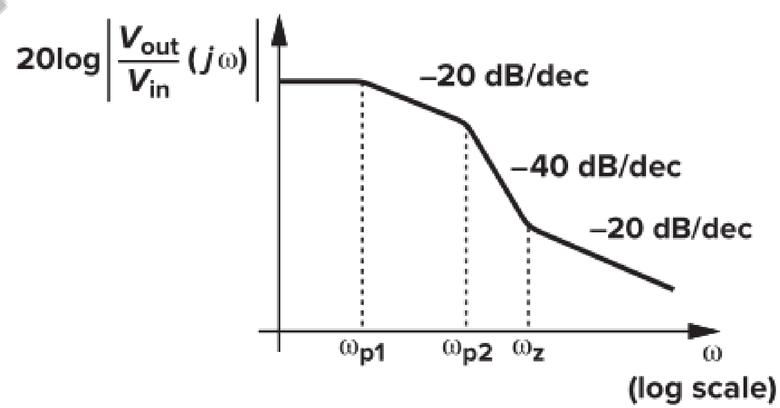
$$\Rightarrow \frac{v_o(s)}{v_i(s)} = -A_0 \frac{1 - \dfrac{s}{g_m / C_{gd}}}{1 + s\left[R_s C_{gs} + R_s C_{gd}\left(1 + g_m R_L'\right) + C_{gd} R_L'\right] + s^2 C_{gs} C_{gd} R_s R_L'}$$

- The exact solution gives a zero (at a high frequency) and two poles
- Notice that the *s* term is the same as the solution using the OCT method

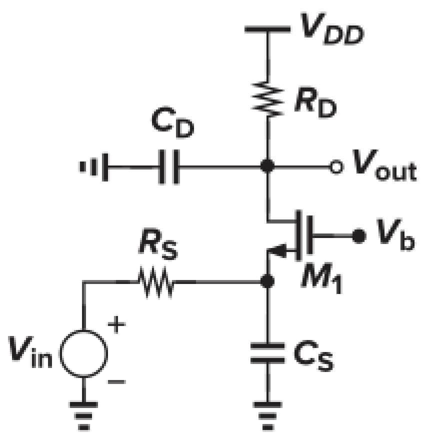
High-Frequency Response of CE Amp







Frequency Response of CG Amplifier

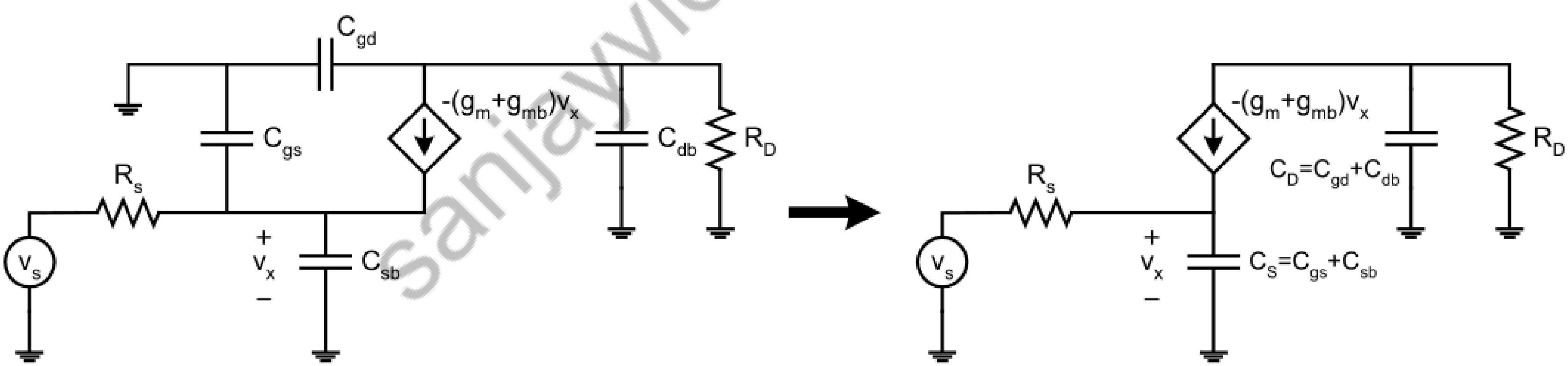


$$I_{CS\_tst} = \frac{V_{CS\_tst}}{R_S} + (g_m + g_{mb})V_{CS\_tst}$$

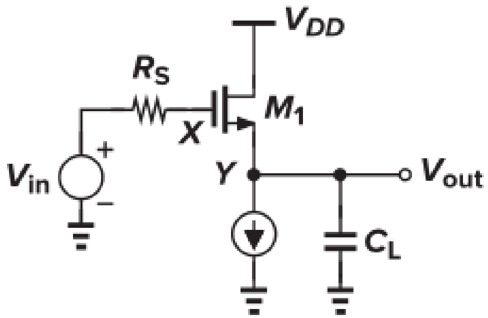
$$\frac{V_{CS\_tst}}{I_{CS\_tst}} = \frac{1}{\frac{1}{R_S} + (g_m + g_{mb})}$$

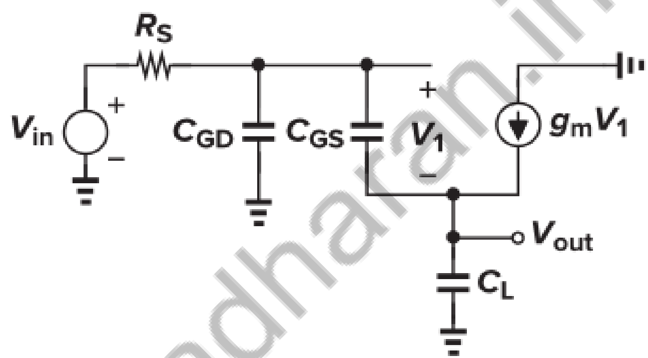
– At the input (source node) $\tau_S = \dfrac{C_S}{1/R_s + g_m + g_{mb}}$

– At the output (drain node) $\tau_D = C_D R_D$



Frequency Response of Source Followers





$$V_1 C_{GS} s + g_m V_1 = V_{out} C_L s$$

$$V_1 = \frac{C_L s}{g_m + C_{GS} s} V_{out}$$

$$V_{in} = R_S[V_1 C_{GS} s + (V_1 + V_{out}) C_{GD} s] + V_1 + V_{out}$$

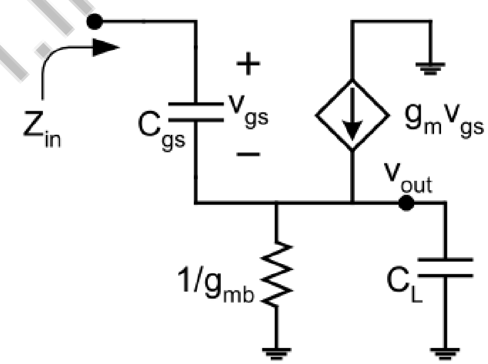
$$\frac{V_{out}}{V_{in}}(s) = \frac{g_m + C_{GS} s}{R_S(C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) s^2 + (g_m R_S C_{GD} + C_L + C_{GS}) s + g_m}$$

$$\omega_{P1} \cong \frac{g_m}{g_m R_s C_{gd} + C_L + C_{gs}}$$

Frequency Response of Source Followers

- Other important aspects of a source follower are its input and output impedances (since they are often used as buffers)
- Let's calculate the input impedance using the high-freq small-signal models

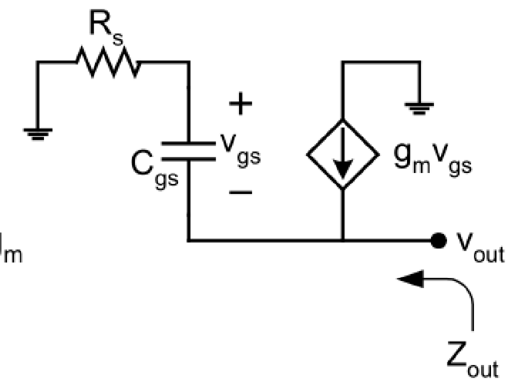
$$Z_{in} = \frac{1}{sC_{gs}} + \left(1 + \frac{g_m}{sC_{gs}}\right)\frac{1}{g_{mb} + sC_L}$$

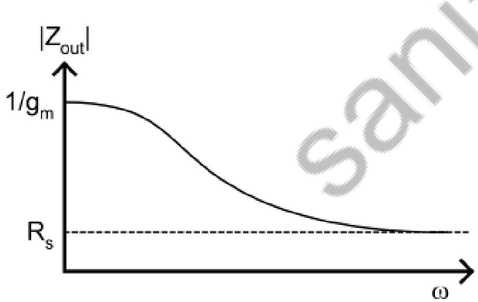


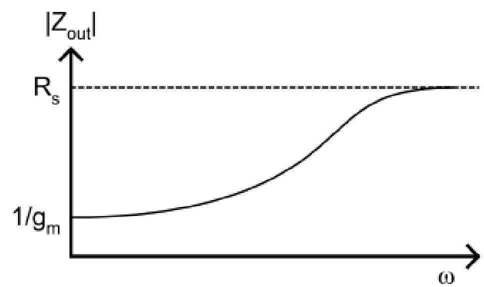
- Now calculate the output impedance (ignoring g_{mb} for simplicity)

$$Z_{out} = \frac{R_s sC_{gs} + 1}{g_m + sC_{gs}}$$

  - At low frequency, $Z_{out} \approx 1/g_m$
  - At high frequency, $Z_{out} \approx R_s$
  - Shape of the response depends on the relative size of R_s and $1/g_m$

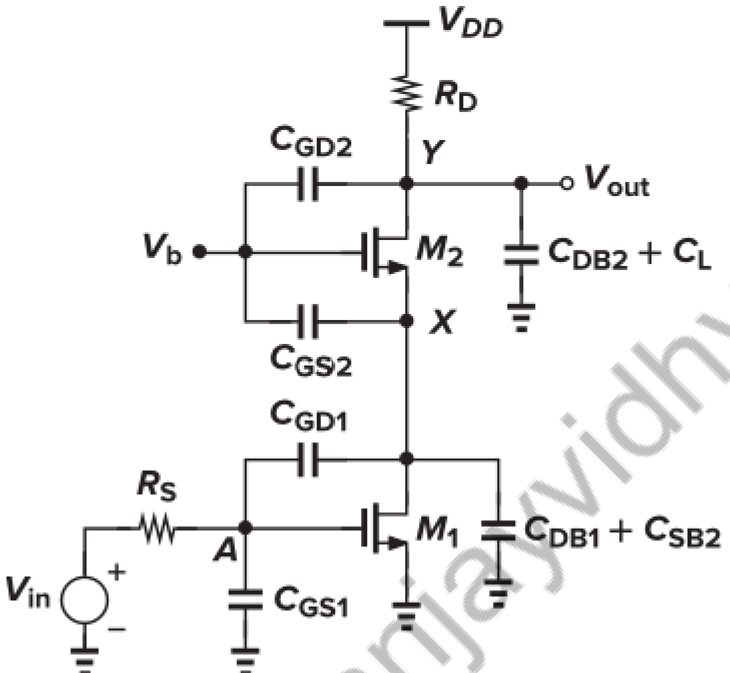






Z_{out} can look inductive or capacitive depending on R_s and $1/g_m$

Frequency Response of Cascode Amplifier

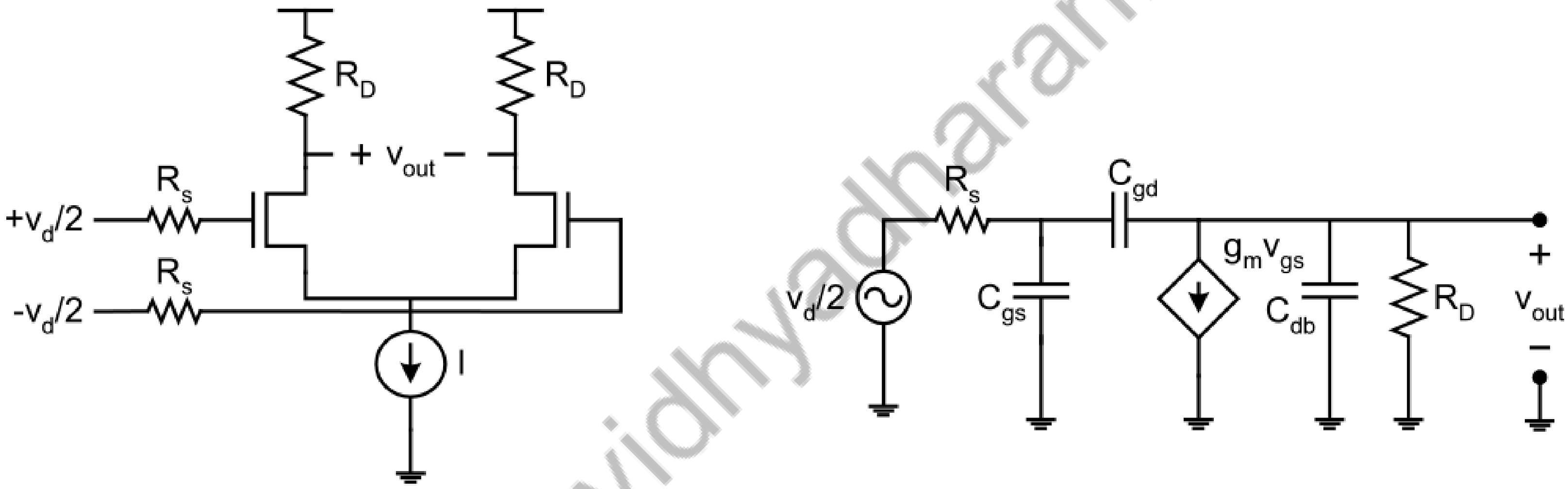


$$\omega_{p,A} = \frac{1}{R_S\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2} + g_{mb2}}\right) C_{GD1}\right]}$$

$$\omega_{p,X} = \frac{g_{m2} + g_{mb2}}{C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2}}$$

$$\omega_{p,Y} = \frac{1}{R_D(C_{DB2} + C_L + C_{GD2})}$$

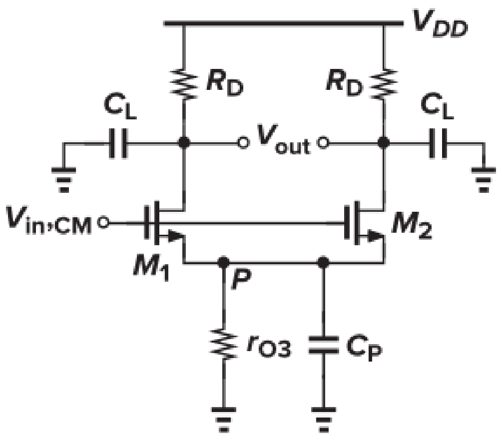
Frequency Response of Differential Amplifier



– The response is identical to that of a common-source stage

Frequency Response of Differential Amplifier

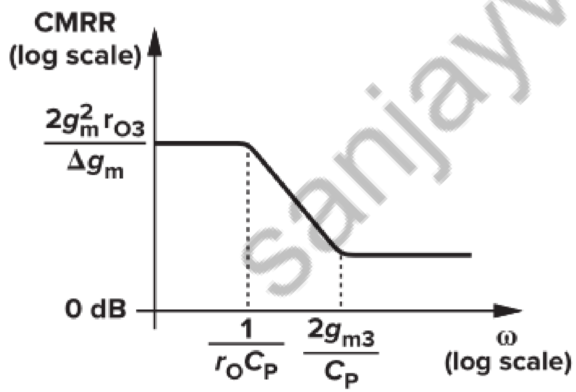
High-Frequency CMRR



$$CM\ Gain = -\frac{R_D/2}{1/(2g_m)+R_{SS}}$$

$$CM\ Gain = \frac{-R_D}{\frac{1}{g_m}+2R_s}$$

$$A_{v,CM} = -\frac{\Delta g_m\left[R_D \Big\| \left(\frac{1}{C_L s}\right)\right]}{(g_{m1}+g_{m2})\left[r_{O3}\Big\|\left(\frac{1}{C_P s}\right)\right]+1}$$



$$\text{CMRR} \approx \frac{g_m}{\Delta g_m}\left[1+2g_m\left(r_{O3}||\frac{1}{C_P s}\right)\right]$$

$$\approx \frac{g_m}{\Delta g_m}\frac{r_{O3}C_P s+1+2g_m r_{O3}}{r_{O3}C_P s+1}$$

Thankyou