



Digital Design : 2021-22

Lecture 10 : Quine-McCluskey (QM) Technique

By Dr. Sanjay Vidhyadharan

QM Technique

QM Method effective for increased number of input variables

Optimal for implementing on a Computer

K-map is a graphical method whereas QM is a Tabular Method

QM Technique

1. Find Prime Implicants of the function
2. Find Essential Prime implicants of the function.
3. Include Essential prime implicants in partial solution and delete them from the prime implicant table
4. Determine and delete dominated rows and dominating columns. Find the (secondary) prime implicants.
5. Repeat steps 3 and 4 as many times as they are applicable until a minimal cover of the function is found

QM Technique

Example 1 $f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$

1. Find all the prime Implicants

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Group the Minterms according to the **number of 1's** in the Minterm

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	
group 0	<u>0 0000</u>		
group 1	1 0001		
	2 0010		
	<u>8 1000</u>		
group 2	5 0101		Combining group 0 and group 1:
	6 0110		
	9 1001		
	<u>10 1010</u>		
group 3	7 0111		
	14 1110		

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001	
	2 0010	
	8 1000	
group 2	<u>5 0101</u>	
	6 0110	
	9 1001	
	10 1010	
group 3	7 0111	
	14 1110	

Find matched pair with only one variable change

$$w'x'y'z' + w'x'y'z = w'x'y'(z+z') = w'x'y'$$

' - ' indicates absence of literal

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	
	8 1000	
group 2	<u>5 0101</u>	
	6 0110	
	9 1001	
	<u>10 1010</u>	
group 3	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	
group 2	5 0101	
	6 0110	
	9 1001	
	10 1010	
group 3	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	
	6 0110	
	9 1001	
group 3	10 1010	
	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	
	6 0110	
	9 1001	
group 3	10 1010	
	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	<hr/>
group 2	5 0101✓	1,5 0-01
	6 0110	1,9 -001
	9 1001✓	
group 3	10 1010	
	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110	
	9 1001✓	
group 3	10 1010	
	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110	
	9 1001✓	
group 3	10 1010	
	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	<hr/>
group 2	5 0101✓	1,5 0-01
	6 0110✓	1,9 -001
	9 1001✓	2,6 0-10
	10 1010	
group 3	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	
group 3	10 1010	
	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	<hr/>
group 2	5 0101✓	1,5 0-01
	6 0110✓	1,9 -001
	9 1001✓	2,6 0-10
	10 1010✓	2,10 -010
group 3	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	
group 3	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
group 3	10 1010✓	8,9 100-
	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	<hr/> 1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
group 3	10 1010✓	8,9 100-
	7 0111	
	14 1110	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	<hr/>
group 2	5 0101✓	1,5 0-01
	6 0110✓	1,9 -001
	9 1001✓	2,6 0-10
	10 1010✓	2,10 -010
group 3	7 0111	8,9 100-
	14 1110	8,10 10-0

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3	7 0111✓	8,10 10-0
	14 1110	5,7 01-1

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3	7 0111✓	8,10 10-0
	14 1110	5,7 01-1

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3	7 0111✓	8,10 10-0
	14 1110	5,7 01-1
		6,7 011-

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3	7 0111✓	8,10 10-0
	14 1110✓	5,7 01-1
		6,7 011-
		6,14 -110

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3	7 0111✓	8,10 10-0
	14 1110✓	5,7 01-1
		6,7 011-
		6,14 -110

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3	7 0111✓	8,10 10-0
	14 1110✓	5,7 01-1
		6,7 011-
		6,14 -110

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3	7 0111✓	8,10 10-0
	14 1110✓	5,7 01-1
		6,7 011-
		6,14 -110

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3	7 0111✓	8,10 10-0
	14 1110✓	5,7 01-1
		6,7 011-
		6,14 -110
		10,14 1-10

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-	
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100-	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-	
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100-	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-	
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100-	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-	
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100-	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-	
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100-	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0 ✓	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010	
	10 1010✓	8,9 100- ✓	
group 3	7 0111✓	8,10 10-0 ✓	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000 ✓	0,8,1,9 -00-
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001 ✓	
	6 0110 ✓	2,6 0-10	
	9 1001 ✓	2,10 -010	
	10 1010 ✓	8,9 100- ✓	
group 3	7 0111 ✓	8,10 10-0 ✓	
	14 1110 ✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

Need not repeat if already Covered

Same as

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000 ✓	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001 ✓	
	6 0110 ✓	2,6 0-10	
	9 1001 ✓	2,10 -010	
	10 1010 ✓	8,9 100- ✓	
group 3	7 0111 ✓	8,10 10-0 ✓	
	14 1110 ✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000 ✓	Already Covered
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001 ✓	
	6 0110 ✓	2,6 0-10	
	9 1001 ✓	2,10 -010 ✓	
	10 1010 ✓	8,9 100- ✓	
group 3	7 0111 ✓	8,10 10-0 ✓	
	14 1110 ✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
	10,14 1-10		

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0 Already Covered
	2 0010✓	0,8 -000 ✓	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001 ✓	
	6 0110 ✓	2,6 0-10	
	9 1001 ✓	2,10 -010 ✓	
	10 1010 ✓	8,9 100- ✓	
group 3	7 0111 ✓	8,10 10-0 ✓	
	14 1110 ✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0 Already Covered
	2 0010✓	0,8 -000 ✓	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001 ✓	
	6 0110 ✓	2,6 0-10	
	9 1001 ✓	2,10 -010 ✓	
	10 1010 ✓	8,9 100- ✓	
group 3	7 0111 ✓	8,10 10-0 ✓	
	14 1110 ✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000 ✓	2,6,10,14 --10
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001 ✓	
	6 0110 ✓	2,6 0-10 ✓	
	9 1001 ✓	2,10 -010 ✓	
	10 1010 ✓	8,9 100- ✓	
group 3	7 0111 ✓	8,10 10-0 ✓	
	14 1110 ✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10 ✓	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000 ✓	2,6,10,14 --10
	8 1000✓	1,5 0-01	Already Covered
group 2	5 0101✓	1,9 -001 ✓	
	6 0110 ✓	2,6 0-10 ✓	
	9 1001 ✓	2,10 -010 ✓	
	10 1010 ✓	8,9 100- ✓	
group 3	7 0111 ✓	8,10 10-0 ✓	
	14 1110 ✓	5,7 01-1	
		6,7 011-	
	6,14 -110 ✓		
	10,14 1-10 ✓		

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000- ✓	0,1,8,9 -00- ← D = x'y'
group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0 ← E = x'z'
	2 0010✓	0,8 -000 ✓	2,6,10,14 --10 ← F = yz'
	8 1000✓	1,5 0-01 ← A = w'y'z	
group 2	5 0101✓	1,9 -001 ✓	
	6 0110 ✓	2,6 0-10 ✓	
	9 1001 ✓	2,10 -010 ✓	
	10 1010 ✓	8,9 100- ✓	
group 3	7 0111 ✓	8,10 10-0 ✓	
	14 1110 ✓	5,7 01-1 ← B = w'xz	
		6,7 011- ← C = w'xy	
		6,14 -110 ✓	
	10,14 1-10 ✓	A, B, C, D, E, F are Prime Implicants	

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

2. Find Essential Prime Implicants

The Minterms 9 and 14 are covered by single terms

		Minterms									
		0	1	2	5	6	7	8	9	10	14
Prime Implicants	(1,5)	A= $w'y'z$		X		X					
	(5,7)	B= $w'xz$				X		X			
	(6,7)	C= $w'xy$					X	X			
	(0,1,8,9)	D= $x'y'$	X	X					X	X	
	(0,2,8,10)	E= $x'z'$	X			X			X		X
	(2,6,10,14)	F= yz'				X		X			X

yz' and $x'y'$

Essential Prime Implicants

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

Partial Solution: D+F + -----

		0	1	2	5	6	7	8	9	10	14
(1,5)	$A=w'y'z$		X		X						
(5,7)	$B=w'xz$				X		X				
(6,7)	$C=w'xy$					X	X				
(0,1,8,9)	$D=x'y'$	X	X					X	X		
(0,2,8,10)	$E=x'z'$	X		X				X		X	
(2,6,10,14)	$F=yz'$			X		X				X	X

Once the Essential Prime Implicants are part of solution, The columns covered by them can be removed

Columns 0, 1, 6, 8, 9, 2, 10, 14 can be removed

QM Technique

Example 1

$$f(w, x, y, z) = \sum (0,1,2,5,6,7,8,9,10,14)$$

		5	7
(1,5)	A=w'y'z	X	
(5,7)	B=w'xz	X	X
(6,7)	C=w'xy		X
(0,2,8,10)	E=x'z'		

↓
Redundant

		5	7
(1,5)	A=w'y'z	X	
(5,7)	B=w'xz	X	X
(6,7)	C=w'xy		X

Row corresponding to B is dominating the other Rows

Including B in the final solution both columns get covered

$$\text{Final Solution: } D + F + B = x'y' + yz' + w'xz$$

QM Technique

Row Dominance

In Prime Implicant table dominated rows can be deleted

Row B is dominating the rows A and C

		5	7
(1,5)	$A=w'y'z$	X	
(5,7)	$B=w'xz$	X	X
(6,7)	$C=w'xy$		X

By making B part of minimal function both columns 5 and 7 get covered

Dominated rows A and C can be deleted (if A and C have higher cost than B)

QM Technique

Column Dominance

In Prime Implicant table dominating columns can be deleted

Column corresponding to 9 dominates
column corresponding to 8

		8	9
(8,9,--, --)	A	X	X
(--,9,--,--)	B		X
(--,8,--,9)	C	X	X

Any set of rows that covers dominated column must also cover dominating column

If there is 8 there will always be 9 with it, covering 8 is sufficient

Dominating column 9 can be deleted

QM Technique

<https://sanjayvidhyadharan.in/courses/digital-design-tutorials/>

Sanjay

Digital Electronics 13 Lessons 6 hours 45 mins

HOME COURSES PORTFOLIO OFFICIAL PAGE YOUTUBE CHANNEL

Free

"This course complements the Digital Design (2020-21) course. The lessons contains topic wise solved problems and solutions enable students clearly understand the concepts of Digital Design".

For Digital Design, Lecture Classes, and Laboratory Classes scroll down to Related Courses at the bottom of this page

Lessons

- Tutorial 1 : Binary Number System**
25 mins Sanjay Vidyadharan
- Tutorial 2 : Boolean Algebra & Logic Gates**
38 mins Sanjay Vidyadharan
- Tutorial 3 : Karnaugh Maps**
46 mins Sanjay Vidyadharan
- Tutorial 4 : Quine-McCluskey (QM) Method**
44 mins Sanjay Vidyadharan

Thank you

sanjayvidhyadharan.in