Electrical Science: 2021-22

Tutorial 8

Three Phase Circuits

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Example 1

A 400-V, 3-ϕ supply is connected across a balanced load of three impedances each consisting of a 32-Ω resistance and 24-Ω inductive reactance in Series. Determine the current drawn from the power mains, if the three impedances and source are

(*a*) Y-connected, and

(*b*) Δ-connected.

Example 1

$$\boldsymbol{Z} = R + jX = (32 + j24)\,\Omega.$$

$$\therefore Z = \sqrt{R^2 + X^2} = \sqrt{32^2 + 24^2} = 40\Omega$$

(*a*) Y-*connection* :

$$V_{\text{ph}} = \frac{V_{\text{L}}}{\sqrt{3}} = \frac{400}{\sqrt{3}}\text{V} \Rightarrow I_{\text{ph}} = \frac{V_{\text{ph}}}{Z} = \frac{400/\sqrt{3}}{40} = \frac{10}{\sqrt{3}}\text{A}$$

$$\therefore I_{\text{L}} = I_{\text{ph}} = \frac{10}{\sqrt{3}} = \mathbf{5.78A}$$

(*b*) *For* Δ-*connection* :

$$V_{\text{ph}} = V_{\text{L}} = 400\text{V} \Rightarrow I_{\text{ph}} = \frac{V_{\text{ph}}}{Z} = \frac{400}{40} = 10\text{A}$$

$$\therefore I_{\text{L}} = \sqrt{3}I_{\text{ph}} = \sqrt{3} \times 10 = \mathbf{17.32A}$$

Example 2

A balanced delta-connected load having an impedance 20-j15 Ω is connected to a delta-connected, positive-sequence generator having $V_{ab} = 330\angle 0°$ V.

Calculate the phase currents of the load and the line currents.

Solution: $Z_\Delta = 20 - j15\ \Omega = 25\angle -36.87°$

$$V_{ab} = 330\angle 0°$$

Phase Currents:

$$\boxed{\begin{aligned} I_{AB} &= \frac{V_{AB}}{Z_\Delta} = \frac{330\angle 0°}{25\angle -36.87°} = 13.2\angle 36.87°\text{A} \\ I_{BC} &= I_{AB}\angle -120° = 13.2\angle\text{-}83.13°\text{A} \\ I_{CA} &= I_{AB}\angle +120° = 13.2\angle 156.87°\text{A} \end{aligned}}$$

Example 2

A balanced delta-connected load having an impedance 20-j15 Ω is connected to a delta-connected, positive-sequence generator having $V_{ab} = 330\angle 0°$ V.

Calculate the phase currents of the load and the line currents.

Line Currents:

$$\begin{aligned} I_a &= I_{AB}\sqrt{3}\angle -30° \\ &= (13.2\angle 36.87°)(\sqrt{3}\angle -30°)A \\ &= 22.86\angle 6.87° \end{aligned}$$

$$I_b = I_a\angle -120° = 22.86\angle -113.13° A$$

$$I_c = I_a\angle +120° = 22.86\angle 126.87° A$$

Example 3

A balanced positive sequence Y-connected source with $V_{an} = 100\angle 10°$ V is connected to a Δ-connected balanced load with impedance $(8 + j4)\ \Omega$ per phase.

Calculate the phase currents of the load and the line currents.

Solution: Balanced Y-source with $V_{an} = 100\angle 10°$ V
Balanced Δ-load with $Z_\Delta = 8+j4\ \Omega$

$$\text{Phase Current } I_{AB} = \frac{V_{AB}}{Z_\Delta}$$

$$\text{Line Voltage } V_{AB} = \sqrt{3}\, V_{an} \angle 30°$$

$$V_{AB} = 173.2\angle 40°\ \text{V}$$

$$\Rightarrow I_{AB} = \frac{173.2\angle 40°}{8 + j4} = 19.36\angle 13.43°$$

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Calculate the phase currents of the load and the line currents.

Phase Currents:

$$I_{AB} = 19.36\angle 13.43° \quad \text{A}$$

$$I_{BC} = I_{AB}\angle -120° = 19.36\angle -106.57° \quad \text{A}$$

$$I_{CA} = I_{AB}\angle +120° = 19.36\angle 133.43° \quad \text{A}$$

Line Currents:

$$I_a = \sqrt{3}\ I_{AB}\ \angle -30° = \sqrt{3}\ (19.36)\ \angle(13.43° - 30°)$$

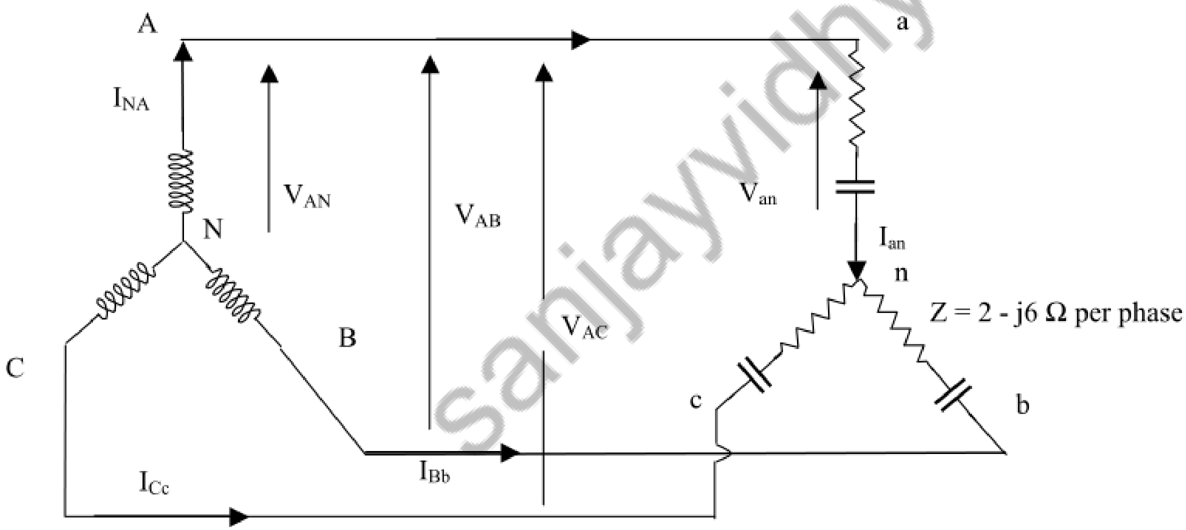
$$I_a = 33.53\ \angle -16.57° \quad \text{A}$$

$$I_b = I_a\angle -120° = 33.53\ \angle -136.57° \quad \text{A}$$

$$I_c = I_a\angle +120° = 33.53\ \angle 103.43° \quad \text{A}$$

Example 4

The supply voltage is 220 V per phase, and again we will use V_{AN} as the reference. The supply is balanced and the load is also balanced with an impedance of 2 – j6 Ω per phase. We will determine the currents in each phase and the total power consumed by the load. The phase rotation is ABC. Again it is assumed that the connection between the source and the load has negligible impedance.

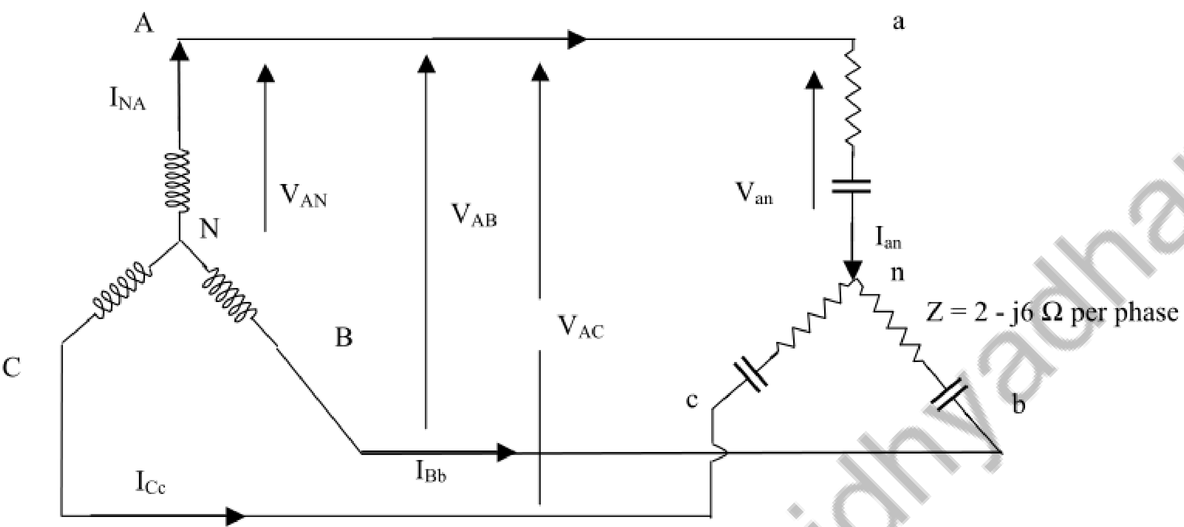


$$V_{an} = 220\angle 0^o$$

$$V_{bn} = 220\angle -120^o$$

$$V_{cn} = 220\angle +120^o$$

Example 4



$$V_{an} = 220\angle 0^o$$

$$V_{bn} = 220\angle -120^o$$

$$V_{cn} = 220\angle +120^o$$

$$I_{an} = \frac{V_{an}}{Z} = \frac{220\angle 0^o}{2 - j6} = \frac{220\angle 0^o}{6{,}325\angle -71{,}57^o} = 34{,}783\angle 71{,}57^o\ Amps$$

$$I_{bn} = 34{,}783\angle -48{,}43^o\ Amps$$

$$I_{cn} = 34{,}783\angle -168{,}43^o\ Amps$$

$$P_{Real} = 3V_{Ph}I_{Ph}\cos\theta = (3)(220)(34{,}783)\cos(71{,}57^o) = 7258\ W$$

Example 5

The input power to a 3-phase a.c. motor is measured as 5 kW. If the voltage and current to the motor are 400V and 8.6 A respectively, determine the power factor of the system?

Power P = 5000W,

Line voltage V_L = 400 V,

Line current, I_L = 8.6 A

$P = \sqrt{3}\, V_L I_L \cos \varphi$

Hence Power factor $= \cos \varphi = P / \sqrt{3}\, V_L I_L$

$= 5000 / \sqrt{3}\, (400)\, (8.6)$

$= 0.839$

Example 6

Two wattmeters are connected to measure the input power to a balanced 3-phase load by the two-wattmeter method. If the instrument readings are 8 kW and 4kW, determine (a) the total power input and (b) the load power factor.

(a)Total input power,

$$P = P1 + P2 = 8 + 4 = 12 \text{ kW}$$

(b) $\tan \varphi = \sqrt{3}(P1 - P2)/(P1 + P2)$

$= \sqrt{3}\,(8 - 4) / (8 + 4)$

$= \sqrt{3}\,(4/12)$

$= \sqrt{3}(1/3)$

$= 1/\sqrt{3}$

Hence $\varphi = \tan^{-1} 1/\sqrt{3} = 30^\circ$

Power factor $= \cos \varphi = \cos 30^\circ = 0.866$

Example 7

Three identical coils, each of resistance 10 ohm and inductance 42 mH are connected (a) in star and (b) in delta to a 415V, 50 Hz, 3-phase supply. Determine the total power dissipated in each case.

(a) Star connection

Inductive reactance, $X_L = 2\pi f L = 2\pi (50) (42\times10-3) = 13.19$

Phase impedance, $Zp = \sqrt{(R^2 + X_L^2)} = \sqrt{(10^2 + 13.19^2)} = 16.55$

Line voltage, $V_L = 415$ V

Phase voltage, $V_P = V_L / \sqrt{3} = 415 / \sqrt{3} = 240$ V.

Phase current, $I_p = V_p / Z_p = 240 / 16.55 = 14.50$ A.

Line current, $I_L = I_p = 14.50$ A.

Power factor = $\cos\varphi = Rp / Zp = 10 / 16.55 = 0.6042$ lagging.

Power dissipated, $P = \sqrt{3}\, V_L I_L \cos\varphi = \sqrt{3} (415) (14.50)(0.6042) = 6.3$ kW

Example 7

(b) Delta connection

$V_L = V_p = 415$ V,

Zp = 16.55 Ohms

Power factor $\cos \varphi = 0.6042$ lagging,

Phase current $I_p = V_p / Z_p = 415 / 16.55 = 25.08$A.

Line current, $I_L = \sqrt{3} I_p = \sqrt{3}(25.08) = 43.44$A.

Power dissipated, $P = \sqrt{3}\, V_L I_L \cos \varphi = \sqrt{3}\,(415)(43.44)(0.6042) = 18.87$ kW

Thank you