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Lecture 3 : Number System Part 2

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ELECTRICAL ELECTRONICS COMMUNICATION INSTRUMENTATION

Number System

Previous Class

- **Decimal System**
- **Binary System**
- **Octal System**
- **Hexadecimal System**
- **Conversion from one system to other**

Representation of Negative Numbers

- Signed Magnitude
- Diminished radix complement
- Radix complement

Representation of Negative Numbers

➢ Signed Magnitude

Limitations

1. Two Zeros

2. Add +2 & -1

$$\begin{array}{r} 0\,1\,0 \\ \underline{1\,0\,1} \\ \underline{1\,1\,1} \end{array}$$

3-bit numbers	Signed magnitude
0 0 0	+0
0 0 1	+1
0 1 0	+2
0 1 1	+3
1 0 0	-0
1 0 1	-1
1 1 0	-2
1 1 1	-3

MSB indicates Sign : 0 indicates positive, 1 indicates negative

Representation of Negative Numbers

Diminished radix complement

Given a number N in base r having n digits (r-1)'s complement is defined as (r^n-1-N)

In case of decimal it is called 9's complement

9's complement of 865 is $10^3 - 1 - 865 = 999 - 865 = 134$

In case of binary it is called 1's complement for 1011

1's complement of 1011 is $2^4 - 1 - 1011 = 1111 - 1011 = 0100$ (or you can simply use the complement ~ 1 for 0 and 0 for 1)

Representation of Negative Numbers

Decimal	S.M.	1's comp.
7	0111	0111
6	0110	0110
5	0101	0101
4	0100	0100
3	0011	0011
2	0010	0010
1	0001	0001
0	0000	0000
-0	1000	1111
-1	1001	1110
-2	1010	1101
-3	1011	1100
-4	1100	1011
-5	1101	1010
-6	1110	1001
-7	1111	1000
-8	–	–

Limitations of 1's Complement

- Two Zeros
- End-around-carry-bit addition

Add 4 & -7

```
 0100
 1000
 ----
 1100
```

Add 4 & -3

```
  0100
  1100
------
1 0000
     1
------
  0001
```

Examples of 1’s Complement

Decimal	S.M.	1’s comp.
7	0111	0111
6	0110	0110
5	0101	0101
4	0100	0100
3	0011	0011
2	0010	0010
1	0001	0001
0	0000	0000
−0	1000	1111
−1	1001	1110
−2	1010	1101
−3	1011	1100
−4	1100	1011
−5	1101	1010
−6	1110	1001
−7	1111	1000
−8	–	–

E.g. 4 + 1

0	1	0	0
0	0	0	1
0	1	0	1

E.g. 4 - 1

	0	1	0	0
	1	1	1	0
1	0	0	1	0
				1
	0	0	1	1

Examples of 1's Complement

Decimal	S.M.	1's comp.
7	0111	0111
6	0110	0110
5	0101	0101
4	0100	0100
3	0011	0011
2	0010	0010
1	0001	0001
0	0000	0000
−0	1000	1111
−1	1001	1110
−2	1010	1101
−3	1011	1100
−4	1100	1011
−5	1101	1010
−6	1110	1001
−7	1111	1000
−8	—	—

E.g. 4 + 1

0	1	0	0
0	0	0	1
0	1	0	1

E.g. 4 - 1

	0	1	0	0
	1	1	1	0
1	0	0	1	0
				1
	0	0	1	1

E.g. 1 - 4

0	0	0	1
1	0	1	1
1	1	0	0

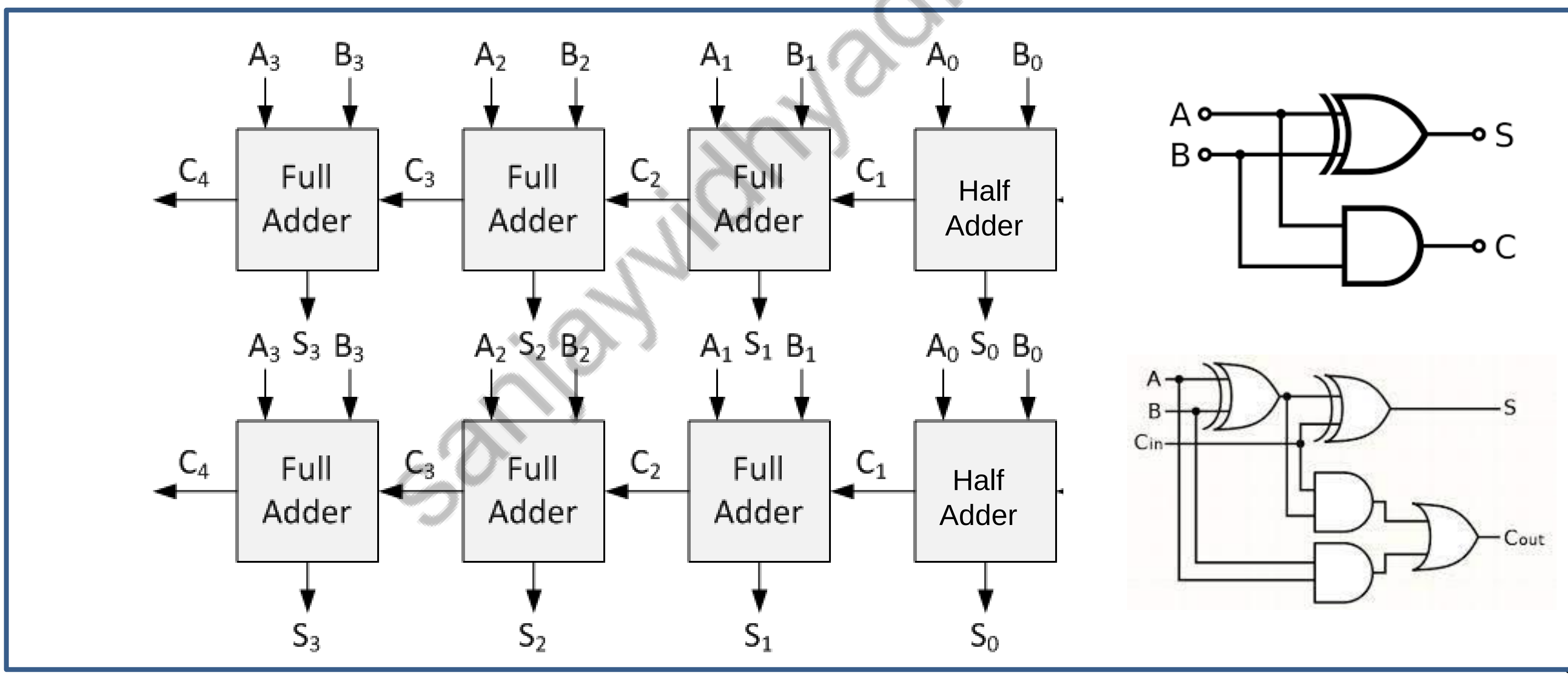
Representation of Negative Numbers

Limitations of 1's Complement

- Two Zeros
- End-around-carry-bit addition

Add 4 & -3

```
  0100
  1100
------
1 0000
     1
------
  0001
```

Representation of Negative Numbers

Radix complement

Given a number N in base r having n digits r’s complement is defined as $(r^n - N)$

In case of decimal it is called 10’s complement $10^3 - 865 = 1000 - 865 = 135$

10’s complement = 9’s complement + 1

In case of binary it is called 2’s complement

2’s complement of 1011 is $2^4 - 1011 = 10000 - 1011 = 0101$

2’s complement = 1’s complement + 1

Representation of Negative Numbers

Decimal	S.M.	1's comp.	2's comp.
7	0111	0111	0111
6	0110	0110	0110
5	0101	0101	0101
4	0100	0100	0100
3	0011	0011	0011
2	0010	0010	0010
1	0001	0001	0001
0	0000	0000	0000
-0	1000	1111	–
-1	1001	1110	1111
-2	1010	1101	1110
-3	1011	1100	1101
-4	1100	1011	1100
-5	1101	1010	1011
-6	1110	1001	1010
-7	1111	1000	1001
-8	–	–	1000

Advantages of 2's Complement

- **One Zero**
- **No End-around-carry-bit addition**

Add 4 & -7

0100
1001
1101

Add 4 & -3

0100
1101
1 0001

Examples of 2's Complement

Decimal	S.M.	1's comp.	2's comp.
7	0111	0111	0111
6	0110	0110	0110
5	0101	0101	0101
4	0100	0100	0100
3	0011	0011	0011
2	0010	0010	0010
1	0001	0001	0001
0	0000	0000	0000
-0	1000	1111	–
-1	1001	1110	1111
-2	1010	1101	1110
-3	1011	1100	1101
-4	1100	1011	1100
-5	1101	1010	1011
-6	1110	1001	1010
-7	1111	1000	1001
-8	–	–	1000

E.g. 4 + 1

0	1	0	0
0	0	0	1
0	1	0	1

E.g. 4 - 1

	0	1	0	0
	1	1	1	1
✖ 1	0	0	1	1

E.g. 1 - 4

0	0	0	1
1	1	0	0
1	1	0	1

EX-OR Gate

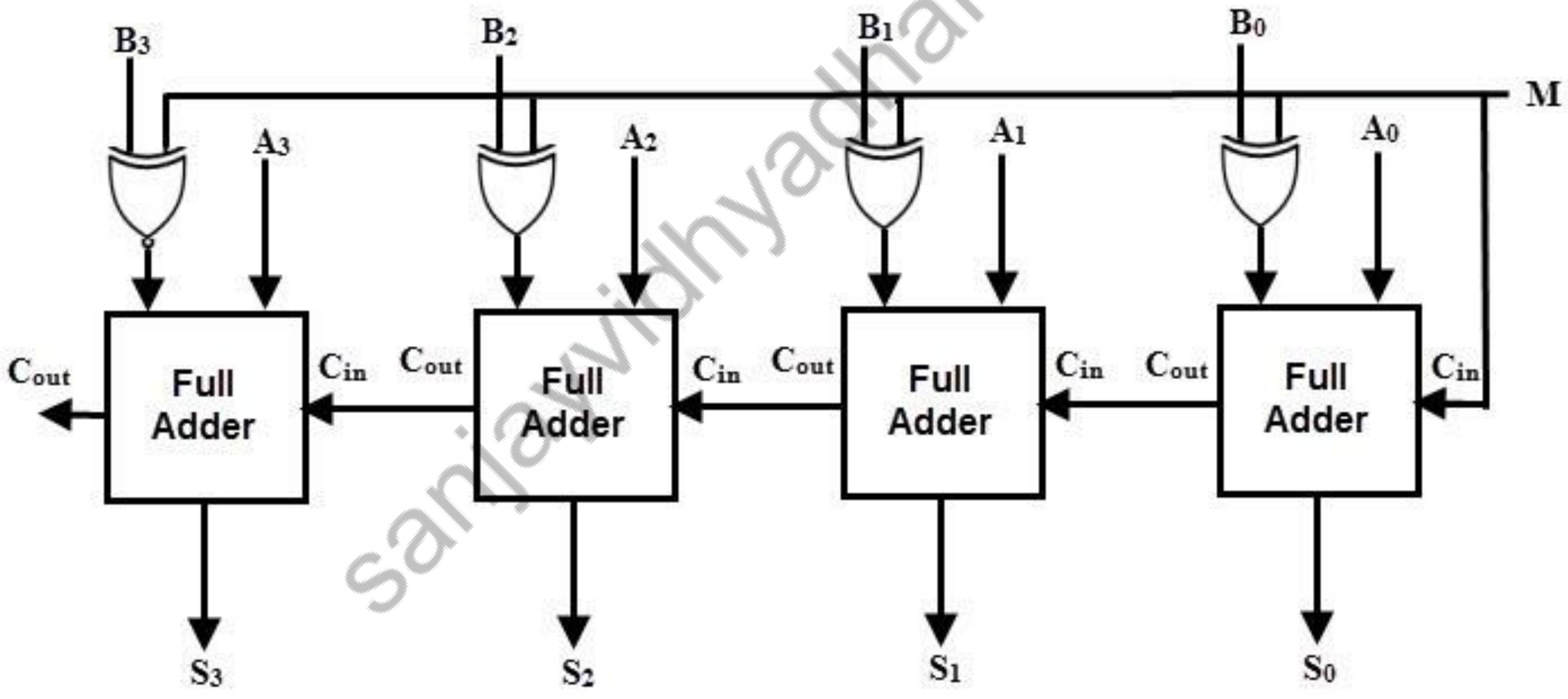
EX-OR Gate As Buffer and Inverter

Control	A	F	
0	0	0	Pass
0	1	1	
1	0	1	Invert
1	1	0	

Implementation of Adder Subtractor

Advantages of 2's Complement

- **Easy Implementation: Adder Subtractor M=0 adder, M=1 Subtractor**



Implementation of Adder Subtractor

Overflow in 2's Complement

Decimal	2's comp.
7	0111
6	0110
5	0101
4	0100
3	0011
2	0010
1	0001
0	0000
-0	–
-1	1111
-2	1110
-3	1101
-4	1100
-5	1011
-6	1010
-7	1001
-8	1000

Add 4 & -3

```
  0100
  1101
1 0001
```

Add -4 & -5

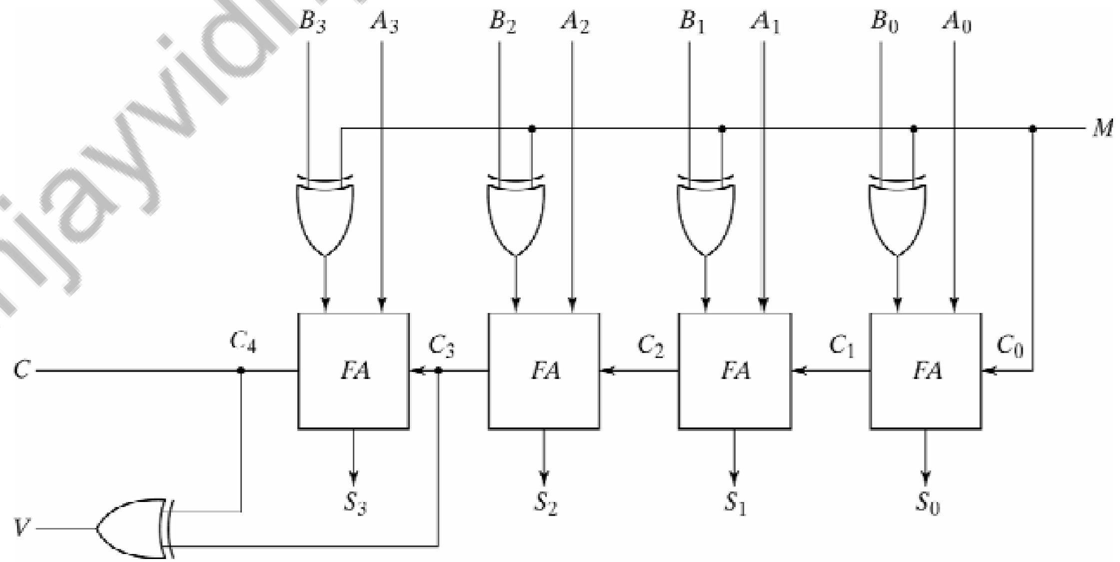
```
  1100
  1011
1 0111
```

Add -8 & 4

```
1000
0100
1100
```

Add 4 & 4

```
0100
0100
1000
```



Thank you