Electrical Science: 2021-22

Lecture 21

Transformer Theory and Operation

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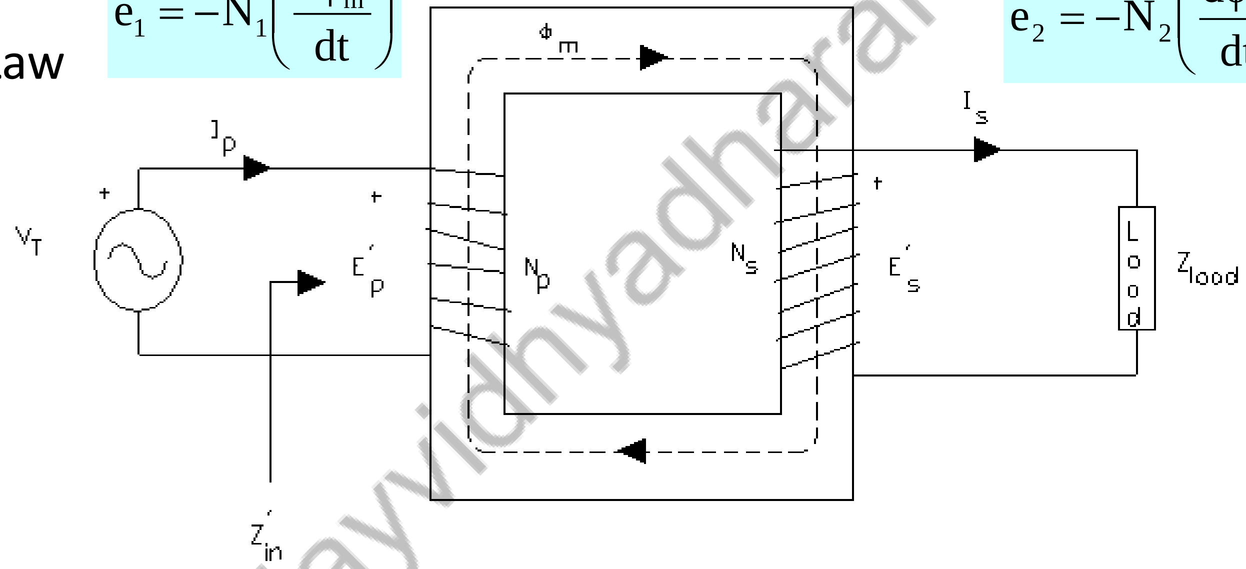
Ideal Transformer

Lentz's Law

$$e_1 = -N_1\left(\frac{d\phi_m}{dt}\right)$$

$$e_2 = -N_2\left(\frac{d\phi_m}{dt}\right)$$



Principle: Stationary coils, time varying flux due to ac current flow. Flux produced by one coil must link to other coil to induce voltage

Ideal Transformer

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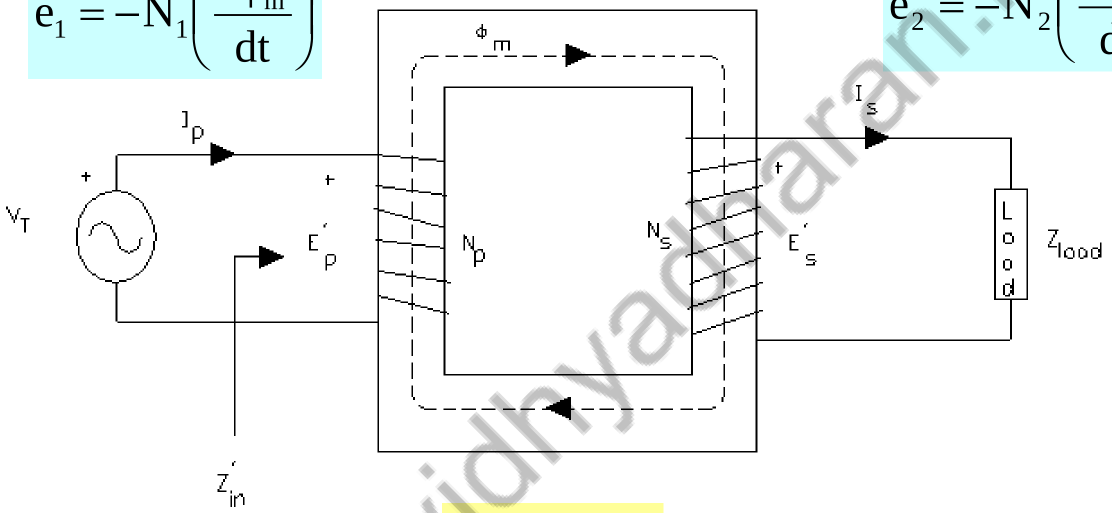

$$\frac{E'_p}{E'_s} = \frac{N_p}{N_s}$$

Voltage relationship for Ideal transformer

Where: E'_p = voltage induced in the primary (V)
E'_s = voltage induced in the secondary (V)
N_p = turns in the primary coil
N_s = turns in the secondary coil

Voltage ratio equals the turns ratio

Ideal Transformer

1) All flux produced in the primary coil links to the secondary coil
2) no core losses due to hysteresis or eddy currents
3) no power losses
4) permeability is infinite (no saturation no magnetizing ϕ)
5) windings have zero resistance
6) no current required to magnetize the iron core

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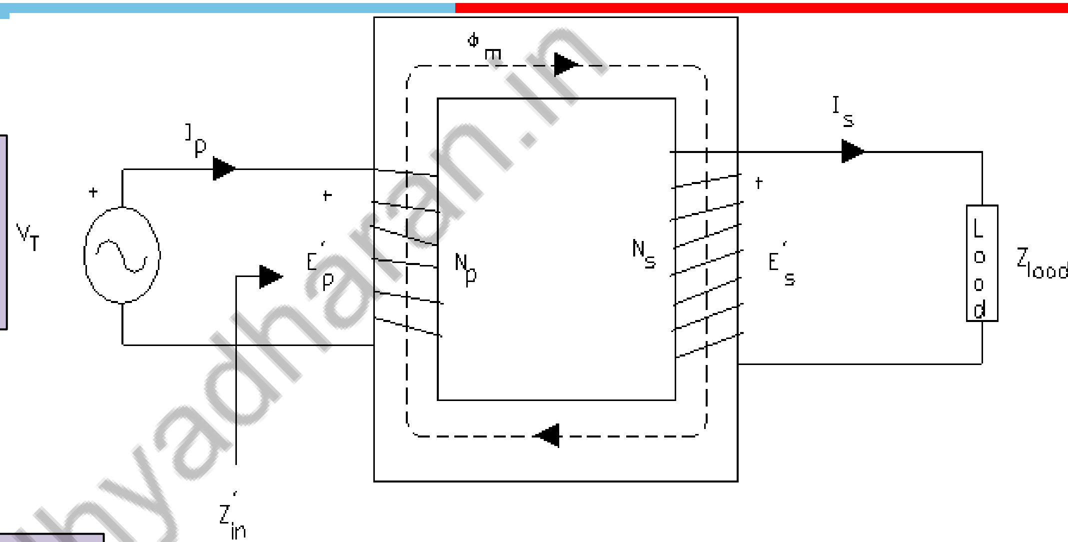
Ideal Transformer Equations

Voltage Ratio

$$a = \frac{E_p}{E_s} = \frac{N_p}{N_s}$$

$$E_p = a \cdot E_s$$

The turns ratio is a scalar. Introduces no phase shift



Apparent Power balance

$$E_p \cdot I_p = E_s \cdot I_s$$

$$S_p = S_s$$

No power losses in idea transformer

Current Ratio

$$\frac{I_p}{I_s} = \frac{1}{a}$$

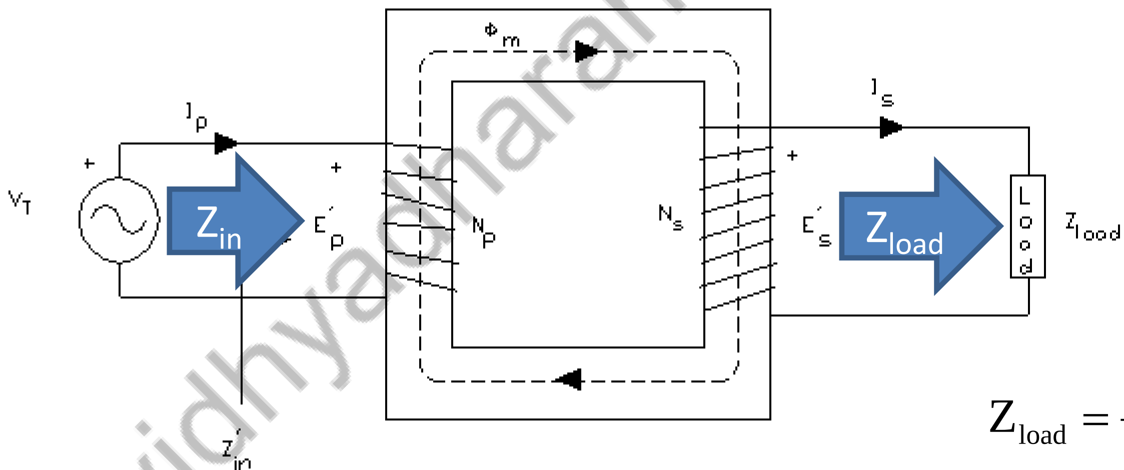
$$I_p = \left(\frac{1}{a}\right) \cdot I_s$$

Current ratio is the inverse of the voltage ratio

Ideal Transformer Equations

Impedances Reflected Through Ideal Transformers

Load impedance as seen from primary side of transformer



By Ohm's Law

$$Z_{in} = \frac{E_p}{I_p}$$

$$Z_{load} = \frac{E_s}{I_s}$$

Write E_s and I_s in terms of primary values

$$E_s = \frac{E_p}{a} \qquad I_s = a \cdot I_p$$

$$\frac{E_s}{I_s} = \frac{\left(\frac{E_p}{a}\right)}{a \cdot I_p} = \left(\frac{E_p}{a}\right)\left(\frac{1}{a \cdot I_p}\right) = \left(\frac{E_p}{I_p}\right)\left(\frac{1}{a^2}\right)$$

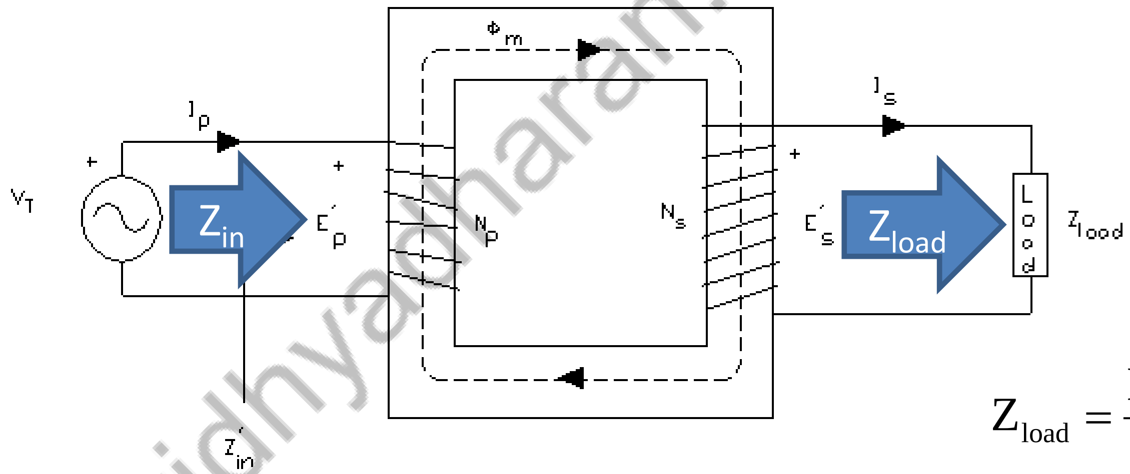
Load impedance is increased when viewed from primary side

$$Z_{load} = Z_{in} \cdot \left(\frac{1}{a^2}\right) \quad \Rightarrow \quad Z_{load} \cdot a^2 = Z_{in}$$

Ideal Transformer Equations

Impedances Reflected Through Ideal Transformers

Load impedance as seen from primary side of transformer



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Ideal Transformer Example

Example 300 kVA 2400-120, 60 Hz single phase transformer operates at 2300 volts on the primary side. It supplies 115 kVA to a load that has a power factor of 0.723 lagging. Assume idea operation and find:

a.) secondary voltage at operating voltage
b.) secondary current
c.) impedance of the load as seen on the secondary side
d.) impedance of the load as seen on the primary side

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$$a = \frac{V_{pr}}{V_{sr}} = \frac{2400}{120} = 20$$

$$V_{sr} = \frac{V_{pr}}{a} = \frac{2300}{20} = 115\,V$$

$$I_s = \frac{115\,KVA}{115\,V} = 1000\,A$$

$$|Z_s| = \frac{115\,V}{1000\,A} = 0.115\,\Omega$$

$$\phi = \cos^{-1}(0.723) = -43.70$$

$$Z_s = 0.115\angle -43.70\,\Omega$$

$$Z_p = a^2 * 0.115\angle -43.70\,\Omega$$
$$= 400 * 0.115\angle -43.70\,\Omega$$
$$= 46\angle -43.70\,\Omega$$

Practical Transformers

Practical transformers draw current with no load connected to secondary winding.
Current caused by two non-ideal conditions: power losses and core magnetization

Active power losses

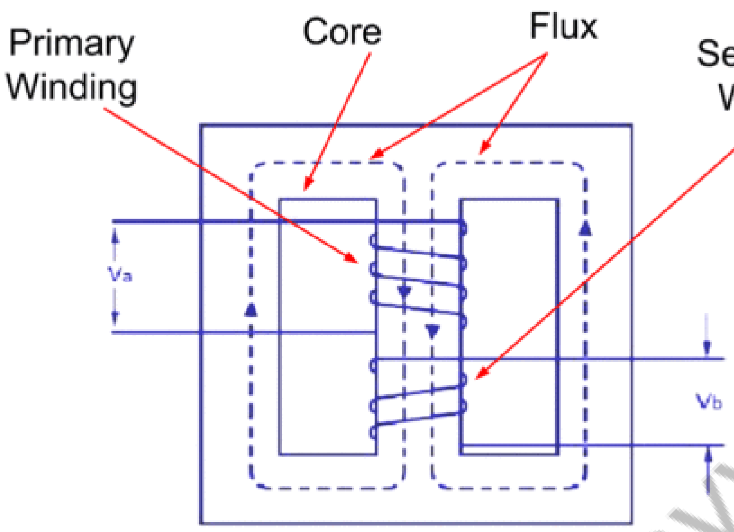
Hysteresis losses - power losses due to repeated change in magnetic polarity. It takes more mmf (NI) to demagnetize core in one direction than the other.

Eddy currents - ac currents induced in iron core due to changing magnetic field

Active power loss Control

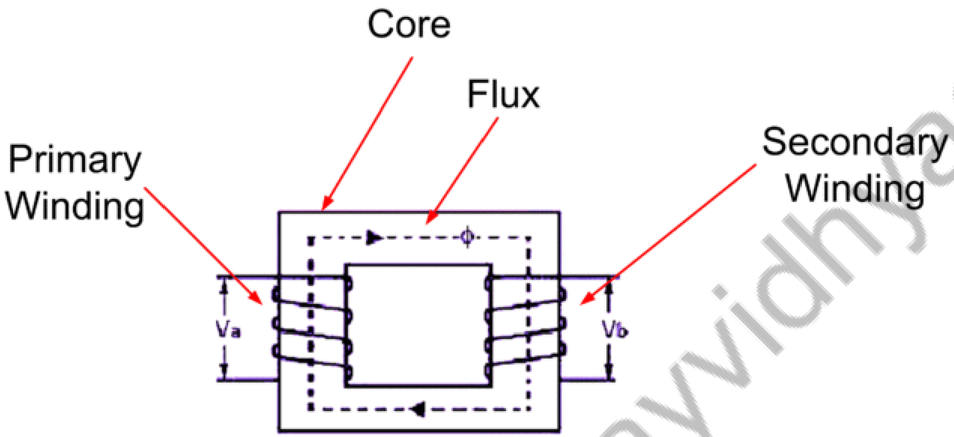
Control hysteresis losses - use alloy steels designed for magnetic circuits
Control eddy current losses - laminate core, insulate laminates

Practical Transformers



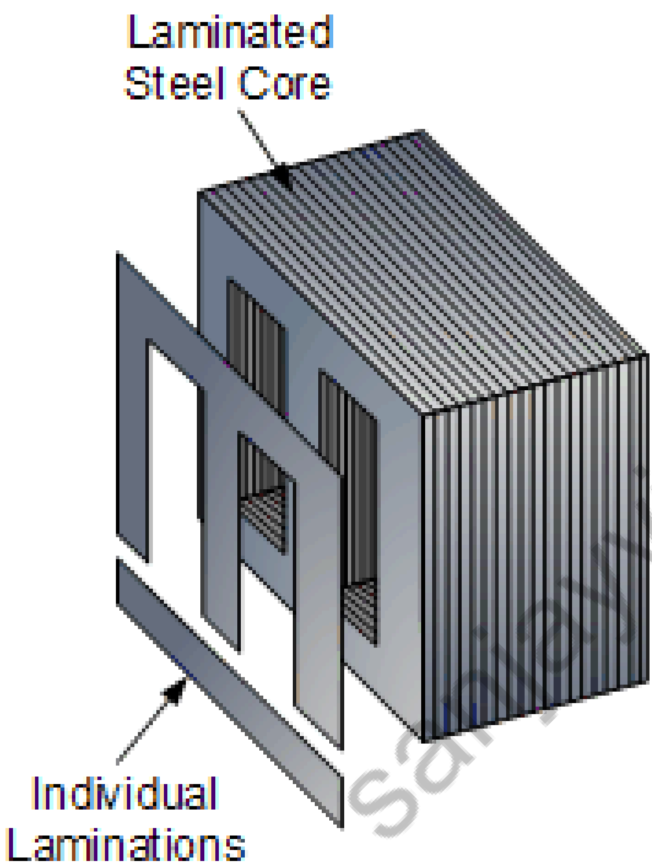
Shell type

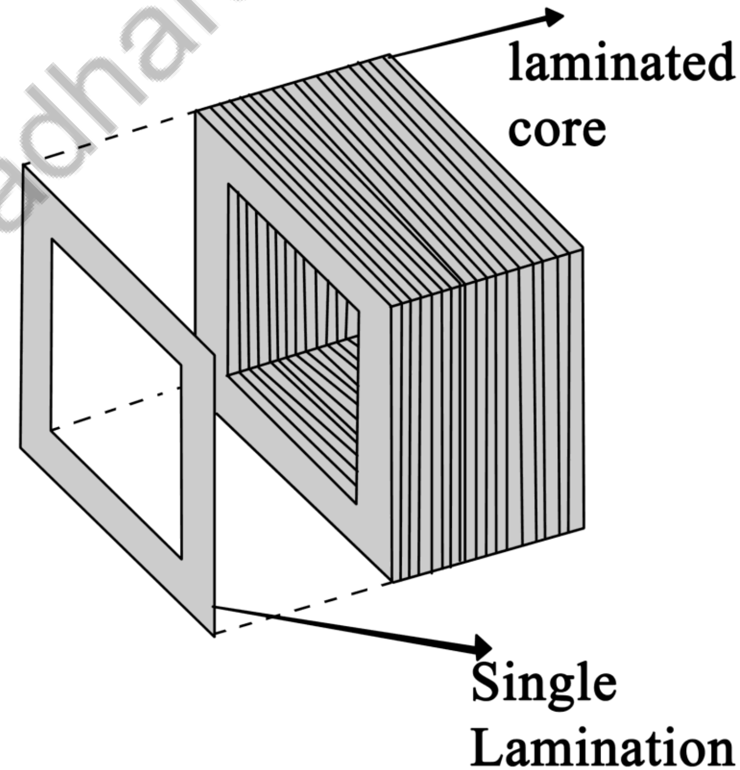
Practical Transformers



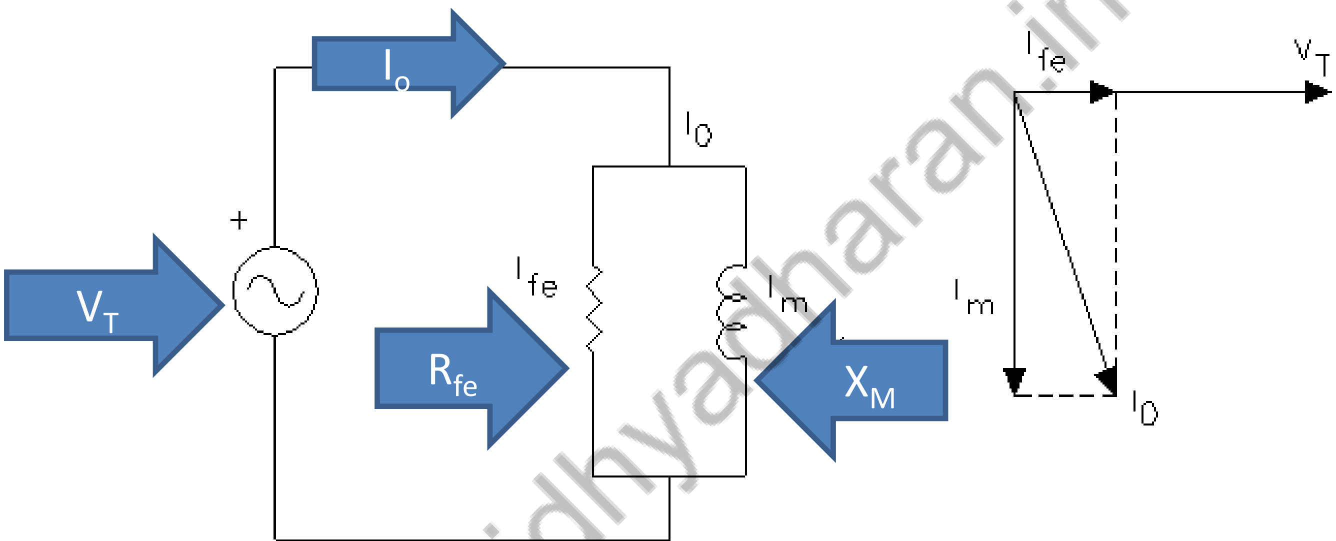
Core type

Practical Transformers





Practical Transformers



V_T = the primary voltage

I_{fe} = core-loss component

I_0 = exciting current

I_M = magnetizing component

R_{fe} = resistance that represents the core losses

X_m = inductive reactance that represents the core magnetizing L

Practical Transformers

Example. A 50 kVA 7200-240 V, 60 Hz single phase transformer is operating with no load. With the primary connected to a 7200 V system, it draws 248 W and has a power factor of 0.187 lagging. Find:

a) the exciting current and its components
b) the magnitudes of magnetizing reactance, X_M and core loss R
c) Repeat parts a and b if the transformer is energized from the secondary (low voltage) side.

Practical Transformers

Example. A 50 kVA 7200-240 V, 60 Hz single phase transformer is operating with no load. With the primary connected to a 7200 V system, it draws 248 W and has a power factor of 0.187 lagging. Find:

a) the exciting current and its components

b) the magnitudes of magnetizing reactance, X_M and core loss R

$$Apparent\ Power = \frac{248}{0.187} = 1326.2\ VA$$

$$|I_0| = \frac{1326.2}{7200} = 0.1842\ A$$

$$\phi = -\cos^{-1}(0.187) = -79.2^0$$

$$I_0 = 0.1842 \angle -79.2^{\,0}\ A$$

$$I_M = 0.1842\ Sin\ 79.2^{\,0}\ A = 0.1809\ A$$

$$I_{fe} = 0.1842\ Cos\ 79.2^{\,0}\ A = 0.0345\ A$$

$$R_{fe} = \frac{7200}{0.0345} = 209032\ \Omega$$

$$X_M = \frac{7200}{0.1809}$$

Practical Transformers

Transformer Voltage Regulation

The output voltage of a transformer varies with the load even if the input voltage remains constant. This is because a real transformer has series impedance within it. Full load Voltage Regulation is a quantity that compares the output voltage at no load with the output voltage at full load, defined by this equation:

$$\text{Regulation up} = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\%$$

$$\text{Regulation down} = \frac{V_{S,nl} - V_{S,fl}}{V_{S,nl}} \times 100\%$$

$$\text{At noload } k = \frac{V_s}{V_p}$$

$$\text{Regulation up} = \frac{(V_P / k) - V_{S,fl}}{V_{S,fl}} x\, 100\%$$

$$\text{Regulation down} = \frac{(V_P / k) - V_{S,fl}}{V_{S,nl}} x\, 100\%$$

Ideal transformer, VR = 0%.

Practical Transformers

Transformer Efficiency

Transformer efficiency is defined as (applies to motors, generators and transformers):

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\%$$

Types of losses incurred in a transformer:

Copper I^2R losses

Hysteresis losses

Eddy current losses

Therefore, for a transformer, efficiency may be calculated using the following:

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} x100\%$$

Thank you