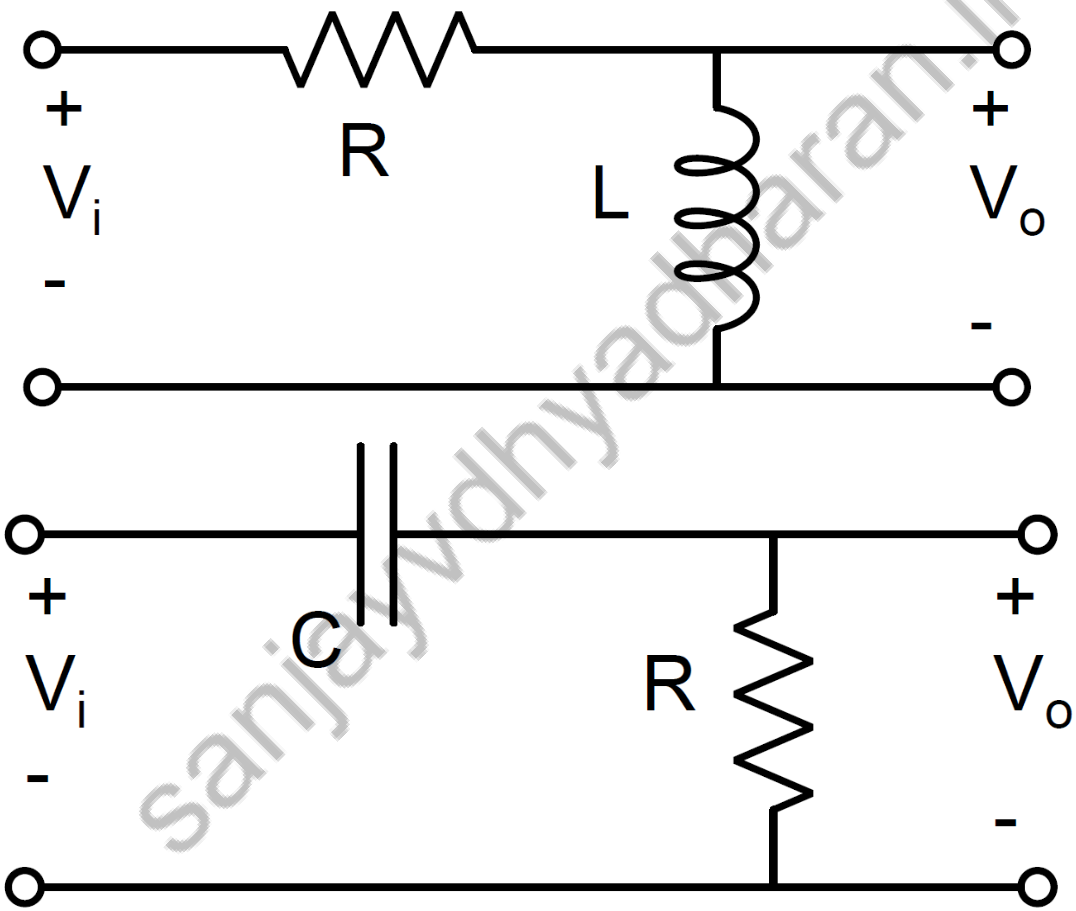
Electrical Science: 2021-22

Lecture 20

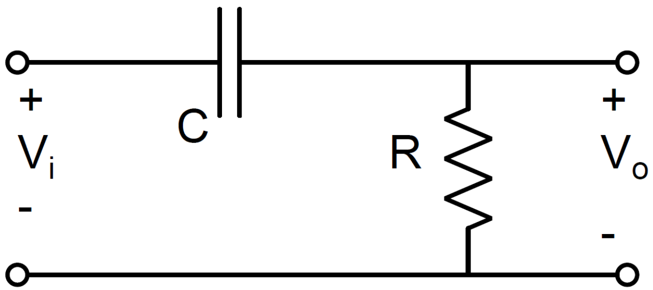
Passive Filters

By Dr. Sanjay Vidhyadharan

Passive High Pass Filters



Passive High Pass Filters



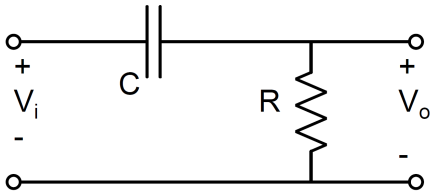
$$V_{out} = V_{in} \frac{R}{R - jXc}$$

$$Critical\ (or)\ Cutoff\ frequency\ \omega_0 = \frac{1}{RC}$$

$$Gain = \frac{V_{out}}{V_{in}} = \frac{R}{R-jXc} = \frac{1}{1-j\frac{1}{RC\omega}} = \frac{1}{1-j\frac{\omega_0}{\omega}}$$

$$Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega_0}{\omega})^2}} \angle \tan^{-1}(\frac{\omega_0}{\omega})$$

Passive High Pass Filters



$$\omega_0 = \frac{1}{RC}$$

$$A_v = Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega_0}{\omega})^2}} \angle \tan^{-1}(\frac{\omega_0}{\omega})$$

$$Voltage\ Gain\ in\ dB = 10 \log(Av)$$

$$Power\ Gain\ in\ dB = 20 \log(Av)$$

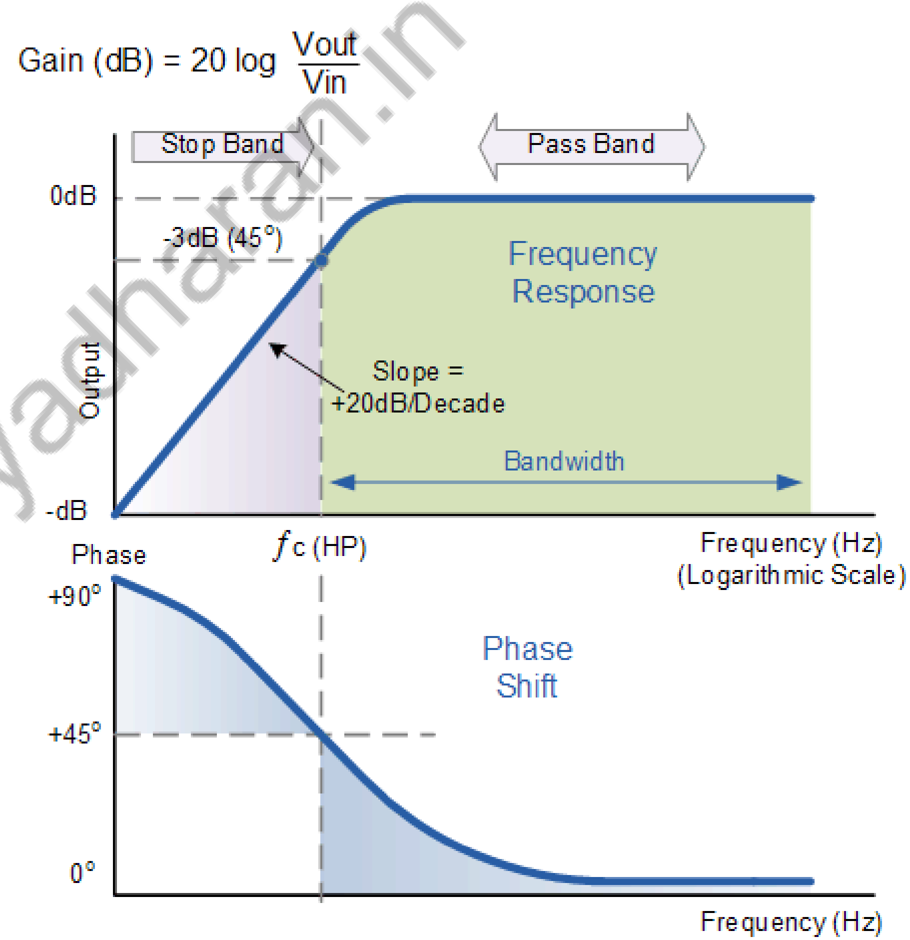
$$Half\ Power\ Gain\ in\ dB = 20 \log(0.5) = -3\ dB$$

$$Half\ Power\ A_v = \frac{1}{\sqrt{2}}$$

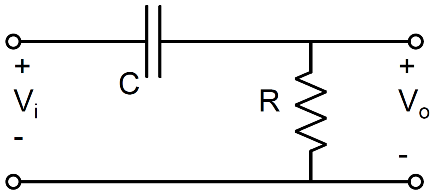
$For\ \omega \gg \omega_0\ \ A_v = 1\ Power\ Gain = 0\ dB, \phi = 0$

$For\ \omega = \omega_0\ A_v = \frac{1}{\sqrt{2}}\ \ Power\ Gain = -3\ dB, \phi = 45^0$

$For\ \omega < \omega_0\ Slope\ 20\ dB/\text{decade}\quad for \omega << \omega_0\ \phi = 90^0$



Passive High Pass Filters



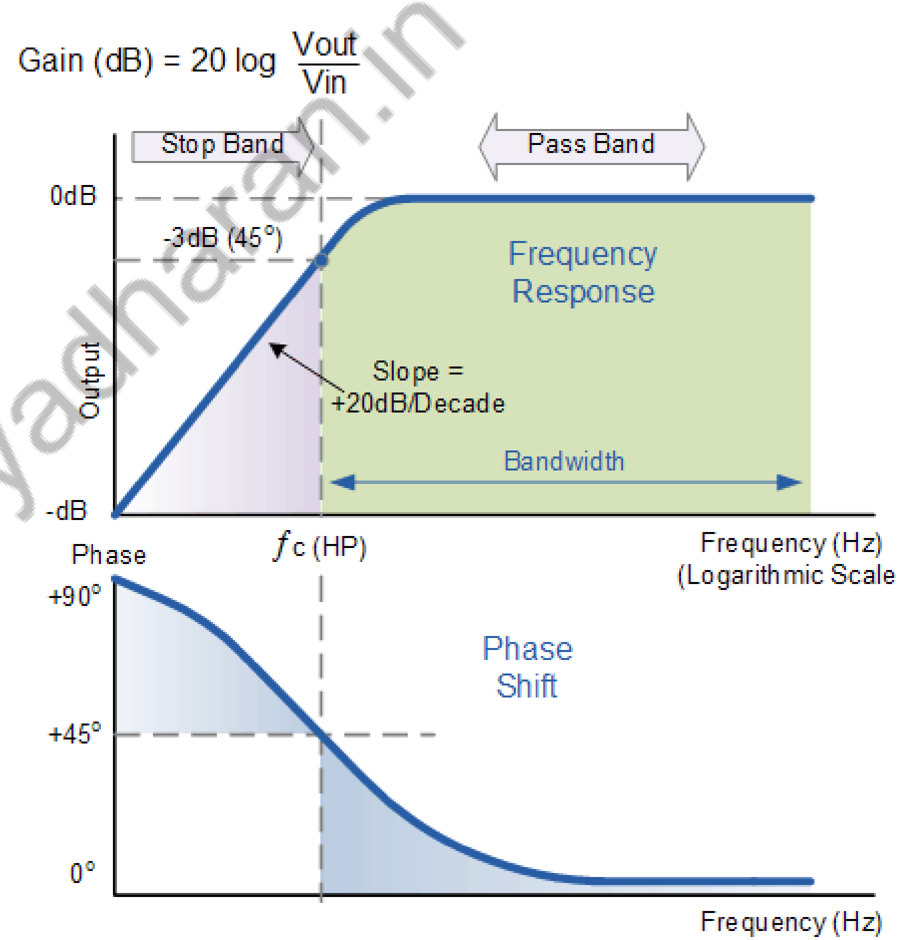
$$\omega_0 = \frac{1}{RC}$$

$$A_v = Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega_0}{\omega})^2}} \angle \tan^{-1}(\frac{\omega_0}{\omega})$$

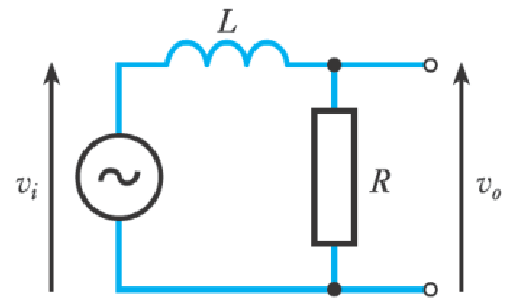
$$A_{v1} = \frac{1}{\sqrt{1+(\frac{\omega_0}{\omega_1})^2}}$$

$$A_{v2} = \frac{1}{\sqrt{1+(\frac{\omega_0}{10\omega_1})^2}} \simeq 10\ A_{v1}$$

$$Gain\ in\ dB = 20 \log 10 = 20\ dB$$



Passive High Pass Filters



$$\frac{v_o}{v_i} = \frac{Z_R}{Z_R + Z_L} = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega\frac{L}{R}} = \frac{1}{1 + j\frac{\omega}{\omega_c}} = \frac{1}{1 + j\frac{f}{f_c}}$$

At low frequencies, ω is small and the voltage gain is approximately 1.

At high frequencies, the magnitude of $\omega L/R$ becomes more significant and the gain of the network decreases.

$$|\text{voltage gain}| = \frac{1}{\sqrt{1 + \left(\omega\frac{L}{R}\right)^2}}$$

When the value of $\omega L/R$ is equal to 1, this gives

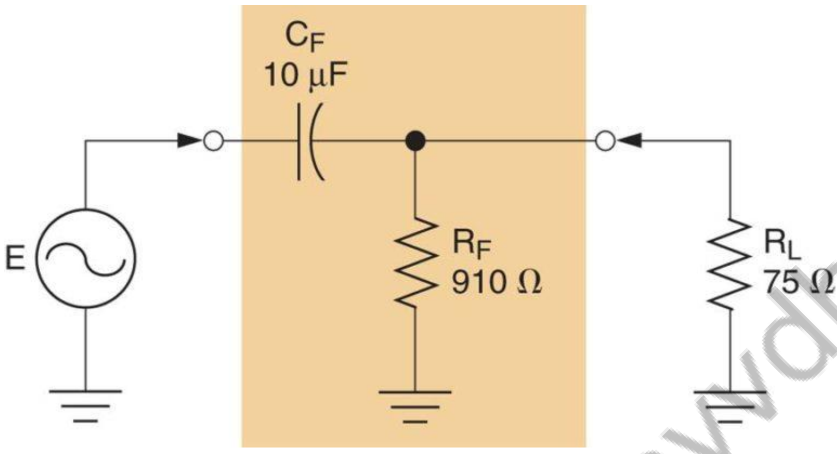
$$|\text{voltage gain}| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

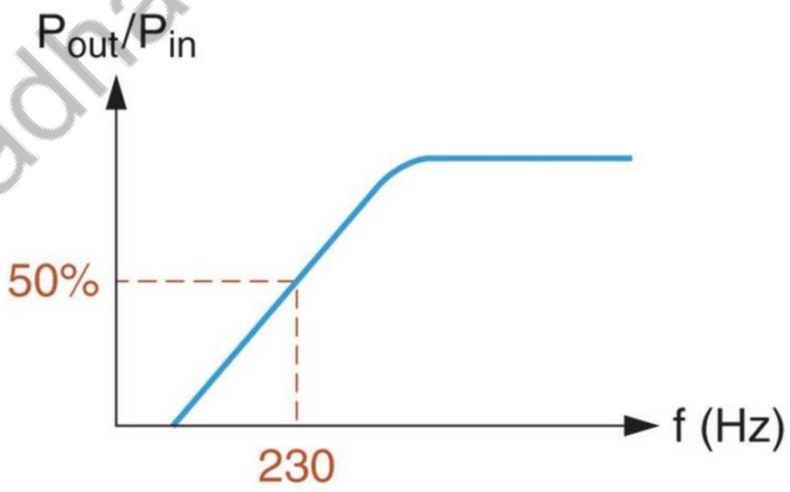
$$A_v = Gain(j\omega) = \frac{1}{\sqrt{1+\left(\frac{\omega_0}{\omega}\right)^2}} \angle \tan^{-1}\left(\frac{\omega_0}{\omega}\right)$$

$$\omega_0 = \frac{R}{L}$$

Passive High Pass Filters

Calculate the cutoff frequency for the RC High-Pass filter in Figure



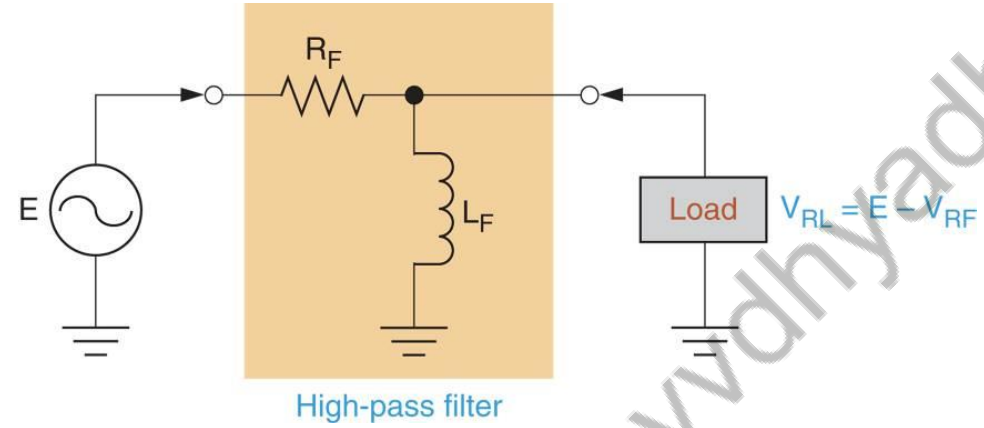


$$R_{EQ} = R_F || R_L = \frac{75 \times 910}{75 + 910} = 69.3\Omega$$

$$f_C = \frac{1}{2\pi RC} = \frac{90.1\Omega}{2\pi \times 69.3\Omega \times 10\mu F} = 230 Hz$$

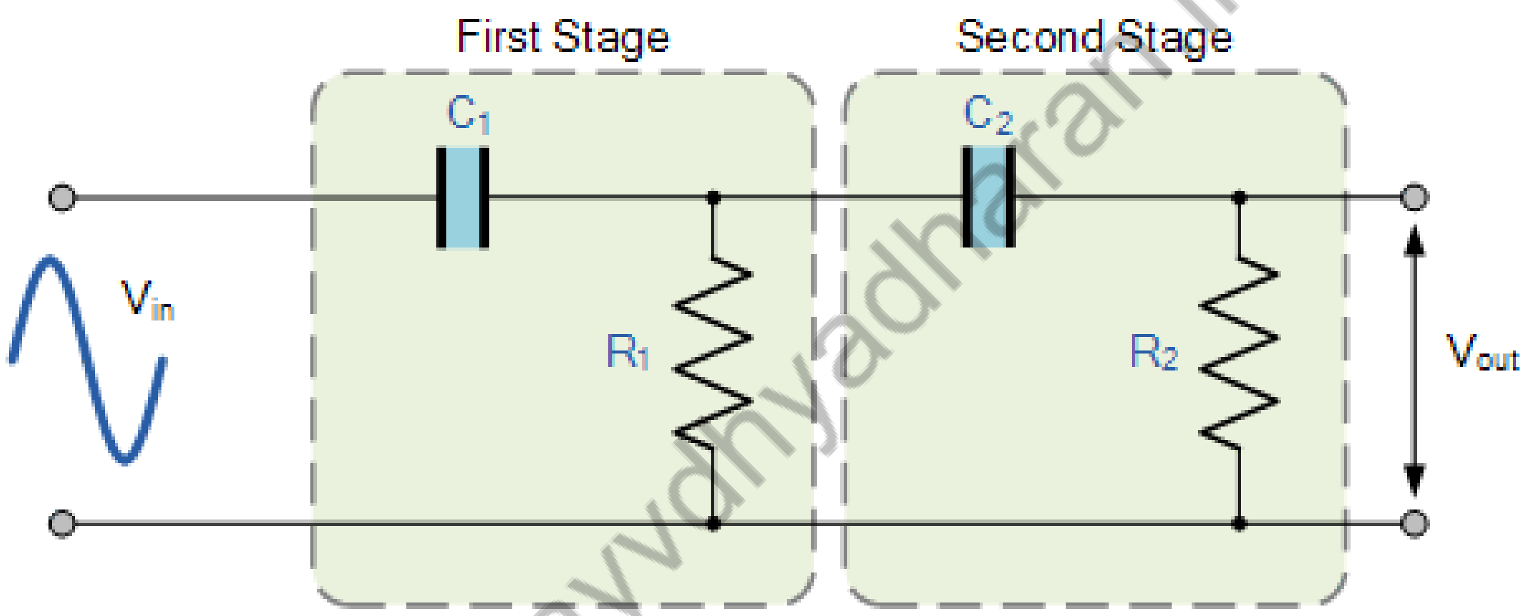
Passive High Pass Filters

Calculate the cutoff frequency for the RL High-Pass filter in Figure.
Resistor = 10KΩ, and Inductor = 470mH



$$f_c = \frac{R}{2\pi L} = \frac{10K\Omega}{2(3.14)(470mH)} = 3,388\ Hz$$

Second Order High Pass Filters

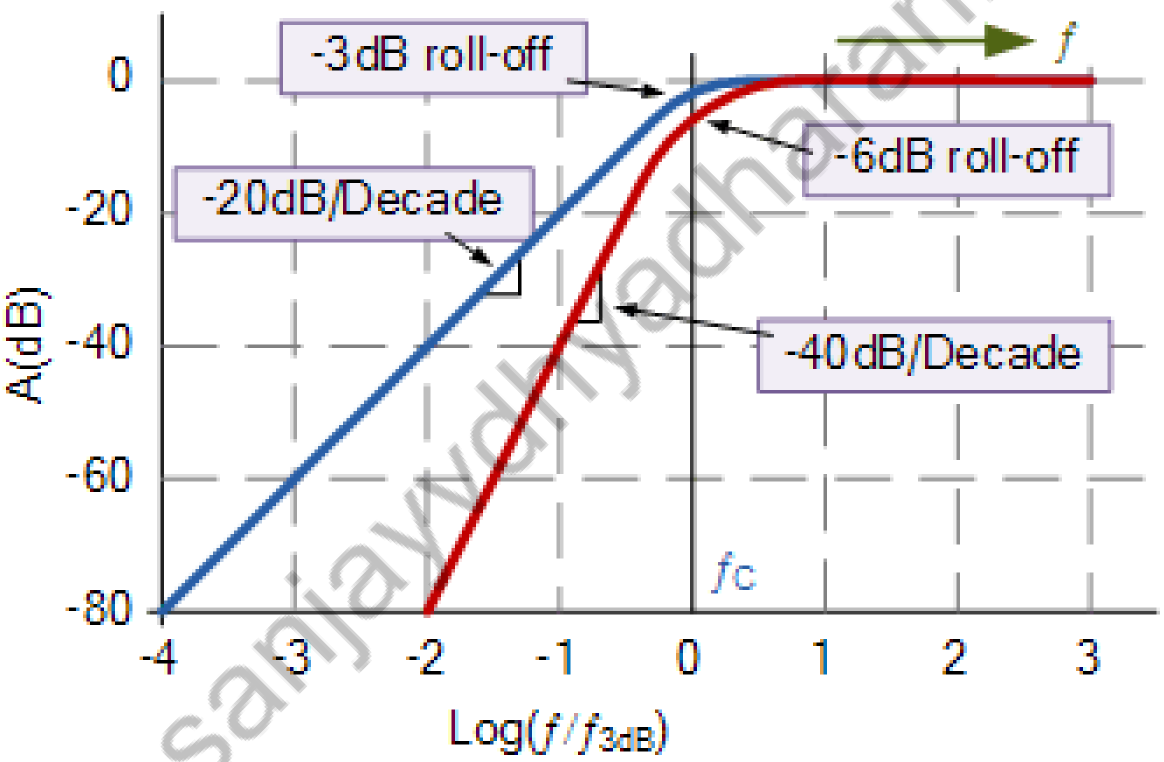


$$f_C = \frac{1}{2\pi\sqrt{R_1 C_1 R_2 C_2}} \ \text{Hz}$$

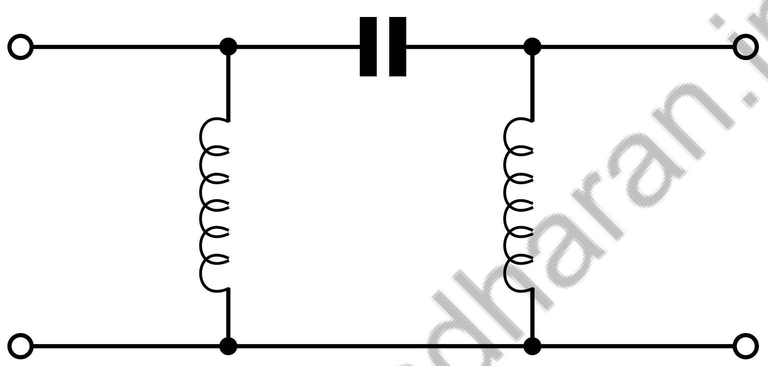
If Resistor and Capacitor values are the same:

$$f_C = \frac{1}{2\pi RC}$$

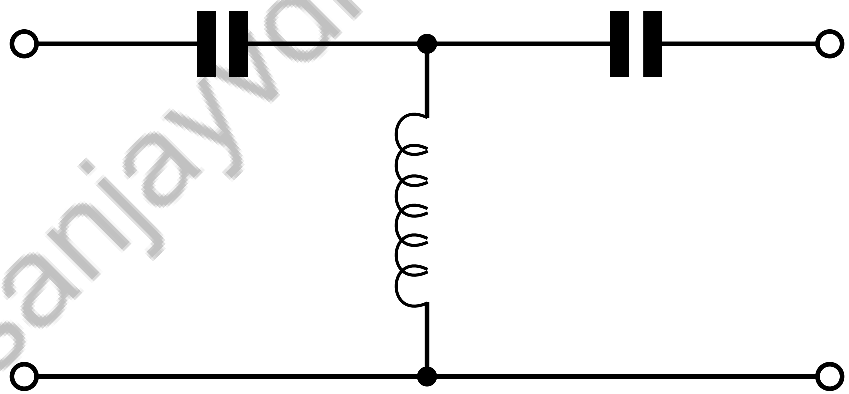
Second Order High Pass Filters



Other High Pass Filters

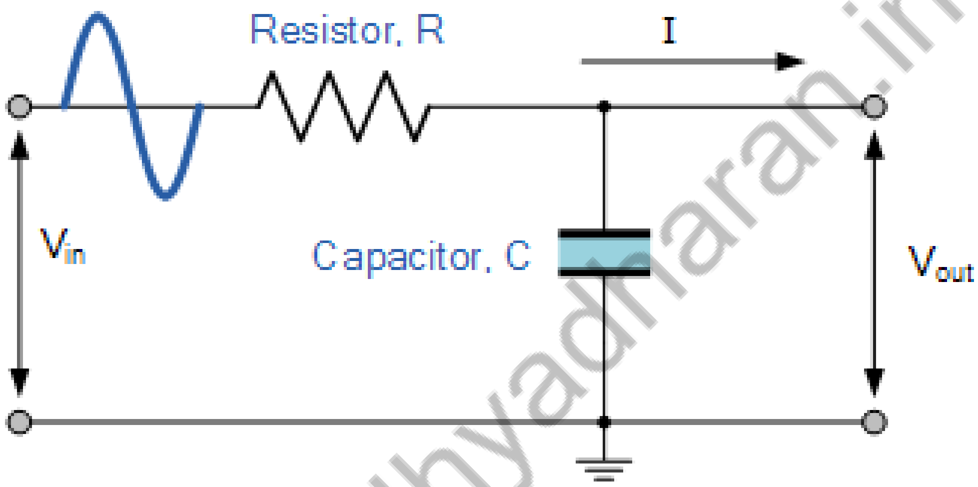


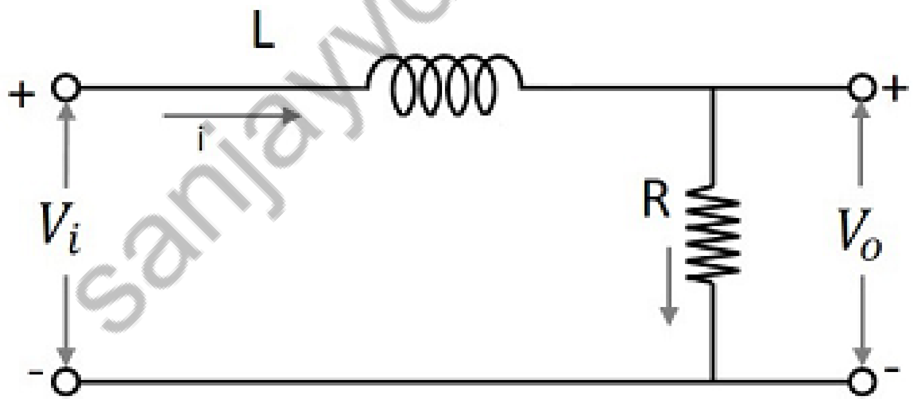
Topology for 3 pole π LC high pass RF filter



3 pole T LC high pass RF filter

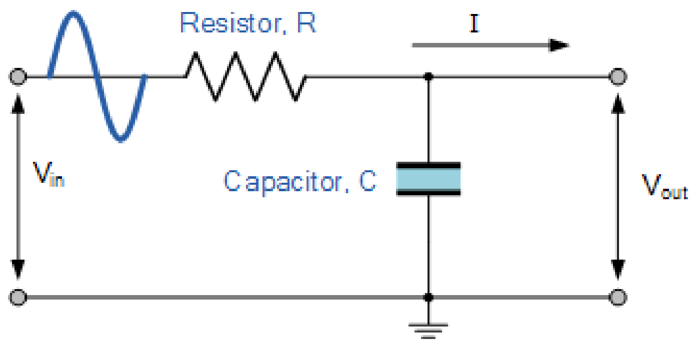
Low Pass Filters





Low Pass RL Circuit

Low Pass Filters

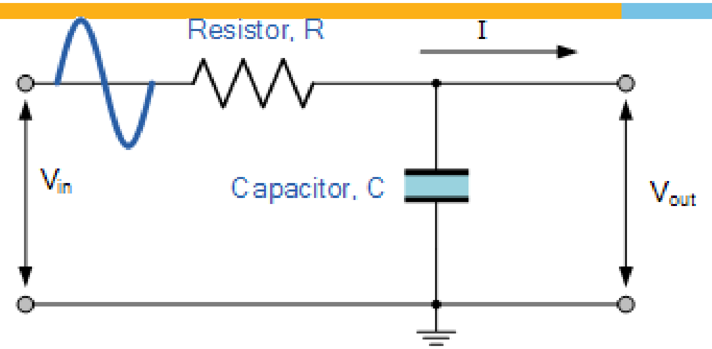


$$V_{out} = V_{in}\frac{-jXc}{R-jXc}$$

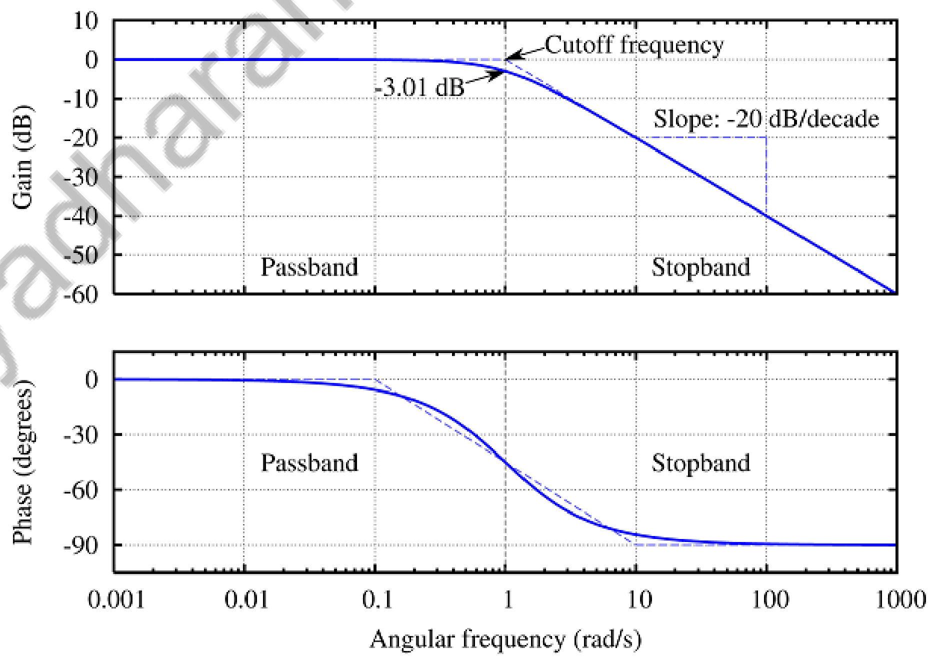
$$Gain = \frac{V_{out}}{V_{in}} = \frac{-jXc}{R-jXc} = \frac{1}{1+jRC\omega} = \frac{1}{1+j\frac{\omega}{\omega_0}} \qquad \omega_0 = \frac{1}{RC}$$

$$Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_0})^2}}\angle -\tan^{-1}(\frac{\omega}{\omega_0})$$

Low Pass Filters



$$Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_0})^2}} \angle -\tan^{-1}(\frac{\omega}{\omega_0}) \qquad \omega_0 = \frac{1}{RC}$$



$$Power\ Gain\ in\ dB = 20\log(Av)$$

$$Half\ Power\ Gain\ in\ dB = 20\log(0.5) = -3\ dB$$

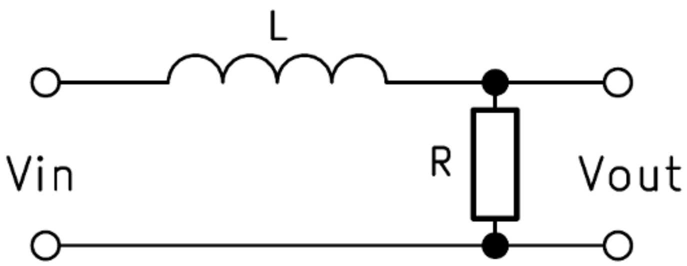
$$Half\ Power Av = \frac{1}{\sqrt{2}}$$

$For\ \omega \ll \omega_0\ \ A_v = 1\ Power\ Gain = 0\ dB, \phi = 0$

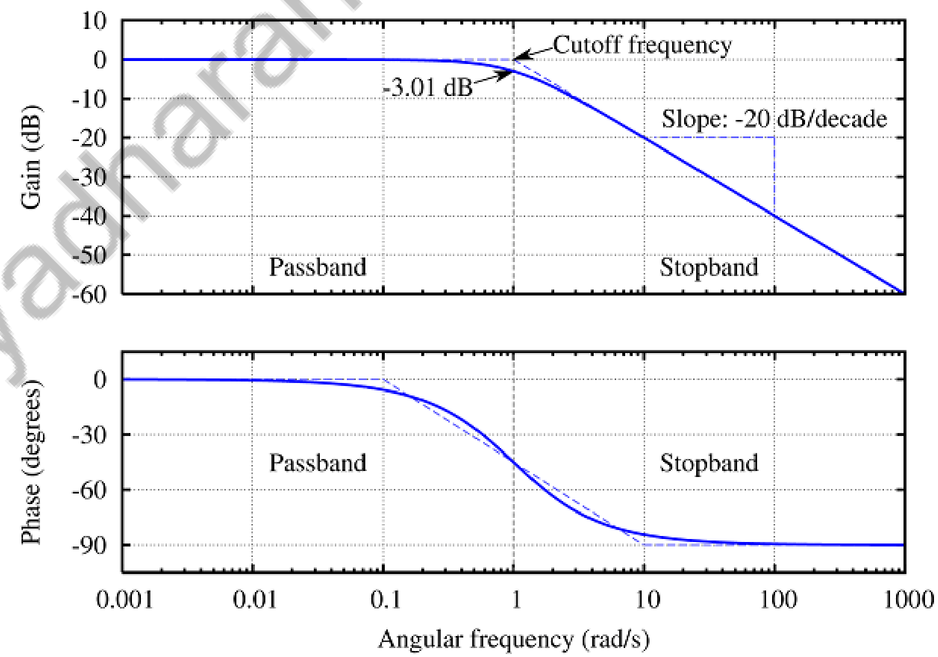
$For\ \omega = \omega_0\ A_v = \frac{1}{\sqrt{2}}\ \ Power\ Gain = -3\ dB, \phi = -45^0$

$For\ \omega > \omega_0\ Slope\ 20\ dB/$decade $\quad for \omega \gg \omega_0\ \phi = -90^0$

Low Pass Filters



$$Gain(j\omega) = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_0})^2}} \angle -\tan^{-1}(\frac{\omega}{\omega_0}) \qquad \omega_0 = \frac{R}{L}$$



$$Power\ Gain\ in\ dB = 20\log(Av)$$

$$Half\ Power\ Gain\ in\ dB = 20\log(0.5) = -3\ dB$$

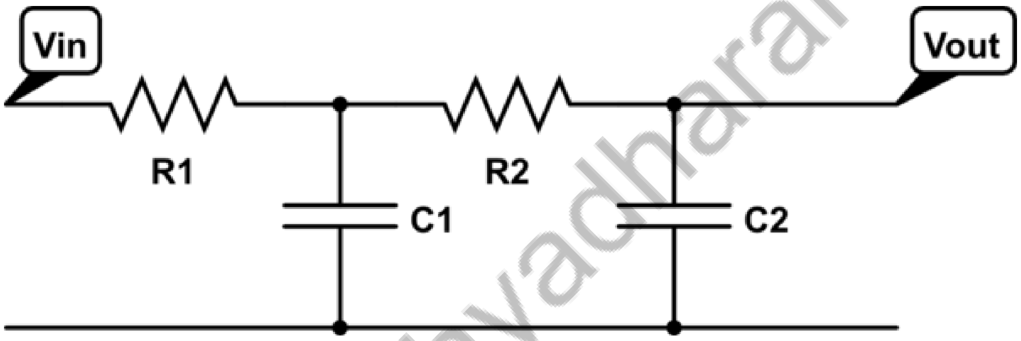
$$Half\ PowerAv = \frac{1}{\sqrt{2}}$$

$$For\ \omega \ll \omega_0\ \ A_v = 1\ Power\ Gain = 0\ dB, \phi = 0$$

$$For\ \omega = \omega_0\, A_v = \frac{1}{\sqrt{2}}\ \ Power\ Gain = -3\ dB, \phi = -45^0$$

$$For\ \omega > \omega_0\ Slope\ 20\ dB/\text{decade} \quad for\omega \gg \omega_0\ \phi = -90^0$$

Second Order Low Pass Filters

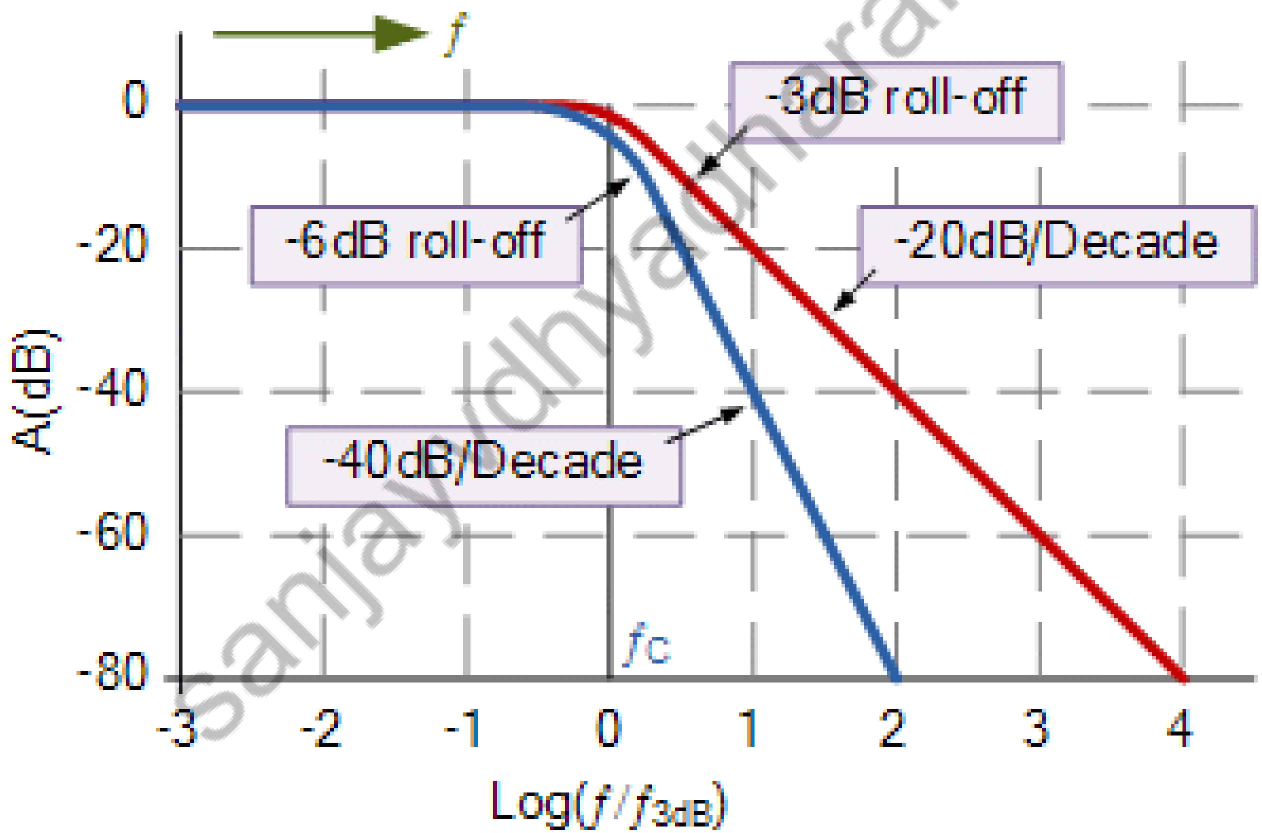


$$\omega_c = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

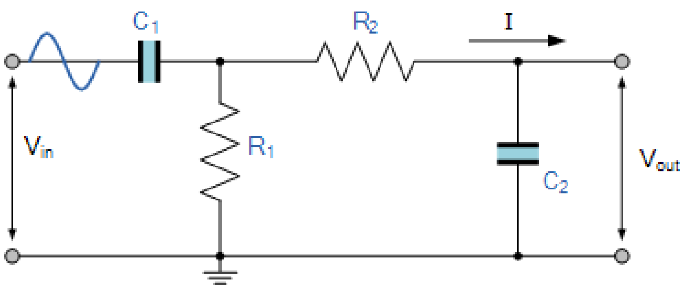
If Resistor and Capacitor values are the same:

$$f_c = \frac{1}{2\pi RC}$$

Second Order Low Pass Filters



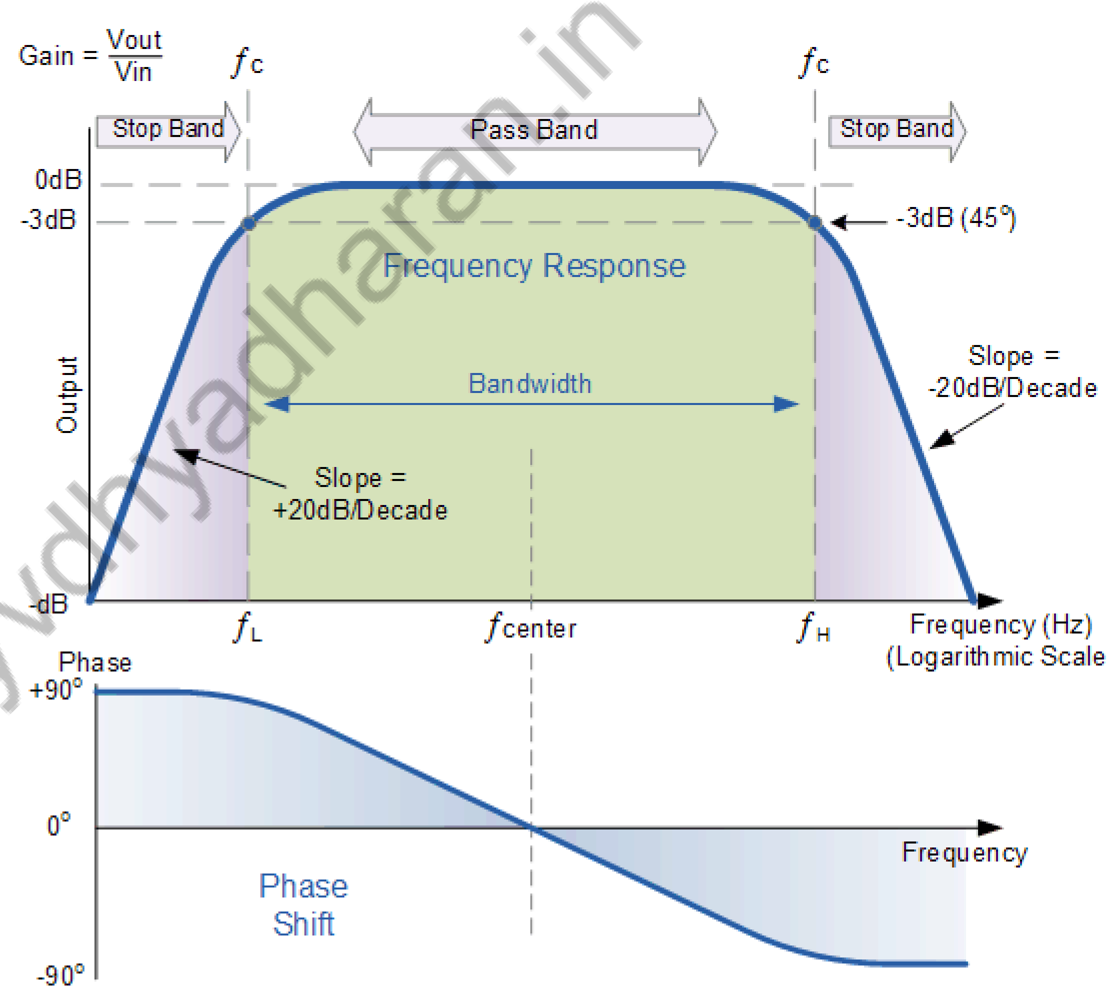
Band Pass Filters



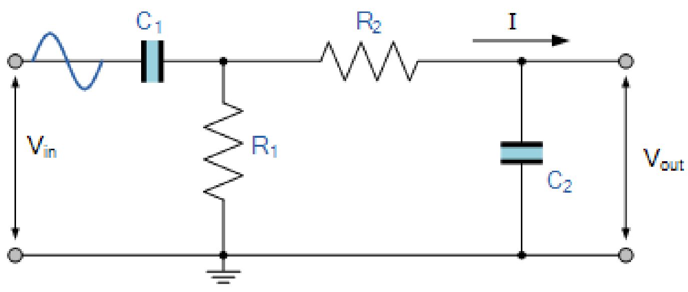
$$BW = f_H - f_L$$

Resonant Frequency (f_r)

$$f_r^2 = f_{(UPPER)} \times f_{(LOWER)}.$$



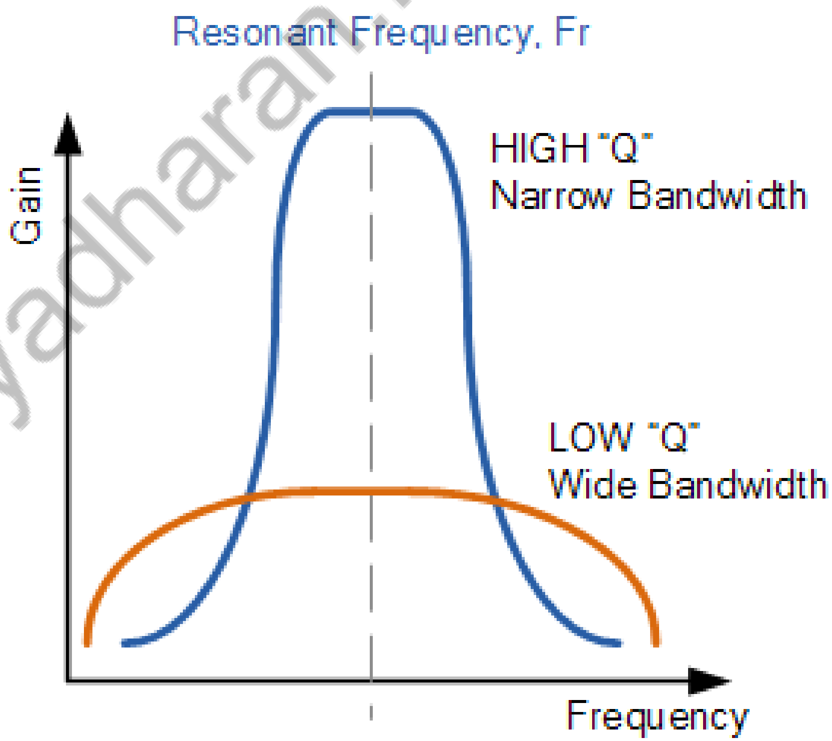
Quality Factor of Band Pass Filters



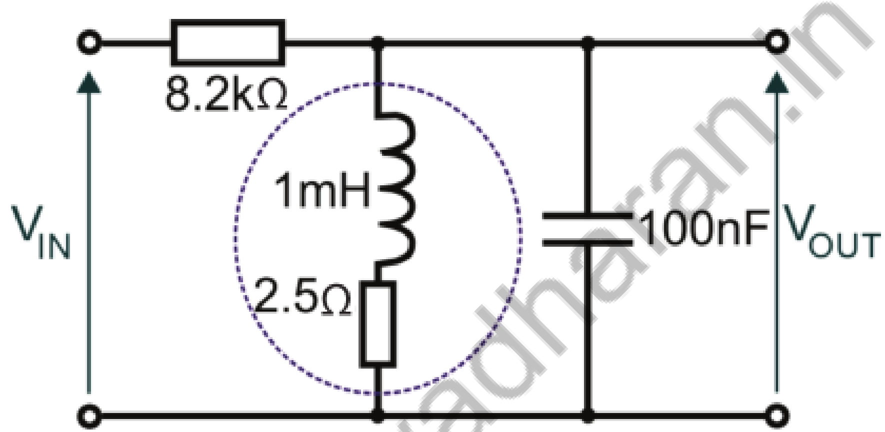
$BW = f_H - f_L$

Resonant Frequency (f_r) or (f_0)
$f^2_r = f_{(UPPER)} \times f_{(LOWER)}$.

$$Q = \frac{f_o}{BW}$$



Band Pass Filters



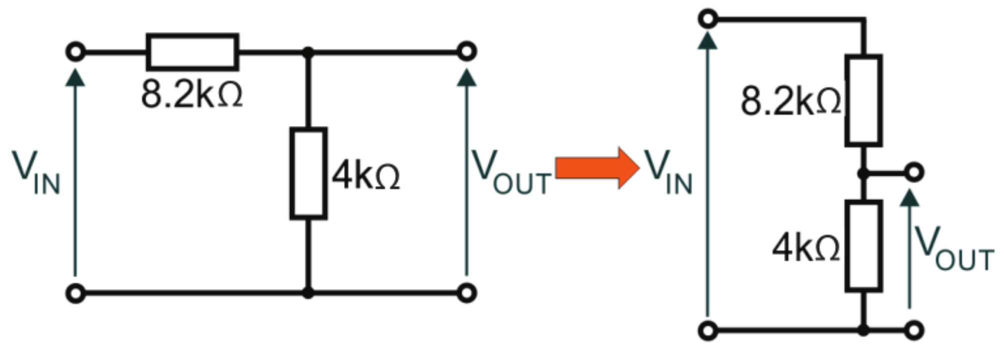
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{1 \times 10^{-3} \times 100 \times 10^{-9}}}$$

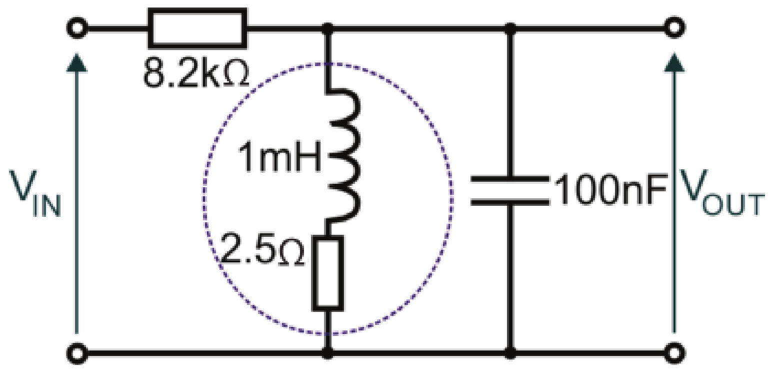
$$= 15915\text{Hz}$$

Dynamic resistance R_D

$$R_D = \frac{L}{r_L C} = \frac{1 \times 10^{-3}}{2.5 \times 100 \times 10^{-9}} = 4000\Omega$$



Band Pass Filters



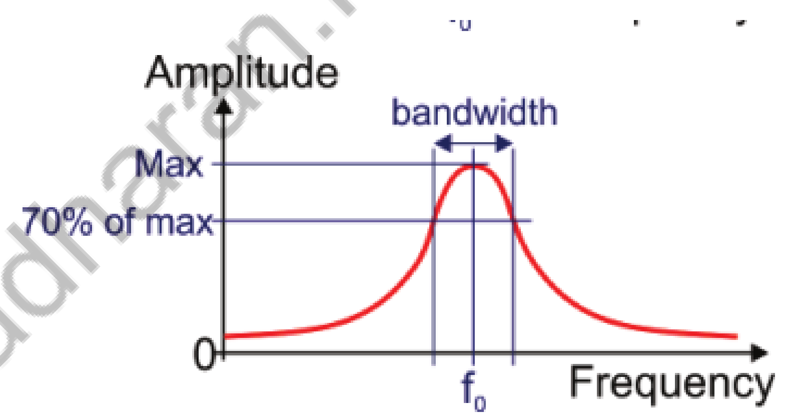
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{1 \times 10^{-3} \times 100 \times 10^{-9}}}$$

$$= 15915\text{Hz}$$

$$R_D = \frac{L}{r_L C} = \frac{1 \times 10^{-3}}{2.5 \times 100 \times 10^{-9}} = 4000\Omega$$

$$BW = \frac{r_L}{L}$$



$$Q = \frac{2\pi f_0 L}{r_L} = \frac{2 \times \pi \times 15915 \times 1 \times 10^{-3}}{2.5} = 40$$

$$Q = \frac{f_0}{\text{bandwidth}} \rightarrow \text{Bandwidth} = \frac{f_0}{Q}$$

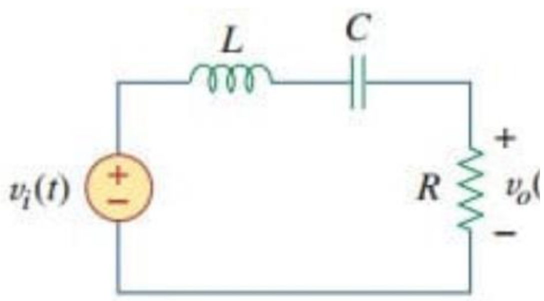
$$\text{Bandwidth} = \frac{f_0}{Q} = \frac{15915}{40} = 397.9\text{Hz}$$

Band Pass Filters

The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{R + j(\omega L - 1/\omega C)}$$

(5)



We observe that $\mathbf{H}(0) = 0$, $\mathbf{H}(\infty) = 0$. Figure.(7) shows the plot of $|\mathbf{H}(\omega)|$.

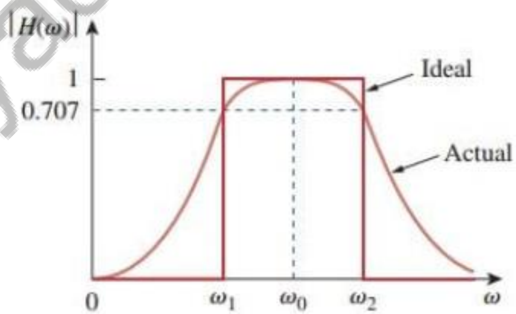


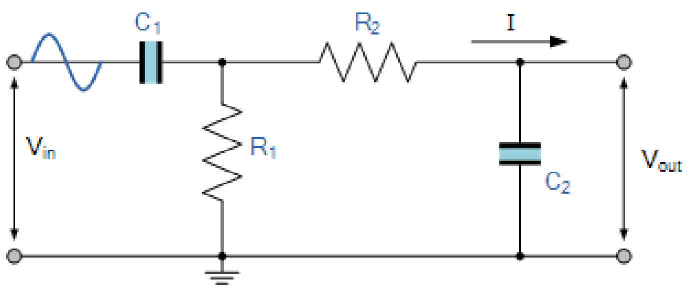
Figure 7. Ideal and actual frequency response of a bandpass filter.

BW= R/L

The bandpass filter passes a band of frequencies ($\omega_1 < \omega < \omega_2$) centered on ω0, the center frequency, which is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

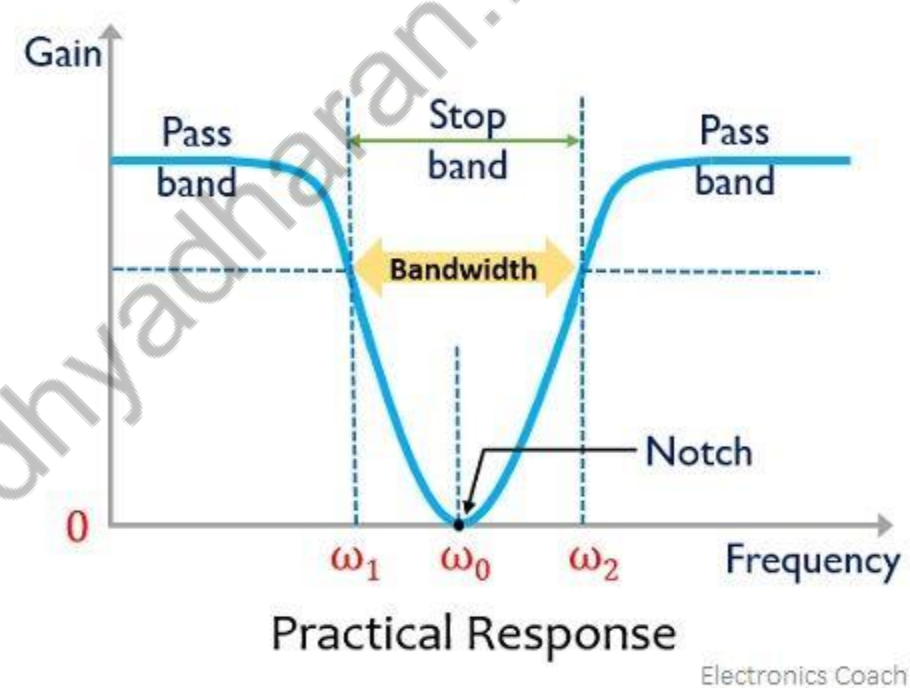
Band Reject Filters



$BW = f_H - f_L$

Resonant Frequency (f_r)

$f^2_r = f_{(UPPER)} \times f_{(LOWER)}$.



Band Reject Filters

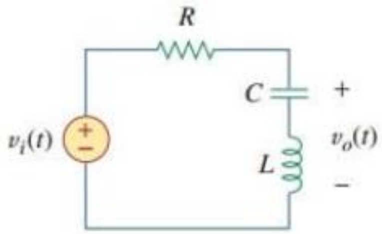


Figure 8. A bandstop filter.

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

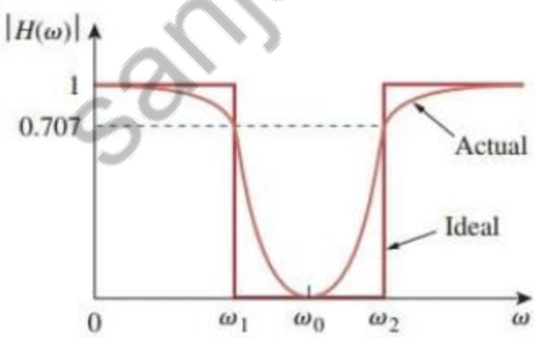
The transfer function is

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{j(\omega L - 1/\omega C)}{R + j(\omega L - 1/\omega C)}$$

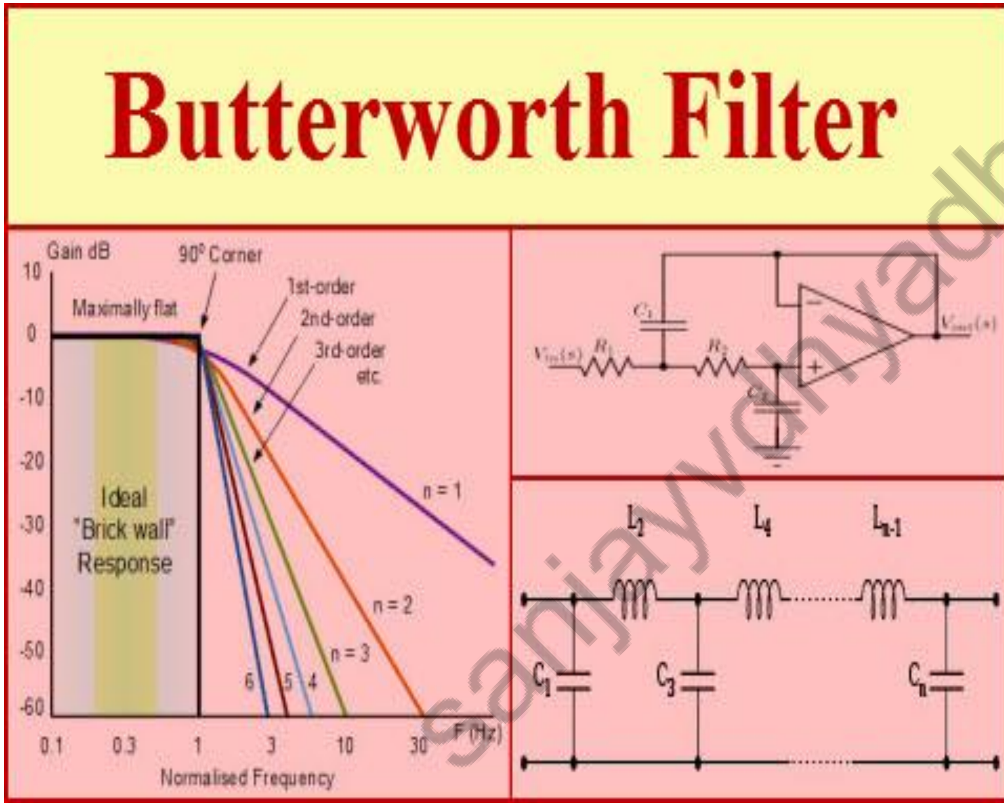
(7)

BW= R/L

Notice that $\mathbf{H}(0) = 1$, $\mathbf{H}(\infty) = 1$. Figure 14.38 shows the plot of $|\mathbf{H}(\omega)|$.



Active Filters



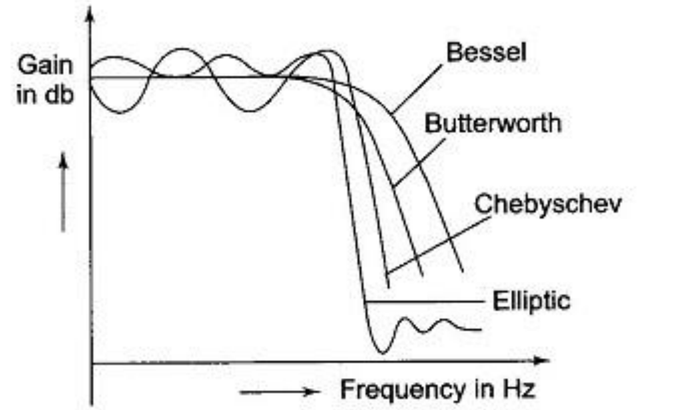


Fig. 15.27 Frequency Response of Various Filters

Thank you