Electrical Science: 2021-22
Lecture 6
Norton’s Theorem &
Max Power Transform Theorem

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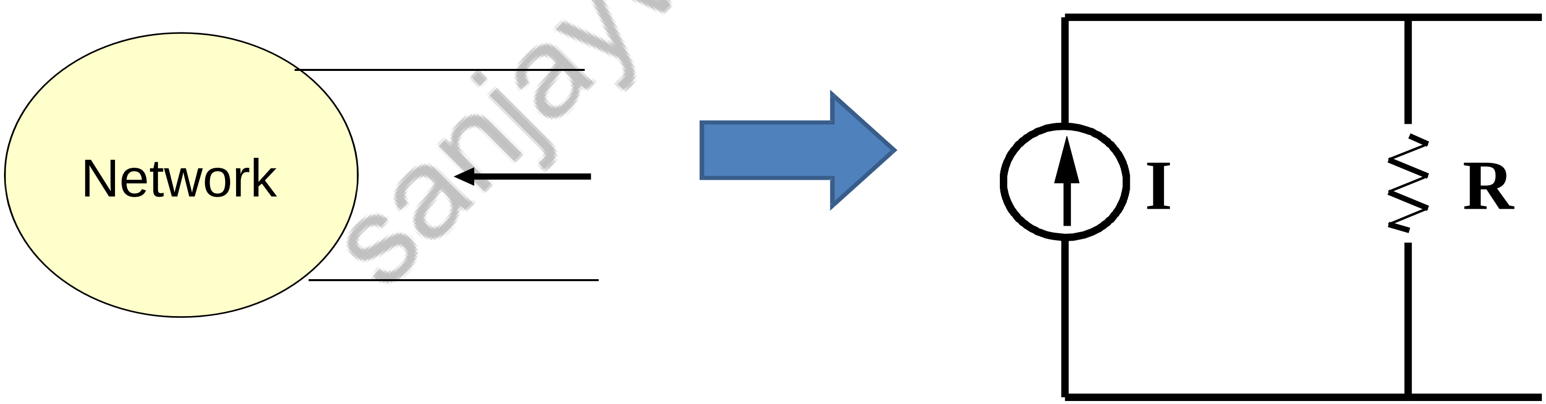
Norton's Theorem

Applications of Norton's Theorem

- Simplifies the network in terms of currents instead of voltages.
- It reduces a network to a simple current source parallel with a resistor.

A **linear** two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



Norton's Theorem

Steps to determine Norton's equivalent Resistance (R_N) and Current (I_N):

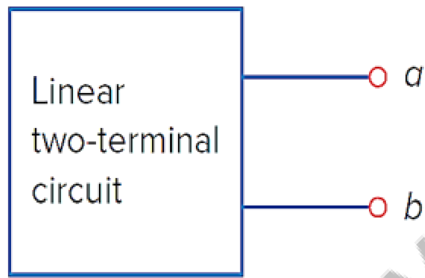
- Calculate R_N in the same way as R_{Th}

- To find the Norton current I_N , we determine the short-circuit current flowing at output terminal

- This short-circuit current is the Norton equivalent current I_N.

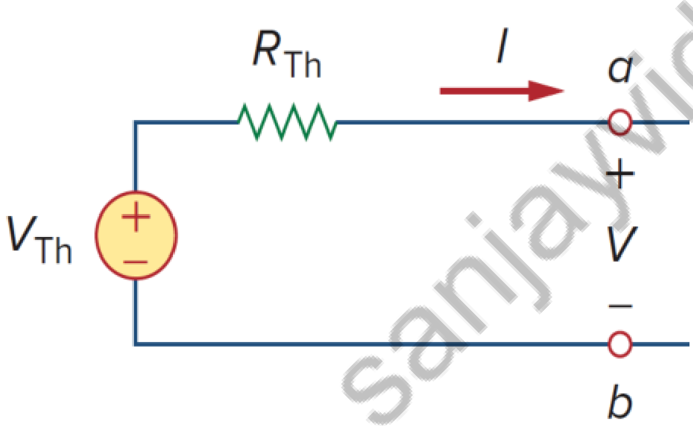
Norton’s Theorem

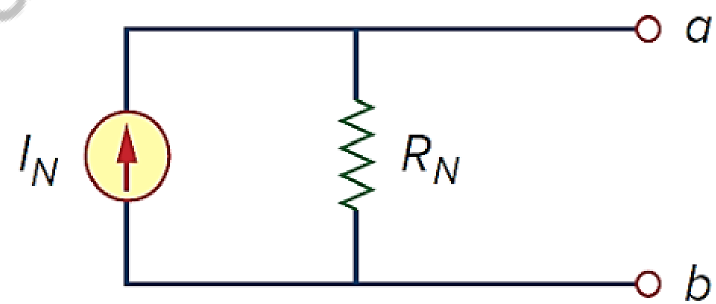
Close relationship between Norton’s and Thevenin’s theorems:



$$R_N = R_{Th}$$

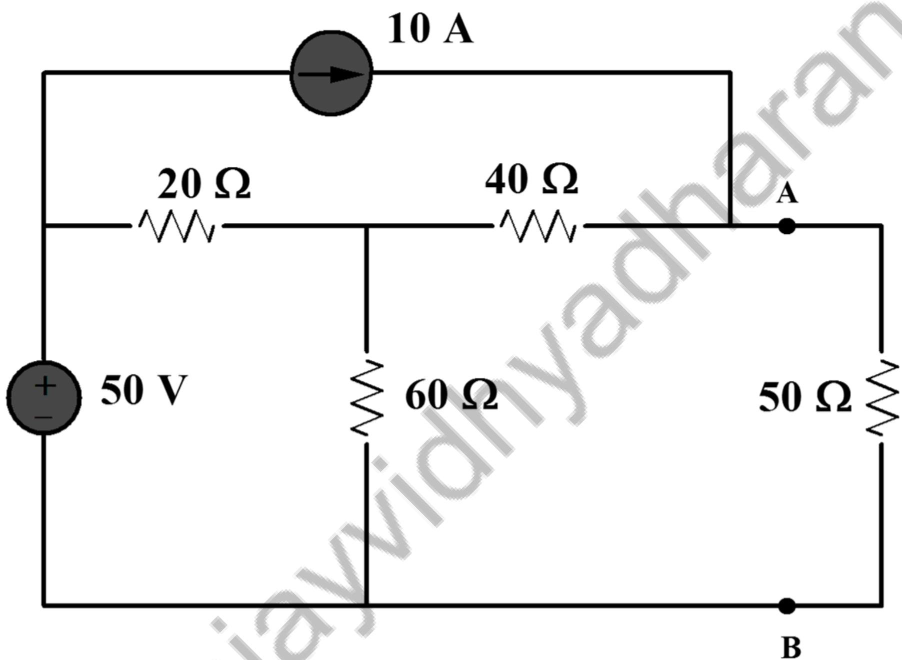
$$I_N = \frac{V_{Th}}{R_{Th}}$$





Norton’s Theorem

Example 1



Example: Find the Norton equivalent circuit to the left of terminals A-B and find the current in the 50 Ω resistor using the equivalent circuit.

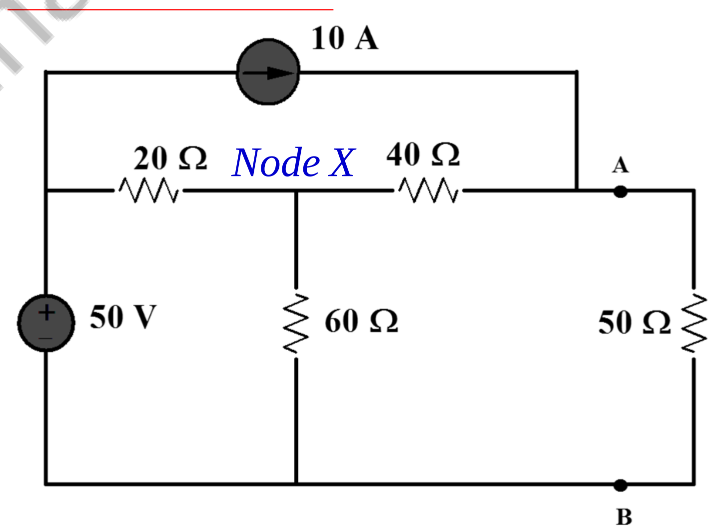
Norton's Theorem

Example 1

KCL at Node X

$$\frac{V_x - 50}{20} + \frac{V_x}{60} + \frac{V_x}{40} = 0$$

$V_x = 27.27$ V



KCL at node A (shorting A and B)

$$\frac{V_x}{40} + 10 = I_{SC}, \quad I_{SC} = 10.681A$$

Norton's Theorem

Example 1

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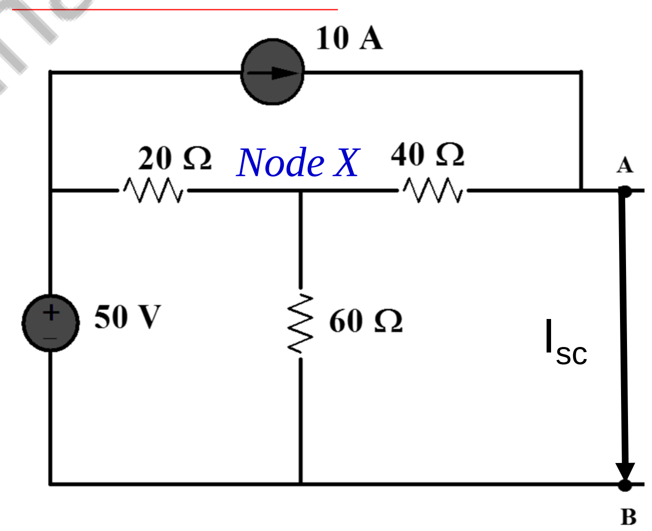
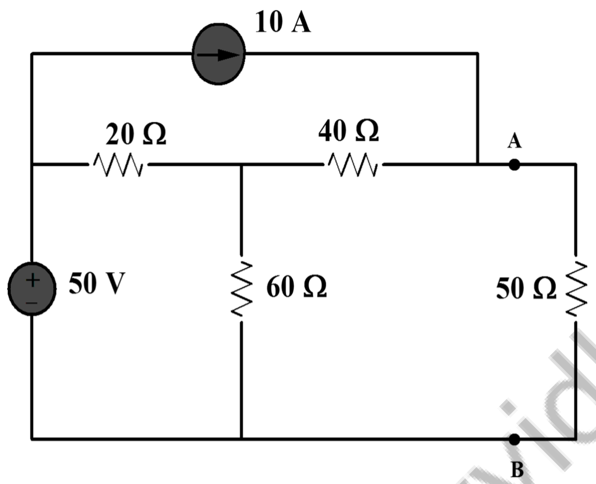
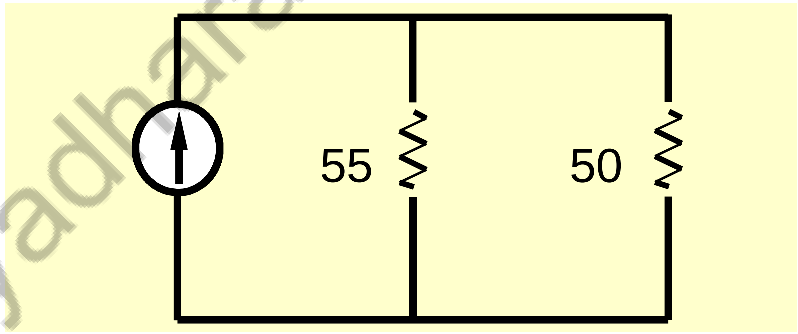


Norton's Theorem

Example 1



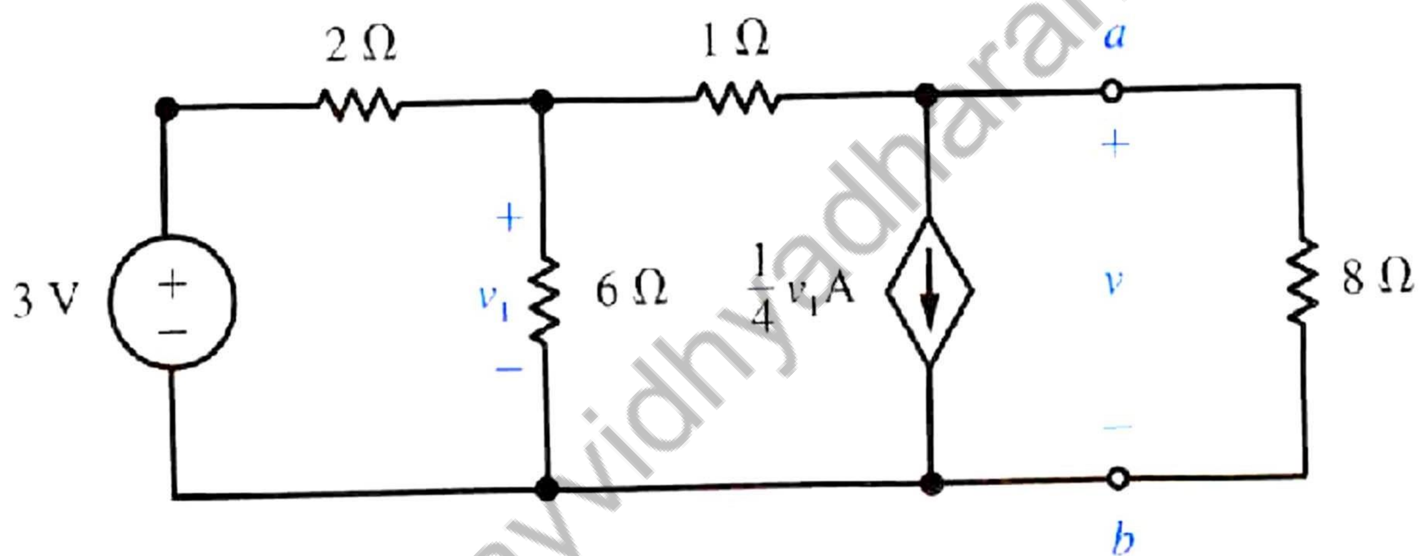


Current through 50 ohm resistor is

(10.7 x 55) / (50 + 55) = 5.6 A

Norton's Theorem

Example 2



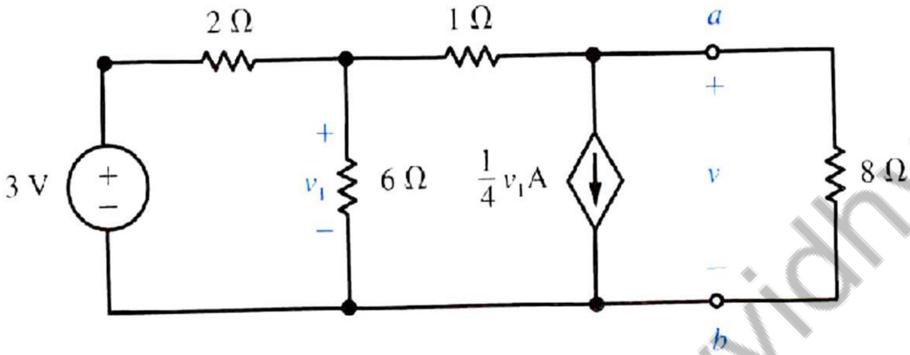
Example of the use of the Norton-equivalent Circuit (Find v).

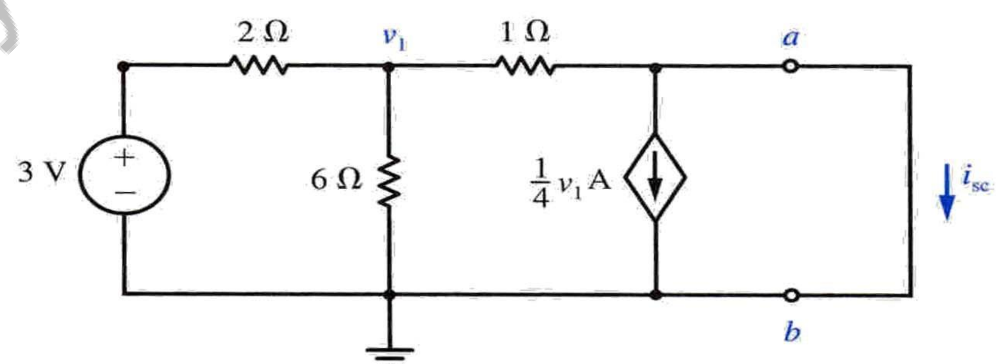
Find $R_{th} = \dfrac{v_{oc}}{i_{sc}}$

Norton's Theorem

Example 2

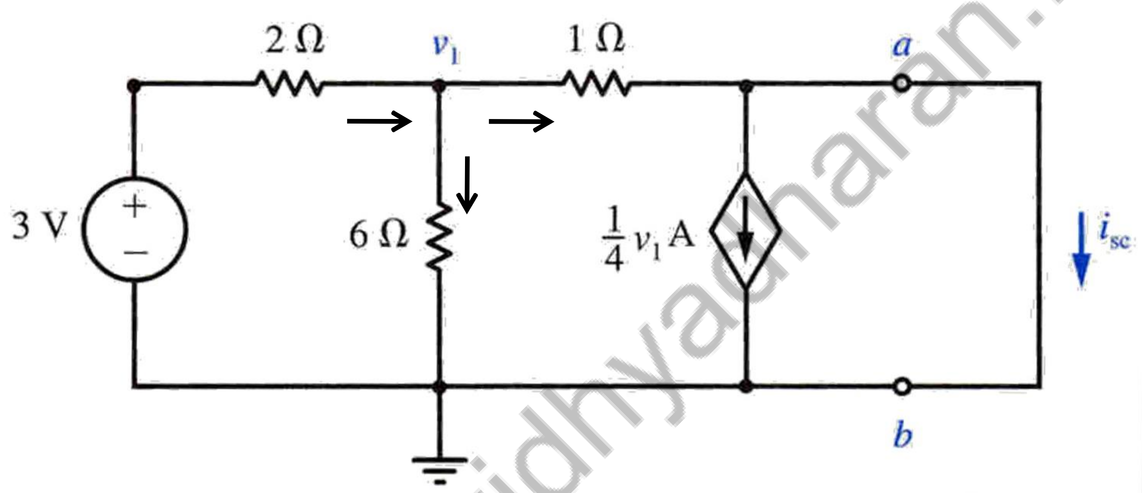
Replace the 8-Ω resistor that is connected between nodes 'a' and 'b' with short circuit, and calculate I_{sc}





Norton's Theorem

Example 2



KCL at node V_1

$$\frac{-v_1+3}{2}=\frac{v_1}{6}+\frac{v_1}{1}$$

$$v_1=0.9$$

KCL at node V_a

$$i_{sc}=\frac{v_1}{1}-\frac{v_1}{4}=\frac{3}{4}v_1$$

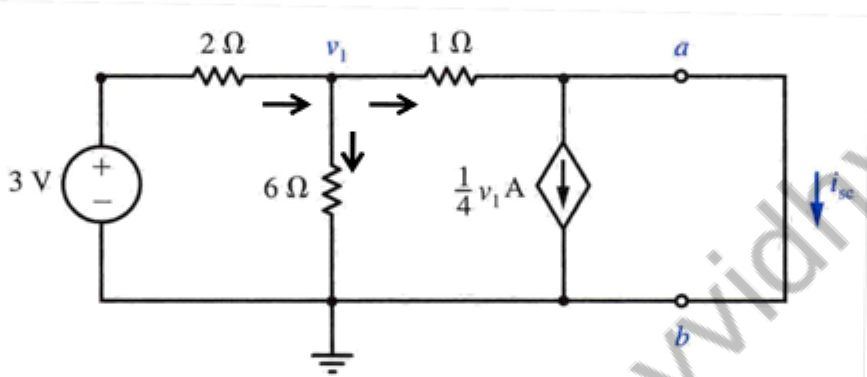
$$=\left(\frac{3}{4}\right)\left(\frac{9}{10}\right)=\frac{27}{40}$$

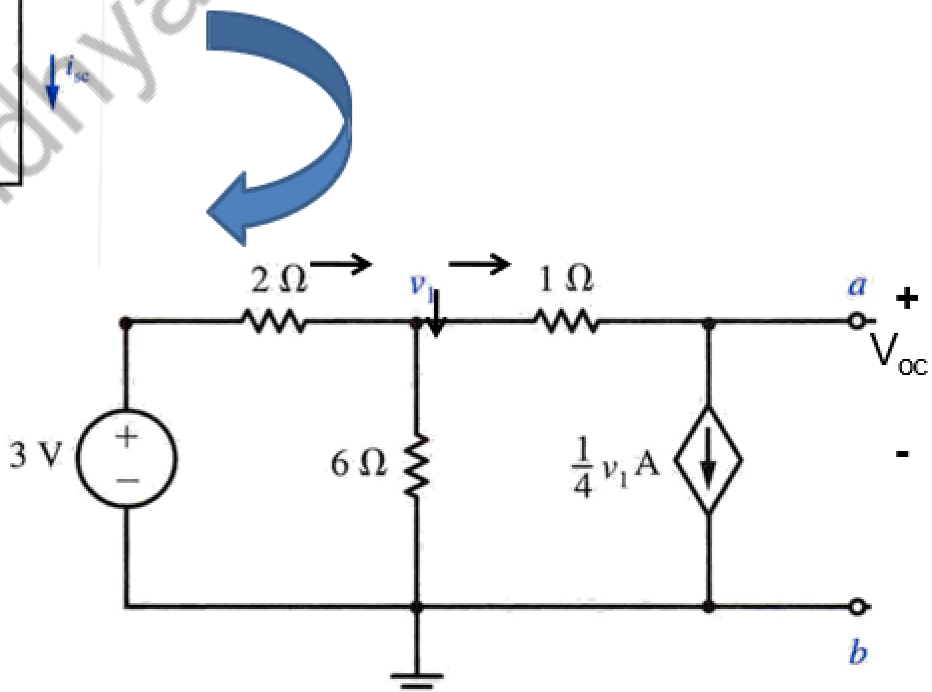
$$=0.675A$$

Norton's Theorem

Example 2

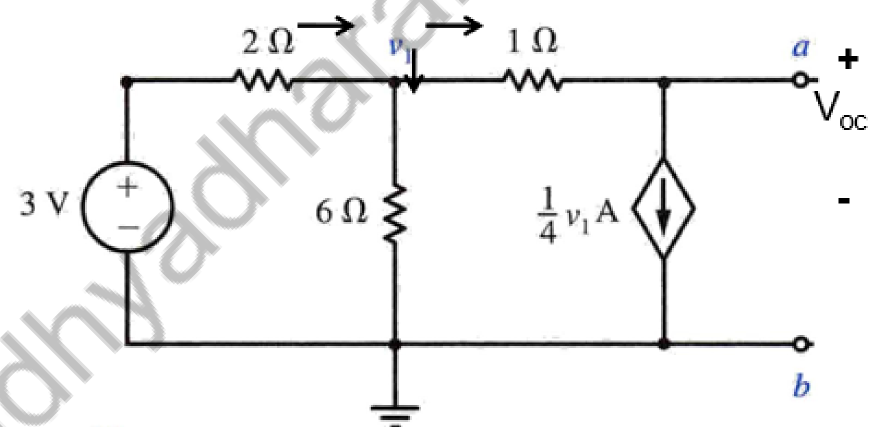
Remove the 8-Ω Load resistor from the circuit. Calculate the open-circuit voltage V_{oc} between nodes a and b.





Norton's Theorem

Example 2



KCL at V_1

$$\frac{v_1 - v_{oc}}{1} + \frac{v_1}{6} = \frac{3 - v_1}{2}$$

KCL at v_{oc}

$$\frac{v_1 - v_{oc}}{1} = \frac{v_1}{4}$$

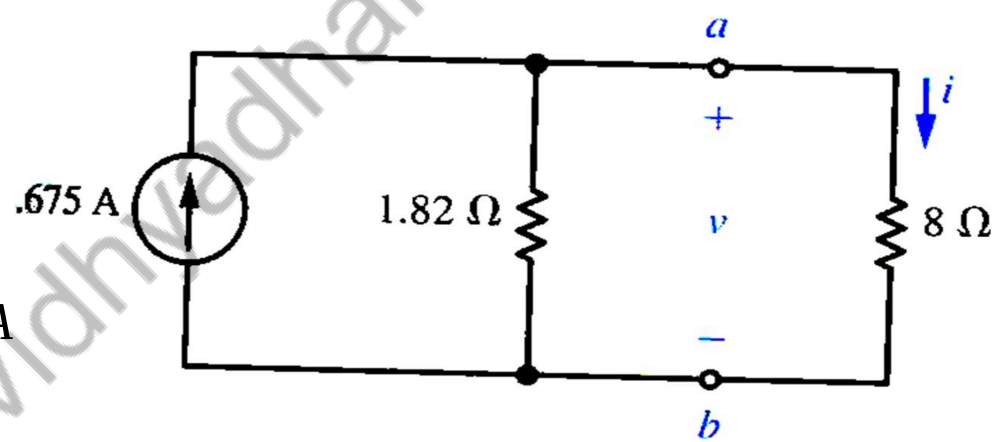
$$v_1 = \frac{18}{11} \quad and \quad v_{oc} = \frac{27}{22}$$

$$R_{th} = \frac{v_{oc}}{i_{sc}} = \frac{27/22}{27/40} = \frac{20}{11} = 1.82\ \Omega$$

Norton's Theorem

Example 2

Using the current-divider formula



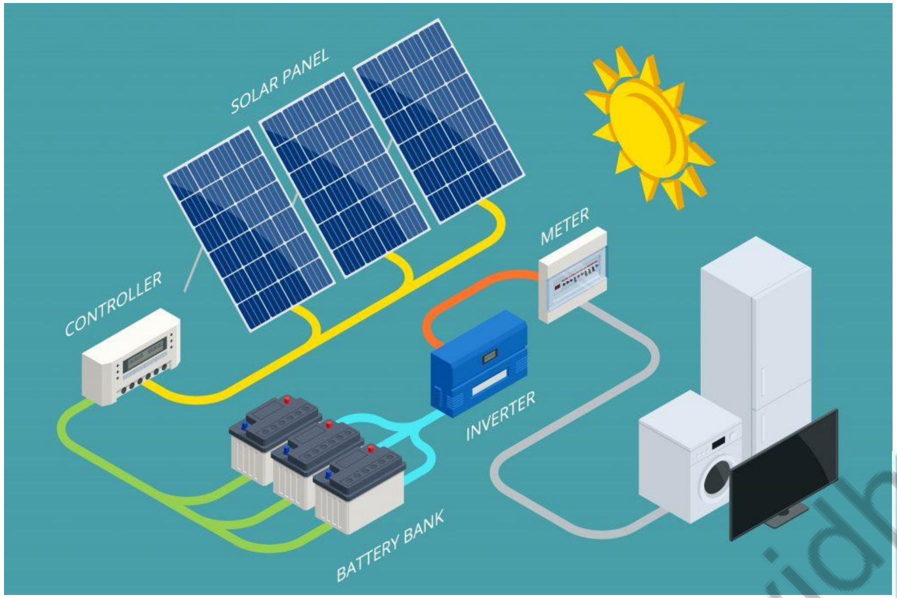
$$i = \frac{1.82}{1.82+8}(0.675) = 0.125A$$

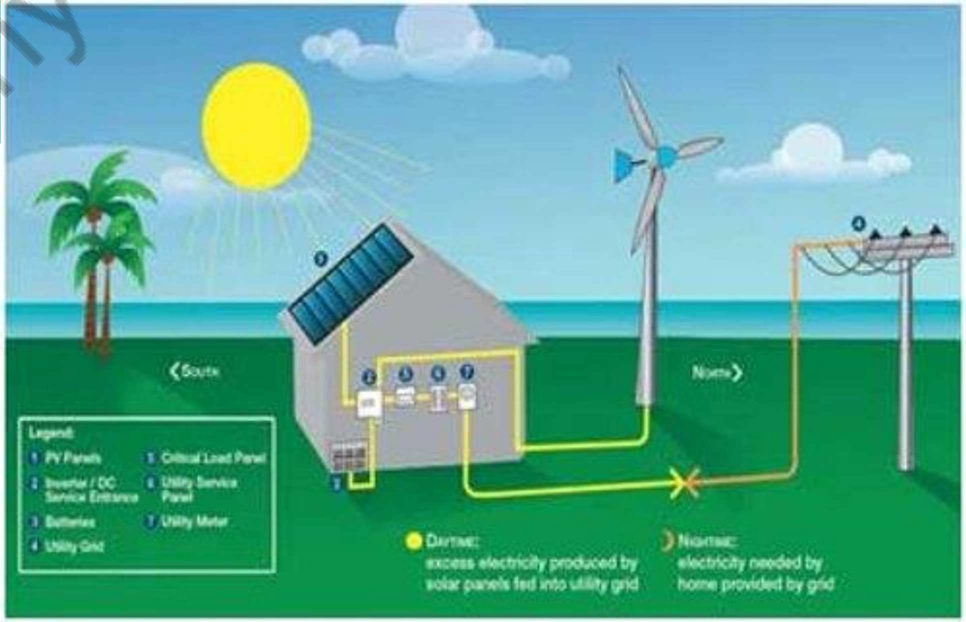
$\Rightarrow$ **V = 8* 0.125 = 1 V**

Limitations of Norton's Theorem

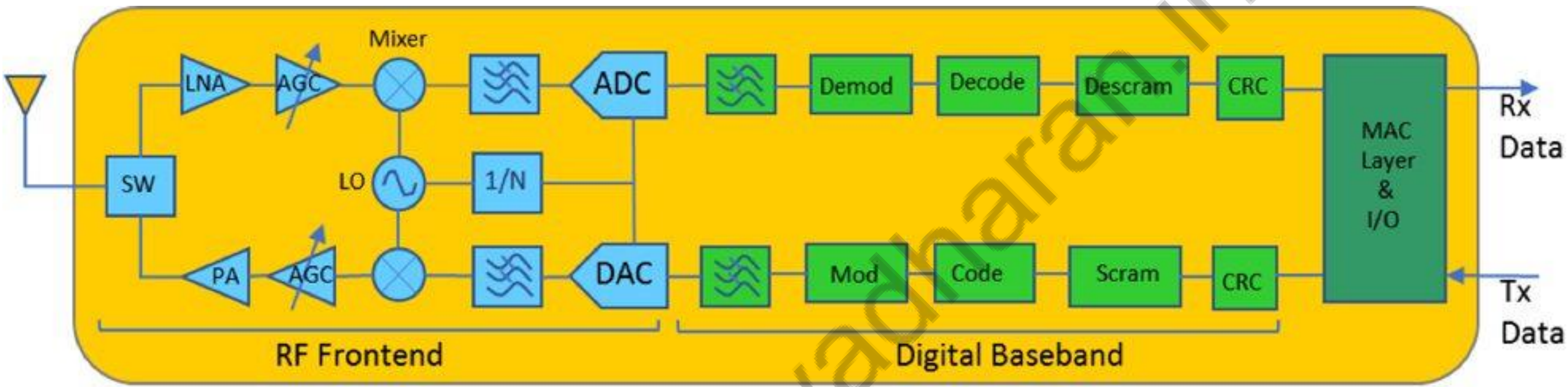
If the circuit consists of nonlinear elements, this theorem is not applicable.

Maximum Power Transfer Theorem



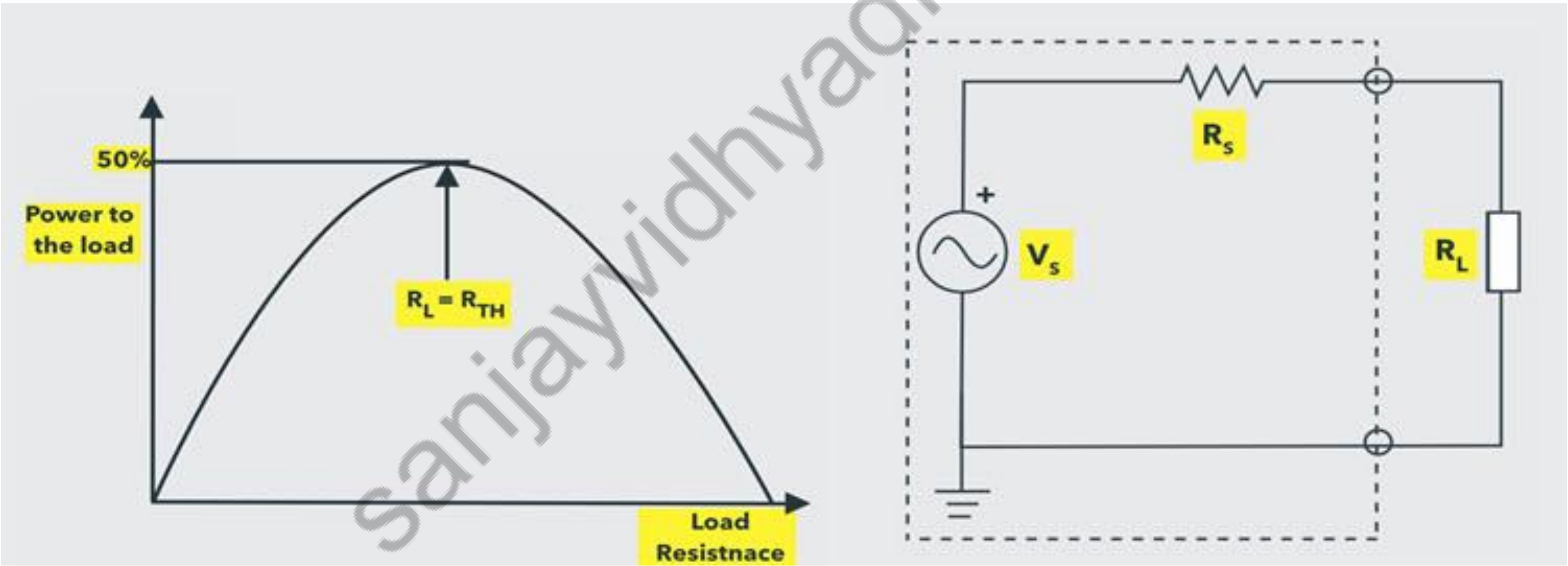


Maximum Power Transfer Theorem



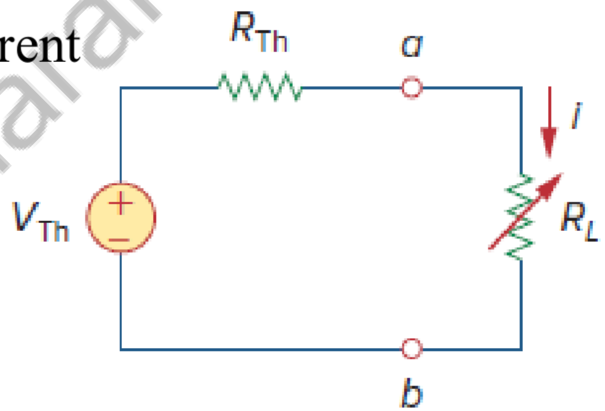
Maximum Power Transfer Theorem

Maximum power is transferred to the load from a network when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).



Maximum Power Transfer Theorem

If the value of the load resistance is R_L, the current flowing through the circuit is $i = \frac{V_{Th}}{R_{Th} + R_L}$



Power transferred to the load is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{th} + R_L} \right)^2 R_L = \frac{V_{Th}{}^2 R_L}{R_{Th}{}^2 + 2R_L R_{Th} + R_L{}^2} = \frac{V_{Th}{}^2}{\left(\frac{R_{Th}{}^2}{R_L} \right) + 2R_{Th} + R_L}$$

Maximum Power Transfer Theorem

Derivation

$$\frac{dp}{dR_L} = V^2{}_{Th}\left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right]$$

$$= V^2{}_{Th}\left[\frac{(R_{Th}+R_L-2R_L)}{(R_{Th}+R_L)^3}\right] = 0$$

$$0 = (R_{Th}+R_L-2R_L) = (R_{Th}-R_L)$$

$$\boxed{R_L = R_{Th}}$$

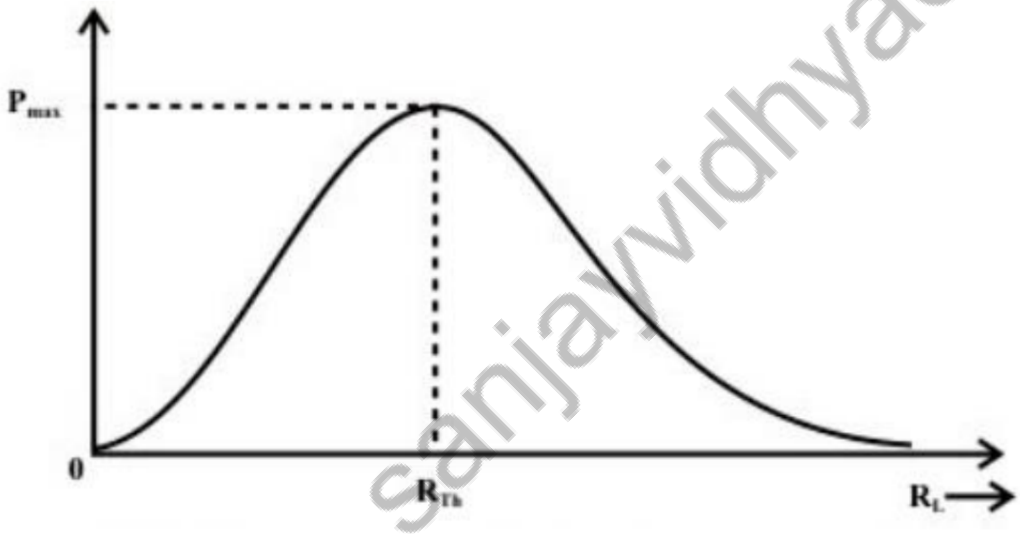
The power transferred from the source to the load is maximum when the resistance of the load is equal to the internal resistance of the source.

This condition is referred to as resistance/impedence matching.

Maximum Power Transfer Theorem

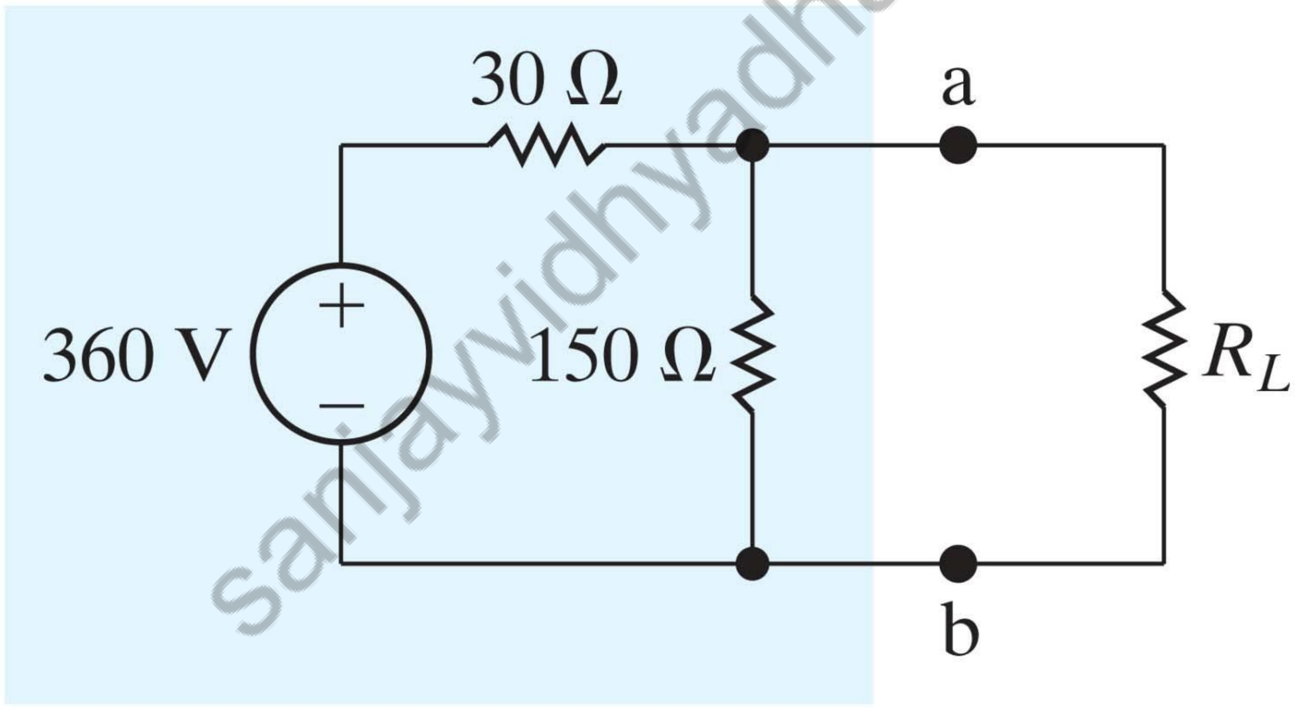
- The maximum power transferred is obtained by $p_{\max} = \dfrac{V_{Th}^{2}}{4R_{Th}}$
- The total power delivered by the source $= I_L^{2}(R_L + R_{Th}) = 2 \times I_L^{2} R_L$
- Efficiency under maximum power transfer condition is given by

$$\frac{I_L^{2} R_L}{2 \times I_L^{2} R_L} \times 100 = 50\%$$



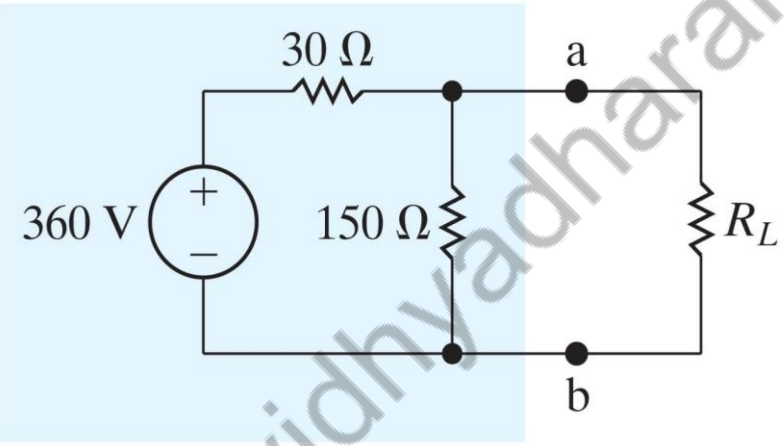
Maximum Power Transfer Theorem

Find the value of R_L for maximum power transfer to R_L.



Maximum Power Transfer Theorem

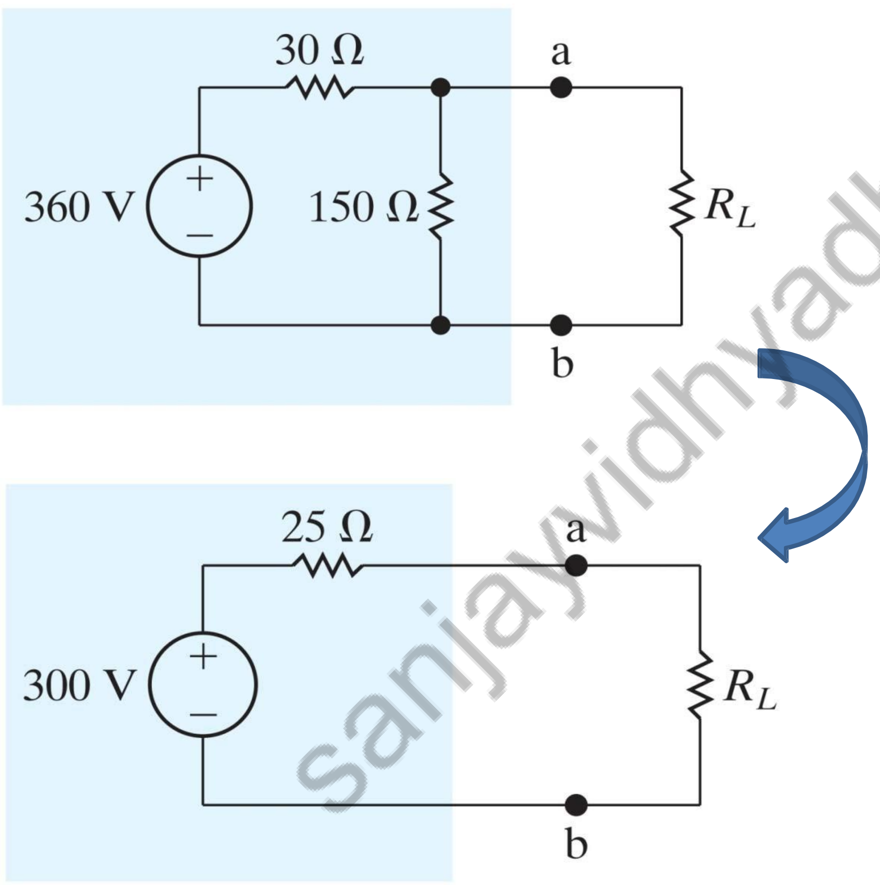
Determine the Thevenin Equivalent



$$V_{Th} = \frac{150}{180}(360) = 300V$$

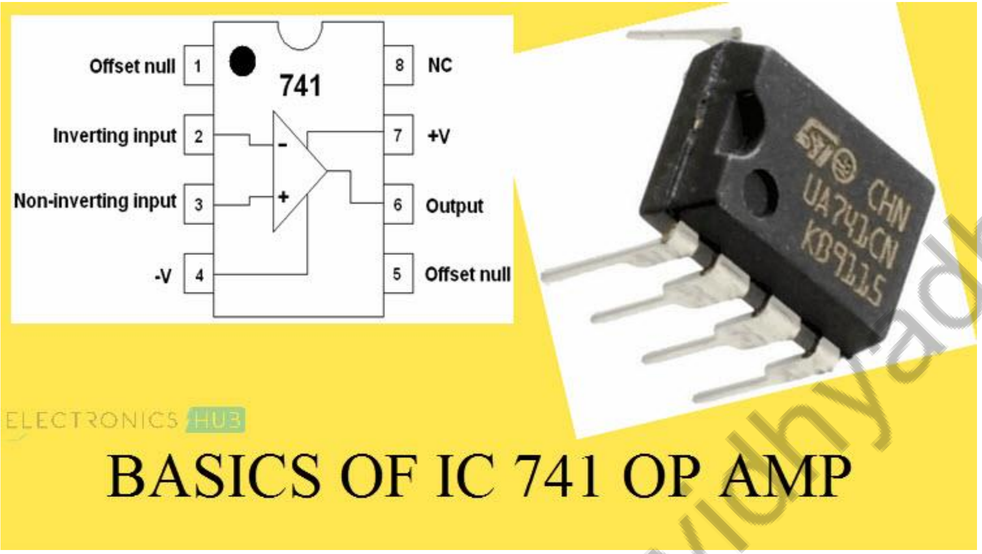
$$R_{Th} = \frac{(150)(30)}{150 + 30} = 25\Omega$$

Maximum Power Transfer Theorem



$R_L = 25\Omega$

Maximum Power Transfer Theorem



$R_{out} \cong 0$

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{th} + R_L} \right)^2 R_L$$

Thank you