Electrical Science: 2021-22

Lecture 15

AC Response for a Parallel RLC Circuits

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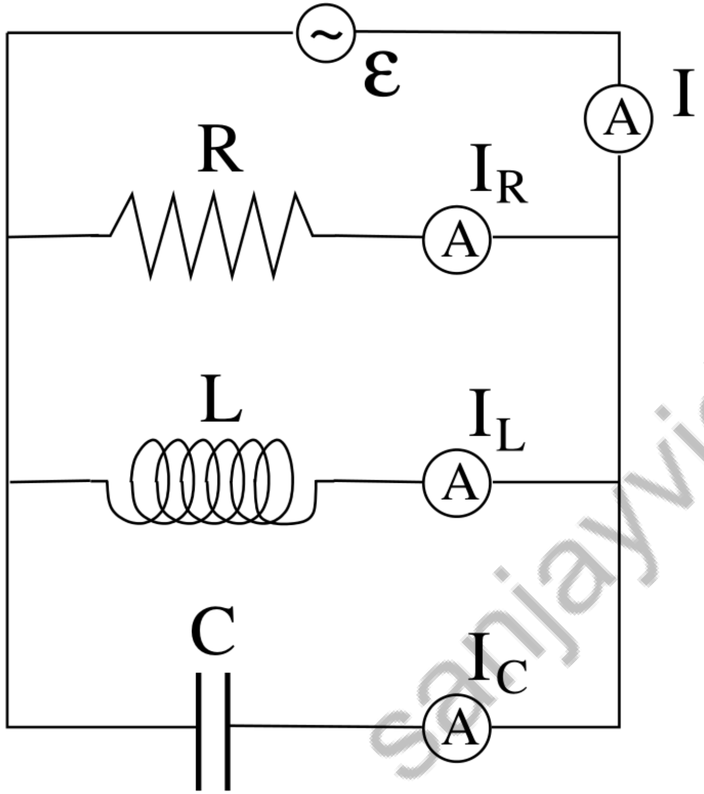
Parallel RLC Circuit



Applied alternating voltage: $\mathcal{E} = \mathcal{E}_{max} \cos \omega t$

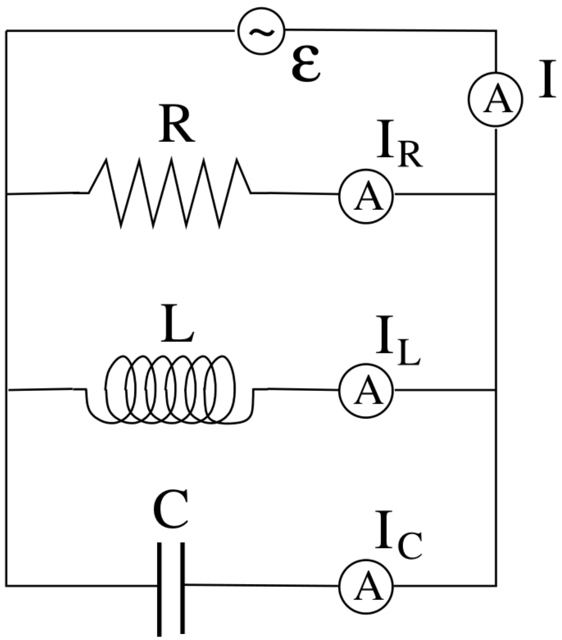
Resulting alternating current: $I = I_{max} \cos(\omega t - \delta)$

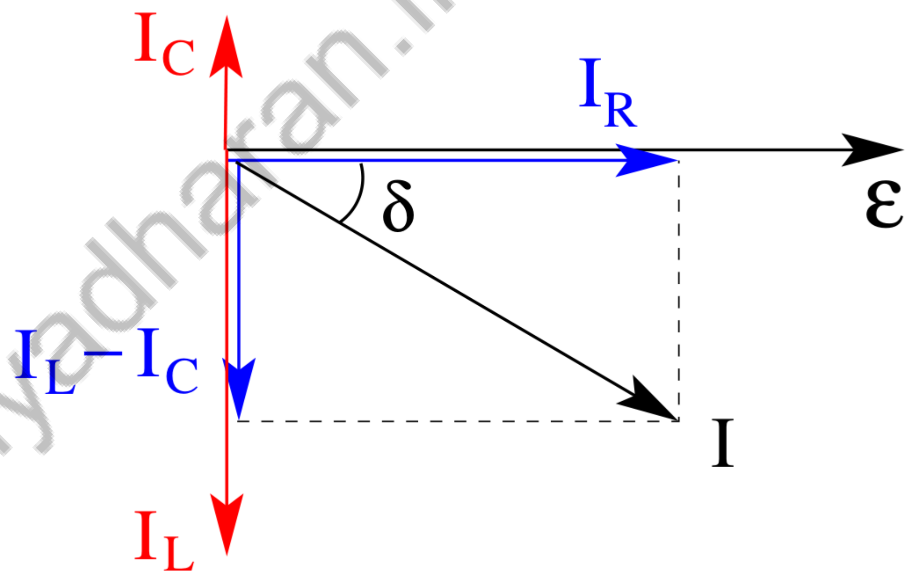
Goals:

- Find I_{max}, δ for given $\mathcal{E}_{max}, \omega$.
- Find currents I_R, I_L, I_C through devices.

Junction rule: $I = I_R + I_L + I_C$

Parallel RLC Circuit





- $I_{R,max} = \frac{\mathcal{E}_{max}}{X_R} = \frac{\mathcal{E}_{max}}{R}$
- $I_{L,max} = \frac{\mathcal{E}_{max}}{X_L} = \frac{\mathcal{E}_{max}}{\omega L}$
- $I_{C,max} = \frac{\mathcal{E}_{max}}{X_C} = \mathcal{E}_{max}\omega C$

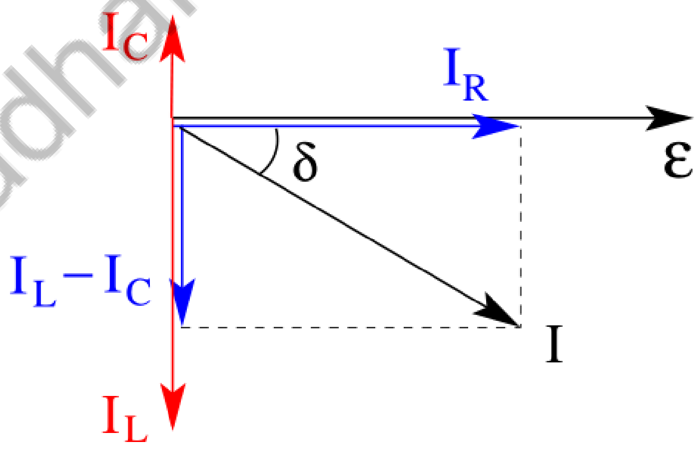
$$I_{max}^2 = I_{R,max}^2 + (I_{L,max} - I_{C,max})^2$$
$$= \mathcal{E}_{max}^2 \left[\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C \right)^2 \right]$$

Parallel RLC Circuit

Impedance: $\frac{1}{Z} \equiv \frac{I_{max}}{\mathcal{E}_{max}} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$

Current amplitude and phase angle:

- $I_{max} = \frac{\mathcal{E}_{max}}{Z} = \mathcal{E}_{max}\sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$
- $\tan\delta = \frac{I_{L,max} - I_{C,max}}{I_{R,max}} = \frac{1/\omega L - \omega C}{1/R}$

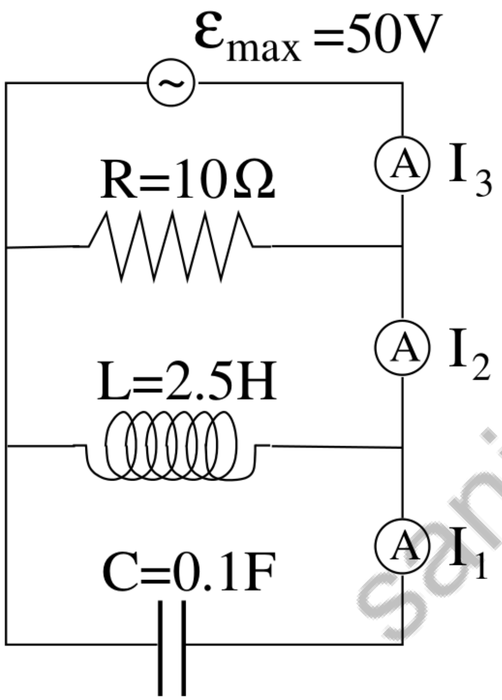


Currents through devices:

- $I_R = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}_{max}}{R}\cos(\omega t) = I_{R,max}\cos(\omega t)$
- $I_L = \frac{1}{L}\int \mathcal{E}dt = \frac{\mathcal{E}_{max}}{\omega L}\sin(\omega t) = I_{L,max}\cos\left(\omega t - \frac{\pi}{2}\right)$
- $I_C = C\frac{d\mathcal{E}}{dt} = -\omega C\mathcal{E}_{max}\sin(\omega t) = I_{C,max}\cos\left(\omega t + \frac{\pi}{2}\right)$

Parallel RLC Circuit

Example 1: Find the current amplitudes I_1, I_2, I_3
(a) for angular frequency ω = 2rad/s,
(b) for angular frequency ω = 4rad/s.



Part (b):

$$X_R \doteq R = 10\Omega, \quad X_L \doteq \omega L = 10\Omega, \quad X_C \doteq \frac{1}{\omega C} = 2.5\Omega.$$

$$I_R^{max} = \frac{\mathcal{E}_{max}}{X_R} = 5\text{A}, \quad I_L^{max} = \frac{\mathcal{E}_{max}}{X_L} = 5\text{A},$$

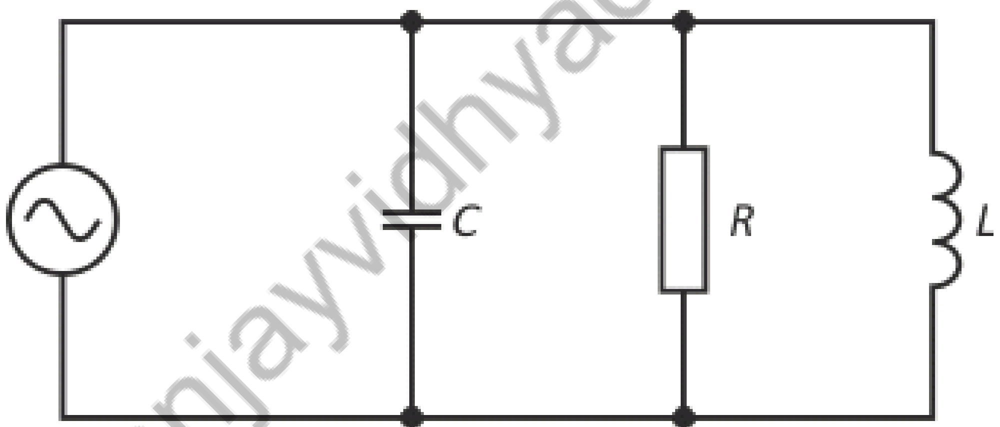
$$I_C^{max} = \frac{\mathcal{E}_{max}}{X_C} = 20\text{A}.$$

$$I_1^{max} = I_C^{max} = 20\text{A}, \quad I_2^{max} = |I_L^{max} - I_C^{max}| = 15\text{A},$$

$$I_3^{max} = \sqrt{(I_R^{max})^2 + (I_2^{max})^2} = 15.8\text{A}.$$

Parallel RLC Circuit

Example 2: A resistor, an ideal capacitor and an ideal inductor are connected in parallel to a source of alternating voltage of 160 V at a frequency of 250 Hz. A current of 2 A flows through the resistor and a current of 0.8 A flows through the inductor. The total current through the circuit is 2.5 A. Assess the resistance of the resistor, the capacity of the ideal capacitor and the inductance of the ideal inductor (presume that $I_C > I_L$).



Parallel RLC Circuit

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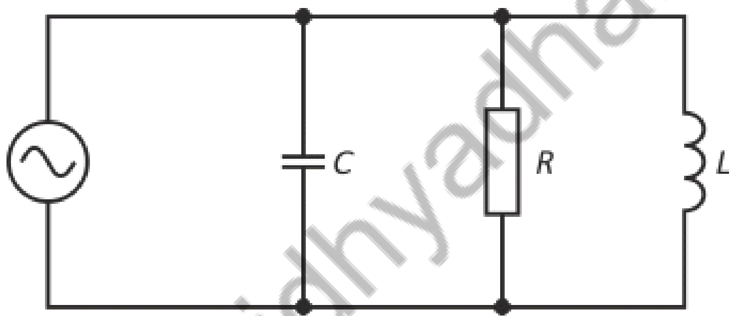


$V = 160$ V
$f = 250$ Hz
$I_R = 2$ A
$I_L = 0.8$ A
$I = 2.5$ A
$R = ?$ [Ω]
$C = ?$ [F]
$L = ?$ [H]

$$V = I_R R,$$

$$R = \frac{V}{I_R}.$$

$$R = \frac{160}{2}\,\Omega = 80\,\Omega.$$

$$V = X_L I_L = 2\pi f L\, I_L,$$

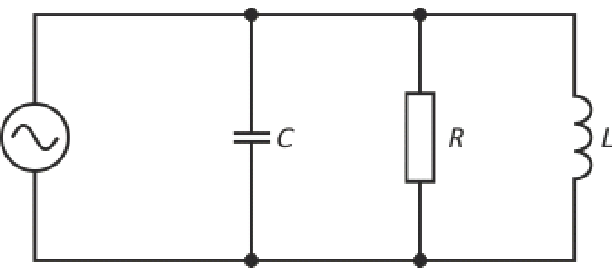
$$L = \frac{V}{2\pi f I_L},$$

$$L = \frac{160}{2\cdot\pi\cdot 250\cdot 0.8}\,\mathrm{H} \doteq 0.13\,\mathrm{H}.$$

$$V = X_C I_C = \frac{I_C}{2\pi f C},$$

$$C = \frac{I_C}{2\pi f V}.$$

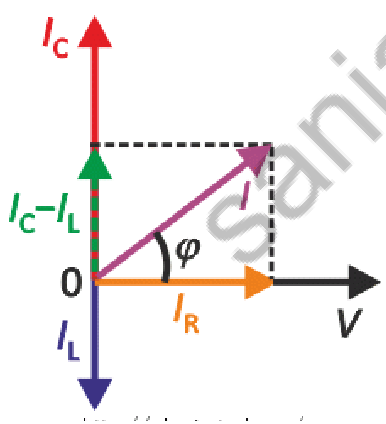
Parallel RLC Circuit



$V = 160$ V
$f = 250$ Hz
$I_R = 2$ A
$I_L = 0.8$ A
$I = 2.5$ A
$R = ?$ [Ω]
$C = ?$ [F]
$L = ?$ [H]

$$V = X_C I_C = \frac{I_C}{2\pi f C},$$

$$C = \frac{I_C}{2\pi f V}.$$



We express the relation between the currents from the phasor diagram:

$$I^2 = (I_C - I_L)^2 + I_R^2.$$

We multiply out and rearrange:

$$I^2 = I_C^2 - 2I_L I_C + I_L^2 + I_R^2,$$

$$0 = I_C^2 - 2I_L I_C + (I_L^2 + I_R^2 - I^2).$$

We solve the quadratic equation for the unknown current I_C:

$$(I_C)_{1,2} = \frac{2I_L \pm \sqrt{(2I_L)^2 - 4(I_L^2 + I_R^2 - I^2)}}{2}.$$

We rearrange:

$$(I_C)_{1,2} = I_L \pm \sqrt{(I_L)^2 - (I_L^2 + I_R^2 - I^2)},$$

$$(I_C)_{1,2} = I_L \pm \sqrt{I^2 - I_R^2}.$$

And we use the assigned values:

$$(I_C)_1 = 0.8 + \sqrt{2.5^2 - 2^2}\,\text{A} = 2.3\,\text{A},$$

$$(I_C)_2 = 0.8 - \sqrt{2.5^2 - 2^2}\,\text{A} = -0.7\,\text{A}.$$

Only the positive value of the effective current $I_C = 2.3$ A is physically meaningful in our case.

We substitute this result into the equation for the capacity of the capacitor (1) that we derived above from Ohm's law:

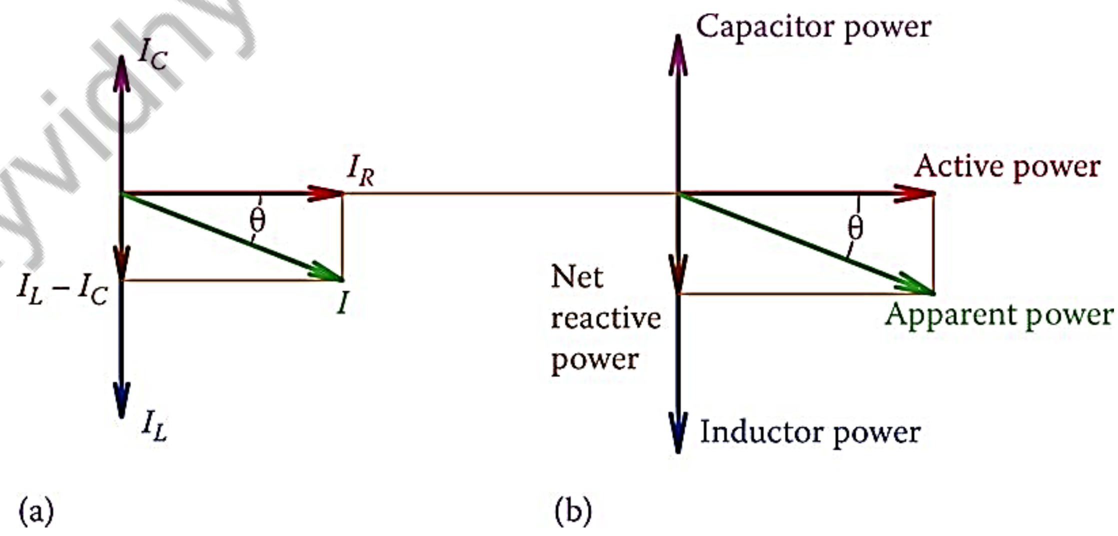
$$C = \frac{I_C}{2\pi f U} = \frac{2.3}{2 \cdot \pi \cdot 250 \cdot 160}\,\text{F} \doteq 9.2 \cdot 10^{-6}\,\text{F} = 9.2\,\mu\text{F}.$$

Power in RLC Parallel AC Circuits

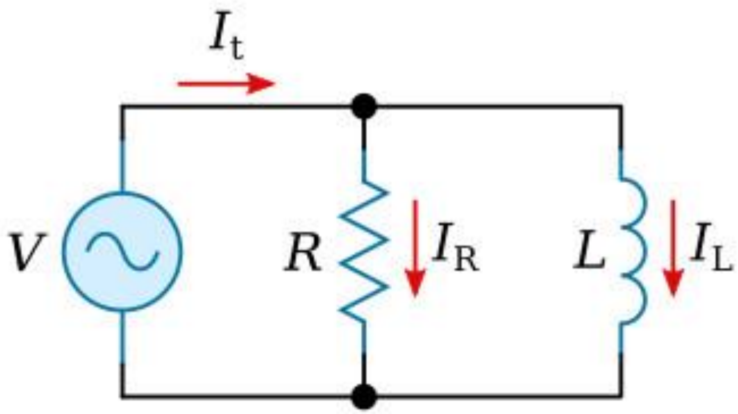
If current varies with frequency in an *RLC* circuit, then the power delivered to it also varies with frequency. But the average power is not simply current times voltage, as it is in purely resistive circuits. As was seen in Figure 2, voltage and current are out of phase in an *RLC* circuit. There is a *phase angle* ϕ between the source voltage V and the current I, which can be found from

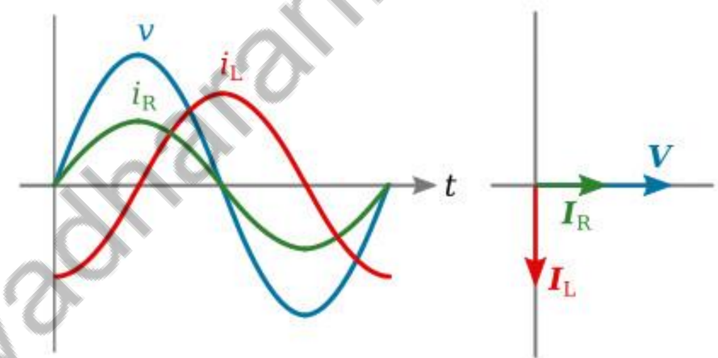
$$\cos\varphi = \frac{R}{Z}$$

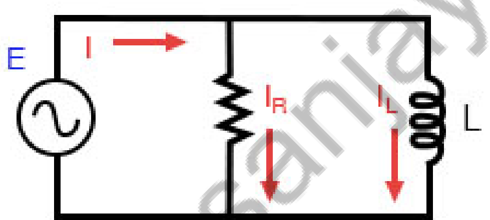
$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos\varphi$$

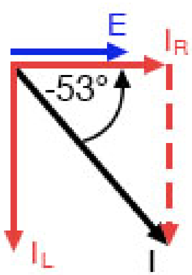


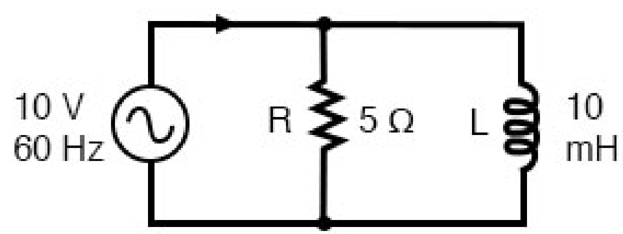
RL Parallel Circuit











$I_T = I_R + I_L$

$E = E_R = E_L$

RL Parallel Circuit

Example 3: Determine the phase angle, $\boldsymbol{Z}_t$, $\boldsymbol{I}_t$, $\boldsymbol{I}_R$, $\boldsymbol{I}_L$, P in a parallel RL circuit containing a 1.4 mH coil, a 25 Ω resistor, and a 10 V source operating at 4 kHz.
Solution: Determine the reactance of coil X_L:

$$X_L = \omega L = 2\pi f L$$

$$X_L = 6.28 \times 4 \times 10^3 \times 1.4 \times 10^{-3} = 35.17\ \Omega$$

Determine current in branch one ($\boldsymbol{I}_R$):

$$\boldsymbol{I}_R = \frac{\boldsymbol{V}}{R}$$

$$\boldsymbol{I}_R = \frac{10}{25} = 0.4\ \text{A}$$

Determine current in branch two ($\boldsymbol{I}_L$): $\boldsymbol{I}_L = \dfrac{\boldsymbol{V}}{jX_L}$ $\quad \boldsymbol{I}_L = \dfrac{10}{j35.17} = -j0.284\ \text{A}$

RL Parallel Circuit

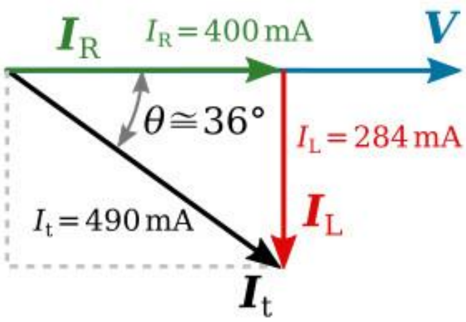
Example 3: Determine the phase angle, $\boldsymbol{Z}_t$, $\boldsymbol{I}_t$, $\boldsymbol{I}_R$, $\boldsymbol{I}_L$, P in a parallel RL circuit containing a 1.4 mH coil, a 25 Ω resistor, and a 10 V source operating at 4 kHz.
Solution: Determine the reactance of coil X_L:

$$\boldsymbol{I}_t = 0.4 - j0.284 = 0.491\angle{-35.4°}$$

This value agrees very closely with the approximation made from the phasors.
Determine total impedance:

$$\boldsymbol{Z}_t = \frac{\boldsymbol{V}}{\boldsymbol{I}_t}$$

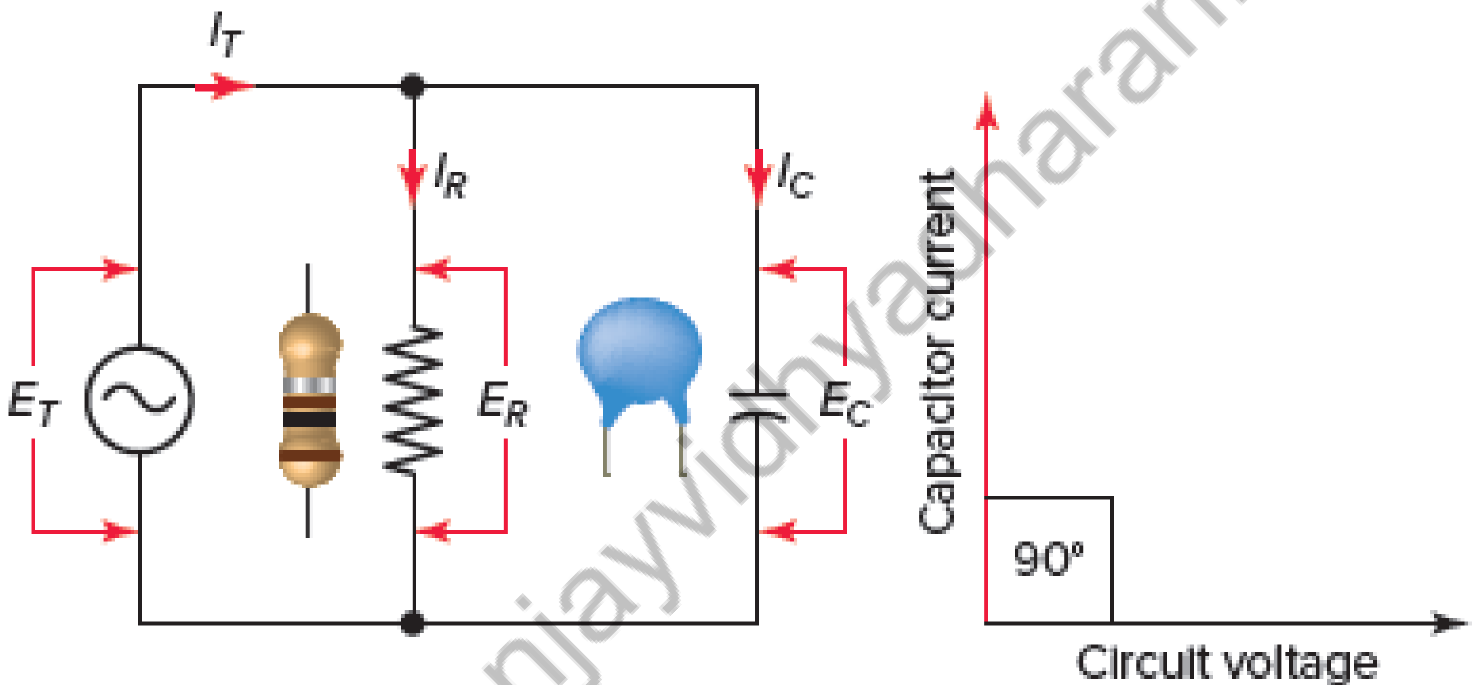
$$\boldsymbol{Z}_t = \frac{10\angle 0°}{0.491\angle{-35.4°}} = 20.37\angle 35.4°\ \Omega$$



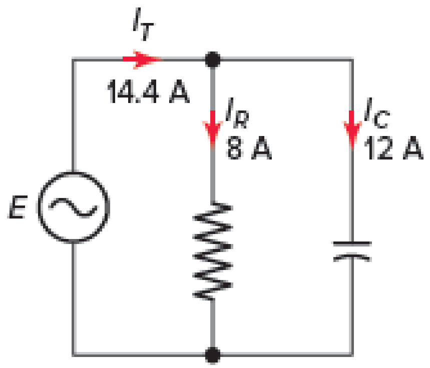
Determine the real power dissipated by the circuit:

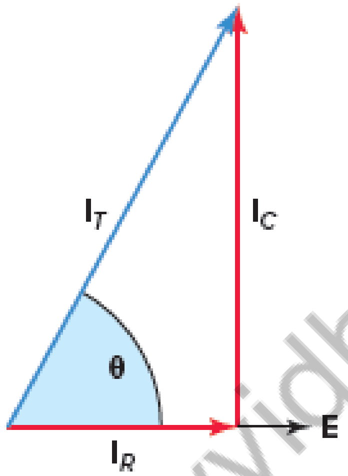
$$P = V \times I_t \times \cos\theta \quad P = 10 \times 0.491 \times \cos 35.4° = 4\ \text{W}$$

RC Parallel Circuit



RC Parallel Circuit



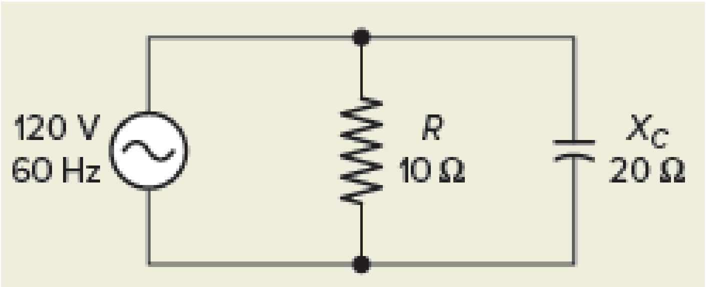


- The ***reference vector is labeled E*** and represents the voltage in the circuit, which is common to all elements.
- Since the current through the resistor is in phase with the voltage across it, I_R (8 A) is shown superimposed on the voltage vector.
- The capacitor current I_C (12 A) leads the voltage by 90 degrees and is positioned in an upward direction, leading the voltage vector by 90 degrees.
- The vector addition of I_R and I_C gives a resultant that represents the total (IT) or line current (14.4 A).

$$\theta = \tan^{-1} \frac{I_C}{I_R}$$

$$\begin{aligned} I_T &= \sqrt{I_R^2 + I_C^2} \\ &= \sqrt{8^2 + 12^2} \\ &= \sqrt{208} \\ &= 14.4 \text{ A} \end{aligned}$$

RC Parallel Circuit



$$a.\ I_R = \frac{E}{R} = \frac{120V}{10\Omega} = 12A$$

$$b.\ I_C = \frac{E}{X_C} = \frac{120V}{20\Omega} = 6A$$

$$c.\ I_T = \sqrt{I_R^2 + I_C^2} = \sqrt{12^2 + 6^2} = 13.4A$$

$$d.\ \theta = \tan^{-1}\left(\frac{I_C}{I_R}\right) = \tan^{-1}\left(\frac{6}{12}\right) = 26.6^o$$

$$e.\ I_T = 13.4\angle 26.6^{oo} \quad I_R = 12\angle 0^o \quad I_C = 6\angle 90^o$$

$$f.\ I_T = 12 + j6 \quad I_R = 12 + j0 \quad I_C = 0 + j6$$

Thank you