Electrical Science: 2021-22

Lecture 14

AC Response for a Series RLC Circuits

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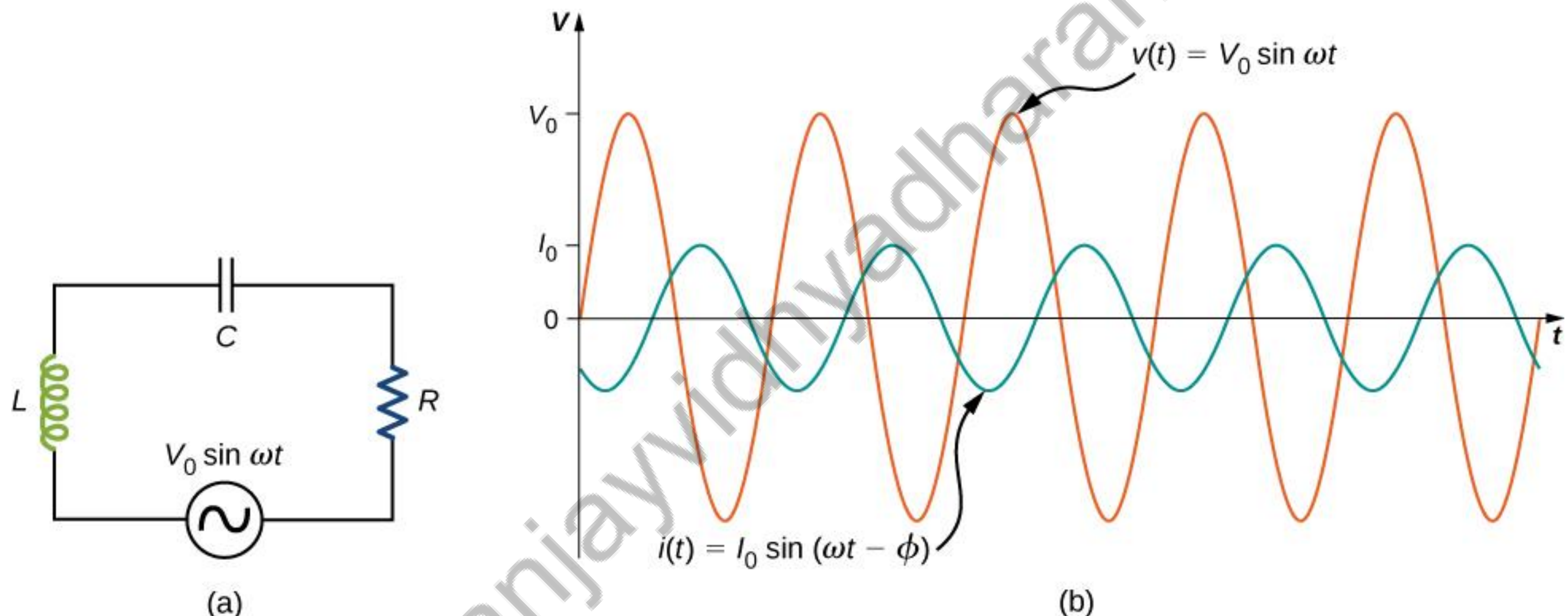
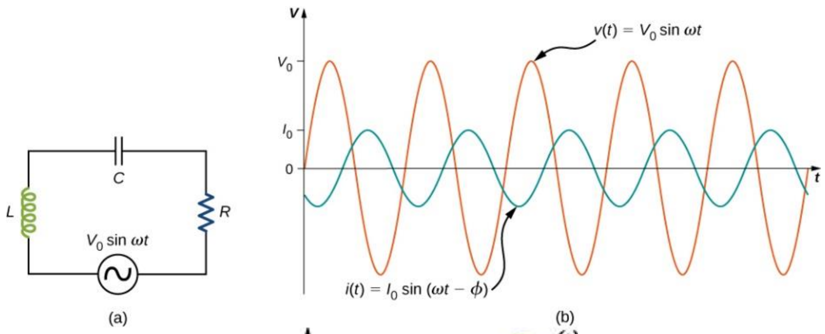
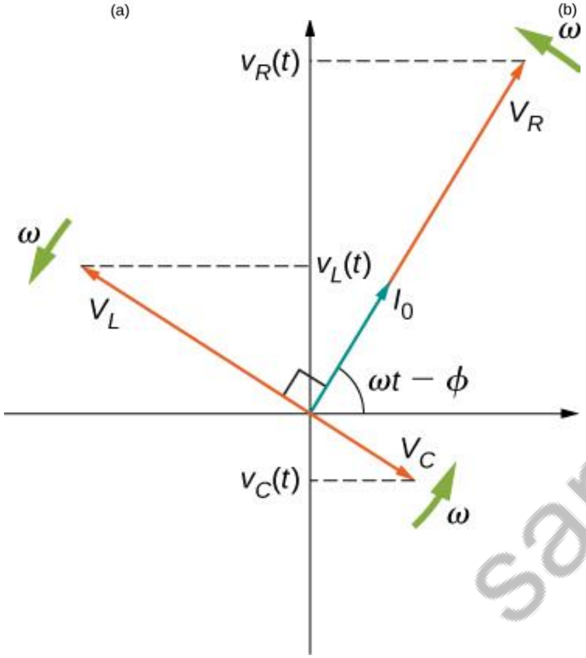
Series RLC Circuit



Series RLC Circuit





$$v_R(t) + v_L(t) + v_C(t) = v(t) = V_0 \sin \omega t.$$

$$\varphi = \tan^{-1}\frac{V_L - V_C}{V_R} = \tan^{-1}\frac{I_0 X_L - I_0 X_C}{I_0 R},$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}.$$

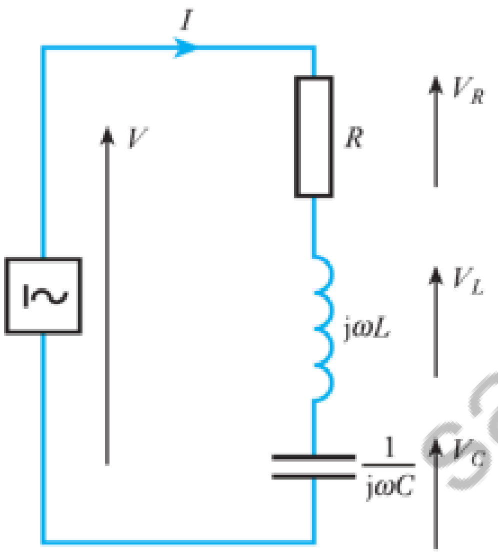
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$V_0 = \sqrt{V_R{}^2 + (V_L - V_C)^2} = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2} = I_0\sqrt{R^2 + (X_L - X_C)^2}.$$

Series RLC Circuit

Variation of Magnitude and Phase of Current with Frequency

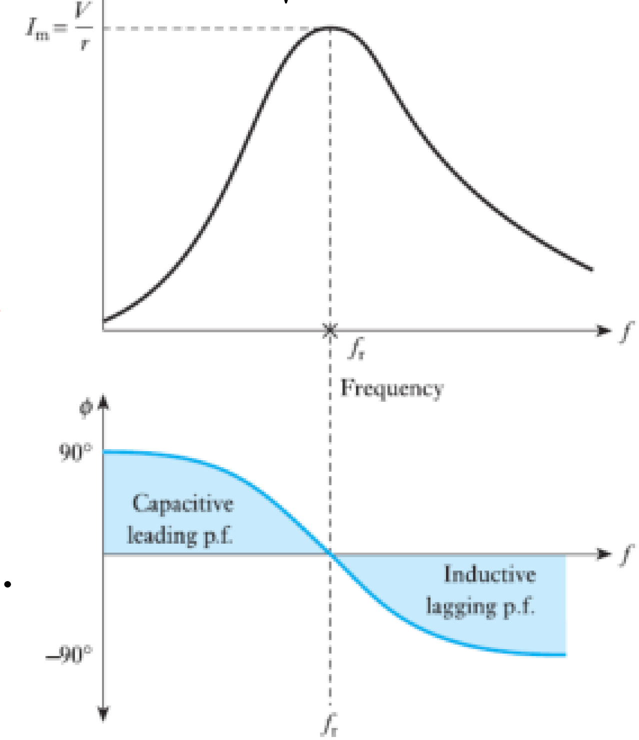
- The current is maximum at resonant frequency (f_r).



Variation of magnitude $|I|$ and phase ϕ of current with frequency in a series RLC circuit

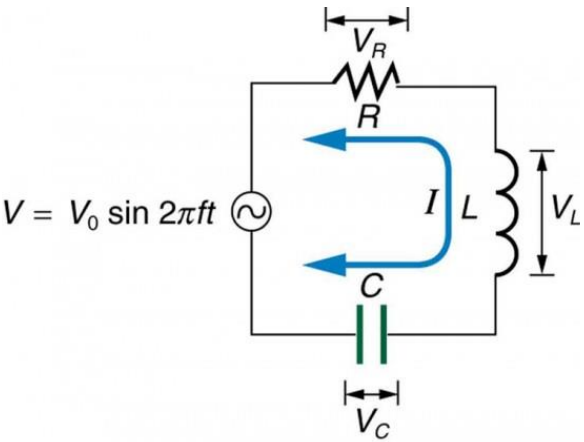
$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}.$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



Series RLC Circuit

Example 1: An *RLC* series circuit has a 40.0 Ω resistor, a 3.00 mH inductor, and a 5.00 μF capacitor. (a) Find the circuit's impedance at 60.0 Hz and 10.0 kHz, (b) If the voltage source has V_{rms} = 120 V, what is I_{rms} at each frequency?



At 60.0 Hz

$$X_L = 2\pi f L = 6.28(60.0/\text{s})(3.00\ \text{mH}) = 1.13\ \Omega\ .$$

$$X_C = 1\ /\ 2\pi f C = 1/\ 6.28(60.0/\text{s})(5.00\ \mu\text{F}) = 531\ \Omega$$

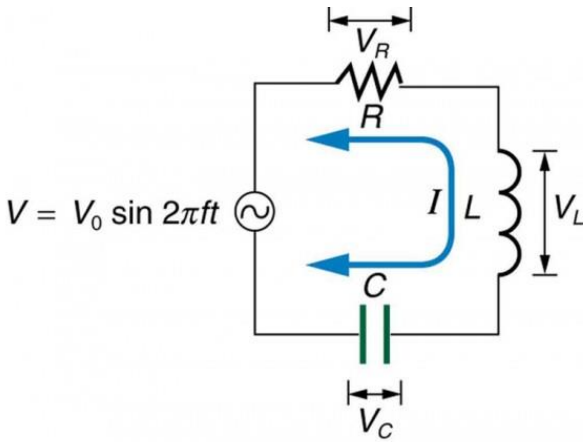
At 10 KHz

$$X_L = 2\pi f L = 6.28(1.00 \times 10^4/\text{s})(3.00\ \text{mH}) = 188\ \Omega$$

$$X_C = 1/2\pi f C = 1/6.28(1.00\times104/\text{s})(5.00\mu\text{F}) = 3.18\ \Omega$$

Series RLC Circuit

Example 1: An *RLC* series circuit has a 40.0 Ω resistor, a 3.00 mH inductor, and a 5.00 μF capacitor. (a) Find the circuit's impedance at 60.0 Hz and 10.0 kHz, (b) If the voltage source has V_{rms} = 120 V, what is I_{rms} at each frequency?



$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

yields

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(40.0\ \Omega)^2 + (1.13\ \Omega - 531\ \Omega)^2} \\ &= 531\ \Omega \text{ at } 60.0 \text{ Hz} \end{aligned}$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{120\text{V}}{531\ \Omega} = 0.226 \text{ A}$$

Similarly, at 10.0 kHz, X_L = 188 Ω and X_C = 3.18 Ω, so that

$$\begin{aligned} Z &= \sqrt{(40.0\ \Omega)^2 + (188\ \Omega - 3.18\ \Omega)^2} \\ &= 190\ \Omega \text{ at } 10.0 \text{ kHz} \end{aligned}$$

Finally, at 10.0 kHz, we find

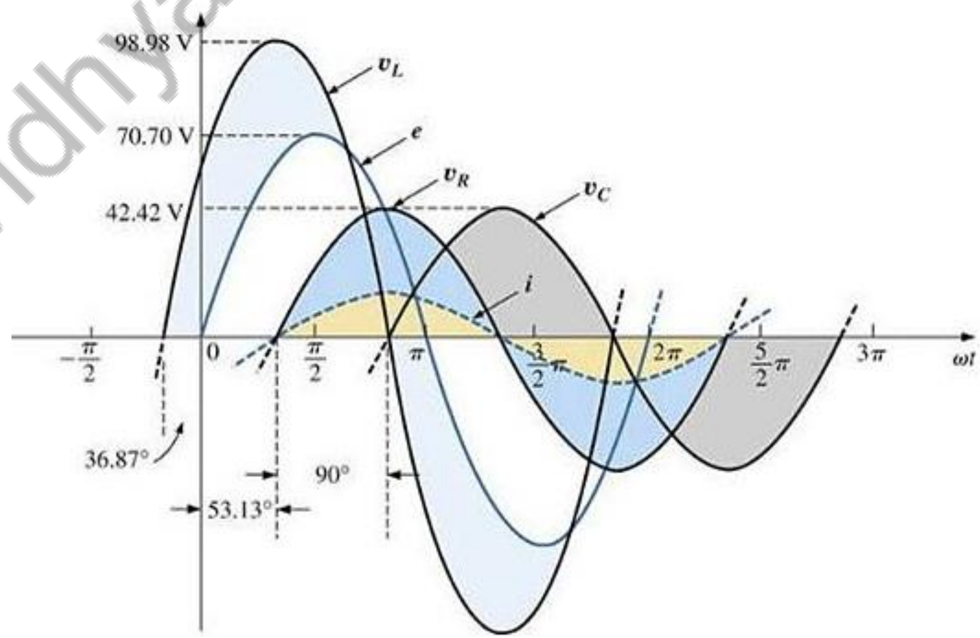
$$I_{rms} = \frac{V_{rms}}{Z} = \frac{120 \text{ V}}{190\ \Omega} = 0.633 \text{ A}$$

Power in RLC Series AC Circuits

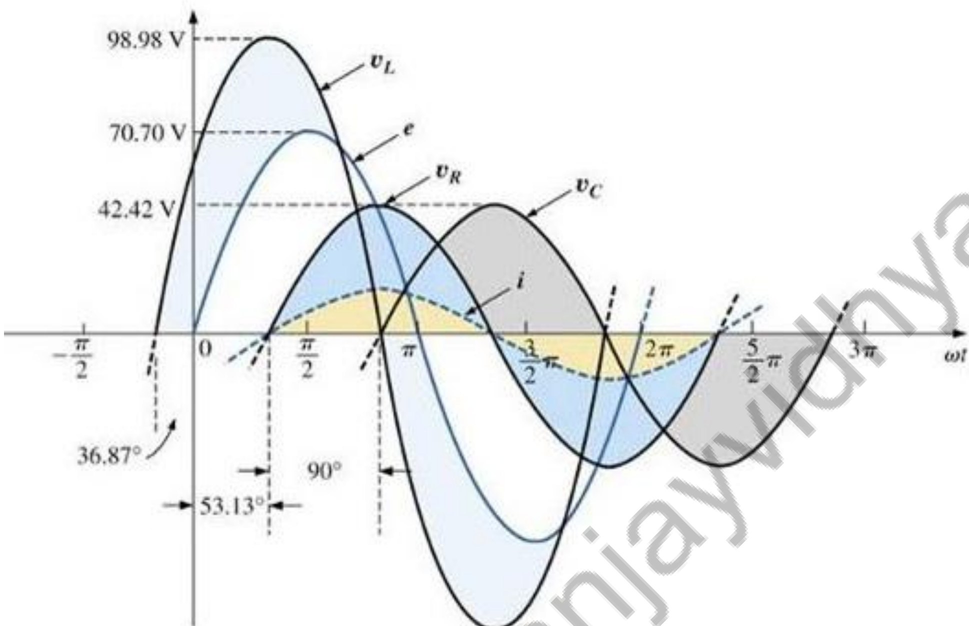
If current varies with frequency in an *RLC* circuit, then the power delivered to it also varies with frequency. But the average power is not simply current times voltage, as it is in purely resistive circuits. As was seen in Figure 2, voltage and current are out of phase in an *RLC* circuit. There is a *phase angle* ϕ between the source voltage V and the current I, which can be found from

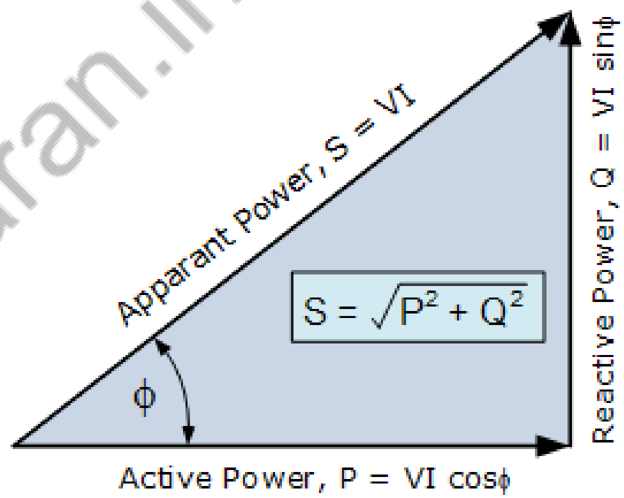
$$\cos\varphi = \frac{R}{Z}$$

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos\varphi$$



Power in RLC Series AC Circuits





PF = P / S

Equation 1

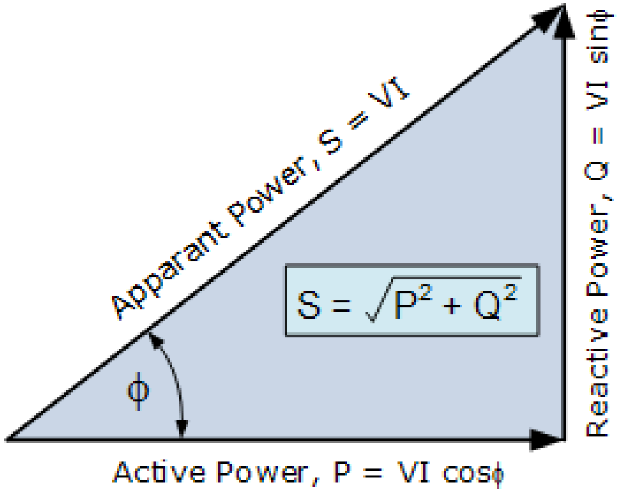
Where:

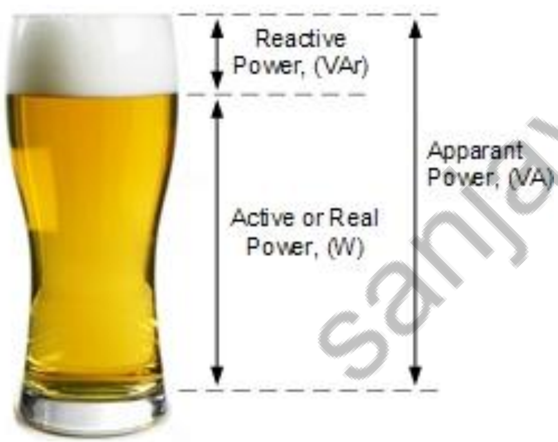
PF = power factor

P = active (real) power (kW)

S = apparent power (VA, volts amps)

Power in RLC Series AC Circuits





Basis for Comparison	Active Power	Reactive Power
Definition	The active power is the real power which is dissipated in the circuit.	The power which moves back and froth between the load and source such type of power is known as the reactive power
Formula	$P = VIcos\emptyset$	$Q = VIsin\emptyset$
Measuring Unit	Watts	VAR
Represented By	P	Q
Causes	Produces heat in heater, light in lamps and torque in motor.	Measures the power factor of the circuit.
Measuring Instrument	Wattmeter	VAR Meter

Reactive currents cause power dissipation in the transmission lines

Calculating the Power Factor and Power

Example 2 For the same *RLC* series circuit having a 40.0 Ω resistor, a 3.00 mH inductor, a 5.00 μF capacitor, and a voltage source with a V_{rms} of 120 V: (a) Calculate the power factor and phase angle for f = 60.0Hz. (b) What is the average power at 60 Hz?

$$\cos\varphi = \frac{R}{Z}$$

.

We know Z = 531 Ω from *Example 1: Calculating Impedance and Current*, so that

$$\cos\varphi = \frac{40.0\ \Omega}{531\ \Omega} = 0.0753 \text{ at } 60.0 \text{ Hz}$$

.

This small value indicates the voltage and current are significantly out of phase. In fact, the phase angle is

$$\varphi = \cos^{-1}0.0753 = 85.7^{\circ} \text{ at } 60.0 \text{ Hz}$$

The average power at 60.0 Hz is

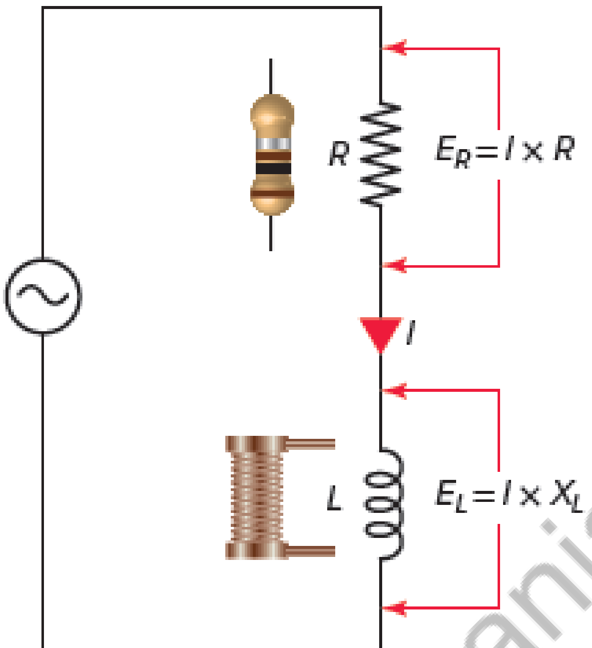
$$P_{ave} = I_{rms}V_{rms}\cos\phi.$$

I_{rms} was found to be 0.226 A in *Example 1: Calculating Impedance and Current*. Entering the known values gives

$$P_{ave} = (0.226 \text{ A})(120 \text{ V})(0.0753) = 2.04 \text{ W at } 60.0 \text{ Hz}.$$

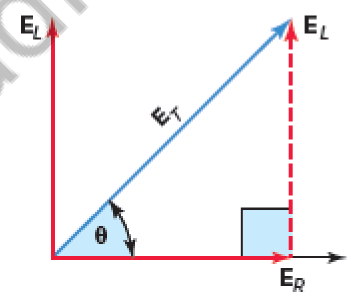
RL Series Circuit

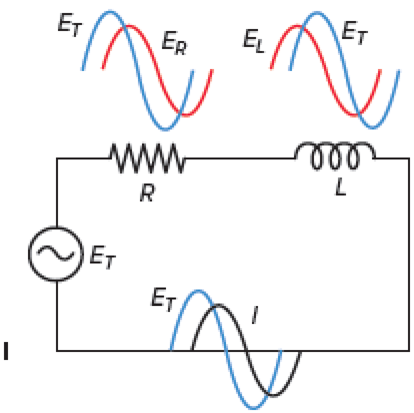
$$E_T = \sqrt{E_R^2 + E_L^2}$$



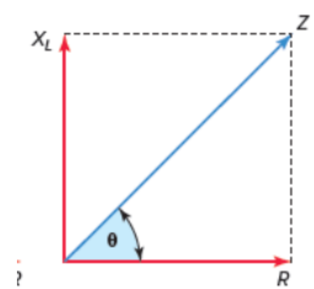
$$E_R = I \times R$$

$$E_L = I \times X_L$$





$$Z = \frac{E_T}{I}$$

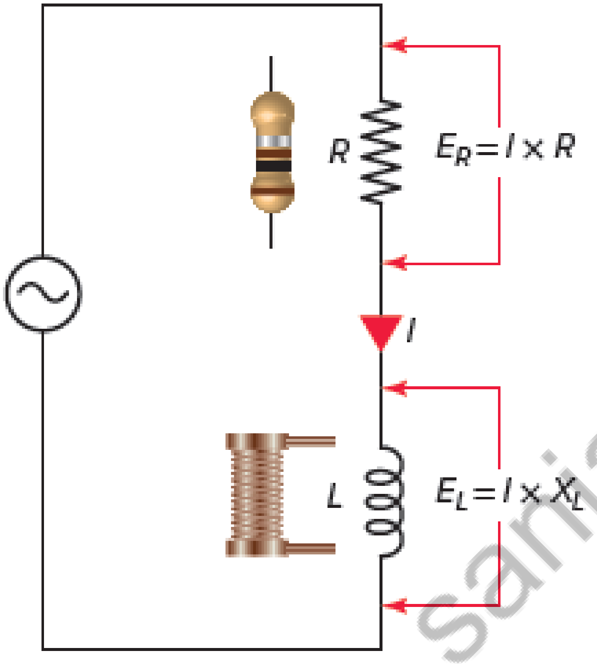


$$Z = \sqrt{R^2 + X_L^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

RL Series Circuit

Example 3: An AC series *RL* circuit is made up of a resistor that has a resistance value of 150 Ω and an inductor that has an inductive reactance value of 100 Ω. Calculate the impedance and the phase angle theta (θ) of the circuit.



$$Z = \sqrt{R^2 + X_L^2}$$
$$= \sqrt{150^2 + 100^2}$$
$$= \sqrt{32{,}500}$$
$$= 180\ \Omega$$

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$
$$= \tan^{-1}\left(\frac{100}{150}\right)$$
$$= \tan^{-1}(0.667)$$
$$= 33.7°$$

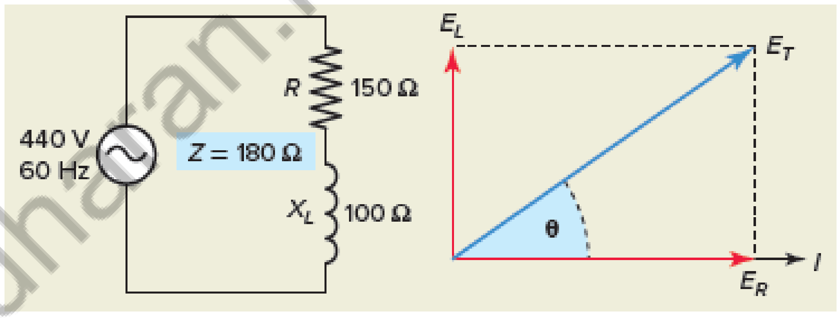
or

$$Z = R + jX_L$$
$$= 150 + j100$$
$$= 180\ \Omega \angle 33.7°$$

RL Series Circuit

Example 4:

1.Calculate the value of the current flow.
2.Calculate the value of the voltage drop across the resistor.
3.Calculate the value of the voltage drop across the inductor.
4.Calculate the circuit phase angle based on the voltage drops across the resistor and inductor.
5.Express all voltages in polar notation.
6.Use a calculator to convert all voltages to rectangular notation.



$$a.\ I = \frac{E_T}{Z} = \frac{440V}{180\Omega} = 2.44A$$

$$b.\ E_R = I \times R = 2.44A \times 150\Omega = 366V$$

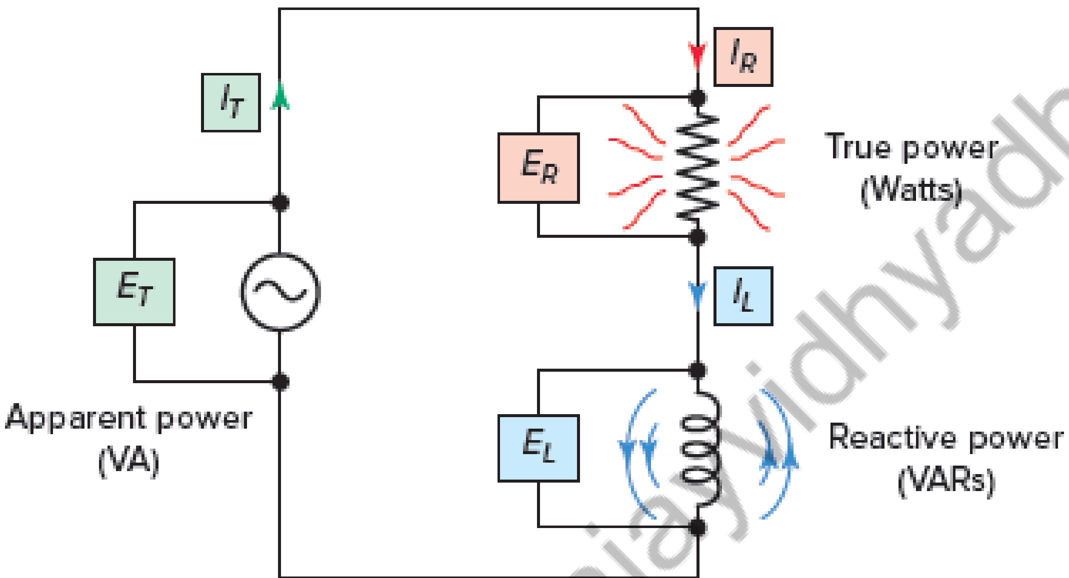
$$c.\ E_L = I \times X_L = 2.44A \times 100\Omega = 244V$$

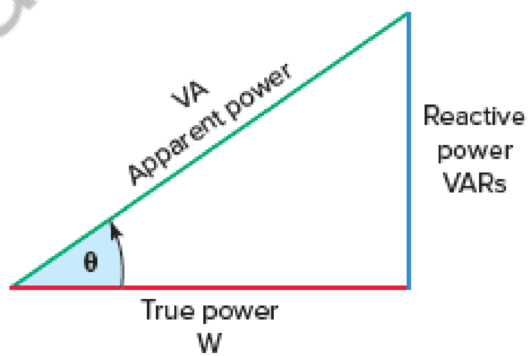
$$d.\ \theta = \tan^{-1}\left(\frac{E_L}{E_R}\right) = \tan^{-1}\left(\frac{244\ V}{366\ V}\right) = \tan^{-1}(0.667) = 33.7°$$

$$e.\ E_T = 440V\angle 33.7° \quad E_R = 366V\angle 0° \quad E_L = 244V\angle 90°$$

$$f.\ E_T = 360 + j24V \quad E_R = 366 + j0V \quad E_L = 0 + j244V$$

Power in RL Series Circuit

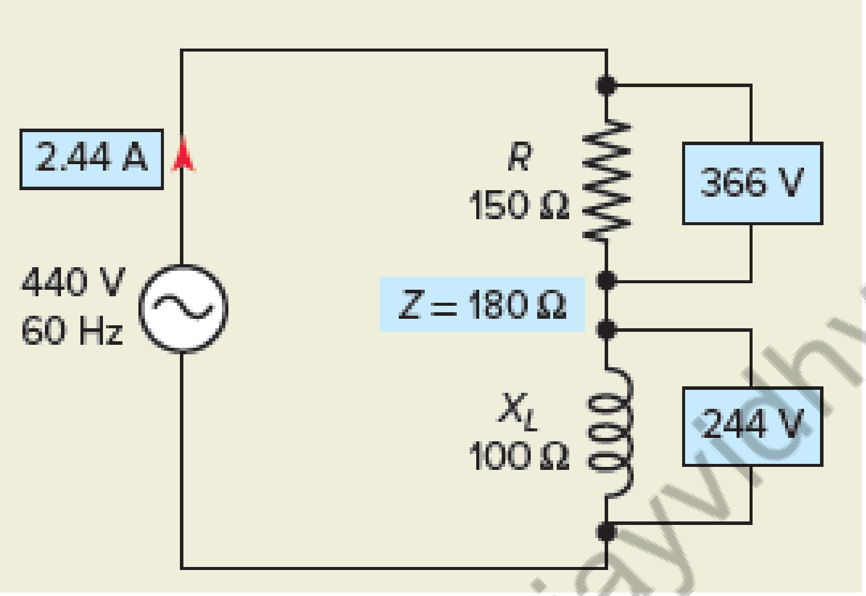




Series *RL* circuit power triangle.

Power in RL Series Circuit

Example 5:



$$\text{True power} = E_R \times I_R = 366V \times 2.44A = 893\ W$$

$$\text{Inductive power} = E_L \times I_L = 244V \times 2.44A = 595\ VARs$$

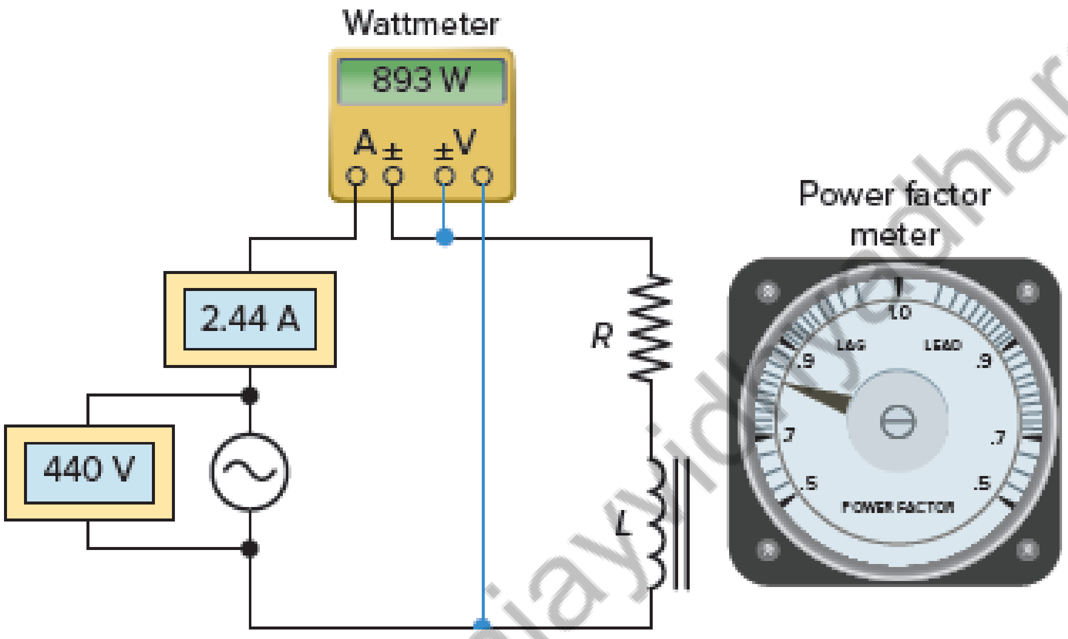
$$\text{Capactive power} = E_R \times I_T = 440V \times 2.44A = 1074\ VA$$

The **power factor (PF)** for any AC circuit is the ratio of the true power (also called real power) to the apparent power:

$$PF = \frac{\text{watts (W)}}{\text{volt-amperes (VA)}} = \frac{\text{true power}}{\text{apparent power}} = \cos \angle\theta$$

Power in RL Series Circuit

Example 5



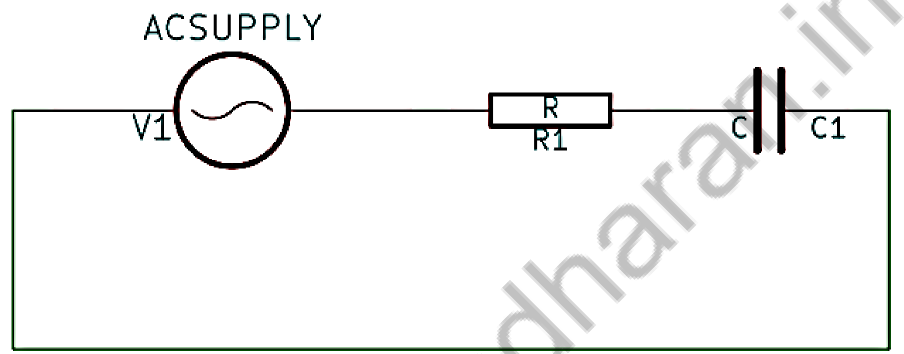
$$PF = \frac{\text{watts (W)}}{\text{volt-amperes (VA)}}$$

$$= \frac{893\ \text{W}}{2.44\ \text{A} \times 440\ \text{V}}$$

$$= 0.832$$

$$= 83.2\%\ \text{(lagging)}$$

RC Series Circuit

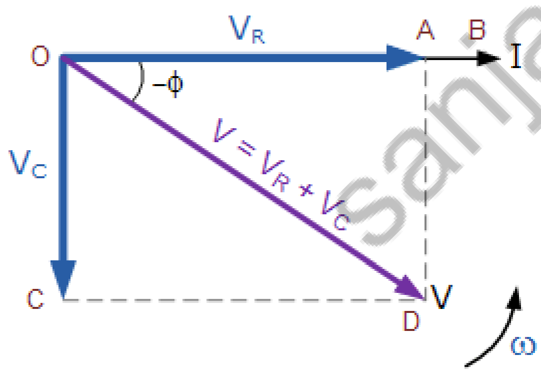


$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

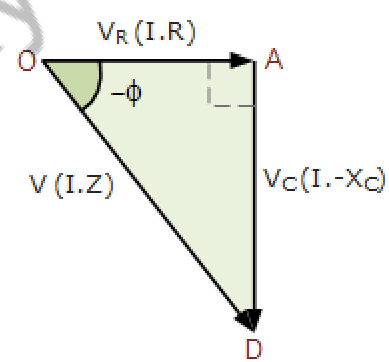
$$Z^2 = R^2 + X_C^2 .$$

$$\tan\phi = \frac{1}{RC\omega} = \frac{X_C}{R} .$$

Vector Diagram



Voltage Triangle



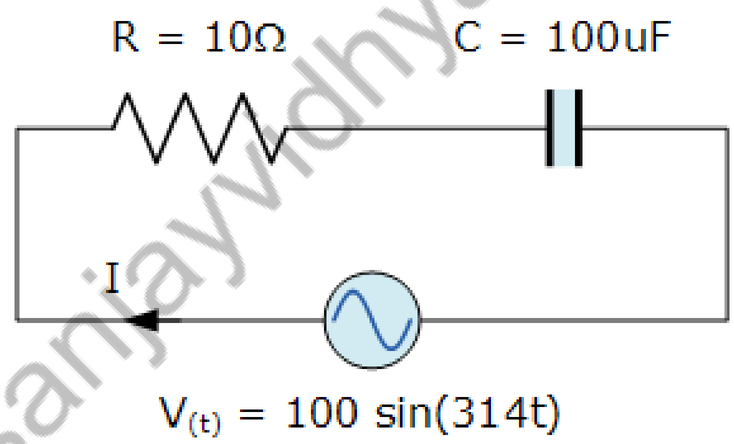
$$V = \sqrt{(I.R)^2 + (I.X_C)^2}$$

$$\therefore I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

RC Series Circuit

Example 6 :

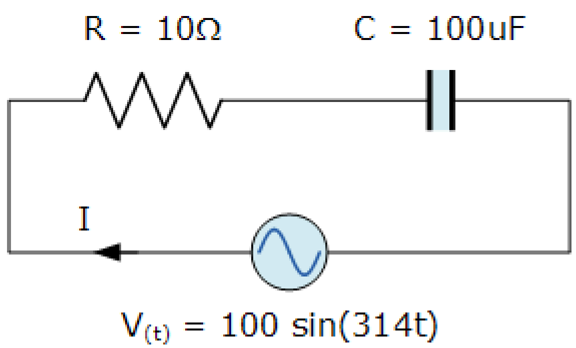
A capacitor which has an internal resistance of 10Ω and a capacitance value of 100uF is connected to a supply voltage given as $V_{(t)} = 100 \sin(314t)$. Calculate the peak current flowing into the capacitor. Also construct a voltage triangle showing the individual voltage drops.



RC Series Circuit

Example 6 :

A capacitor which has an internal resistance of 10Ω and a capacitance value of 100uF is connected to a supply voltage given as $V_{(t)} = 100 \sin(314t)$. Calculate the peak current flowing into the capacitor. Also construct a voltage triangle showing the individual voltage drops.



The capacitive reactance and circuit impedance is calculated as:

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100uF} = 31.85\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^2 + 31.85^2} = 33.4\Omega$$

Then the current flowing into the capacitor and the circuit is given as:

$$I = \frac{V_C}{Z} = \frac{100}{33.4} = 3\text{Amps}$$

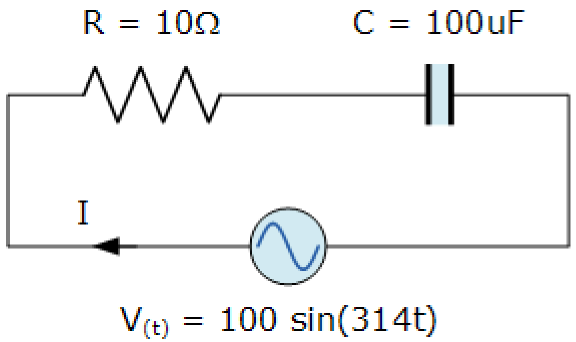
The phase angle between the current and voltage is calculated from the impedance triangle above as:

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \frac{31.85}{10} = 72.6^\circ \text{ leading}$$

RC Series Circuit

Example 6 :

A capacitor which has an internal resistance of 10Ω and a capacitance value of 100uF is connected to a supply voltage given as $V_{(t)} = 100 \sin(314t)$. Calculate the peak current flowing into the capacitor. Also construct a voltage triangle showing the individual voltage drops.



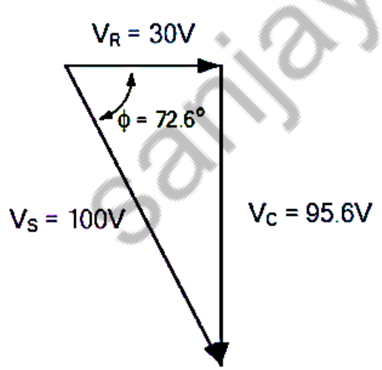
Then the individual voltage drops around the circuit are calculated as:

$$V_R = I \times R = 3 \times 10 = 30V$$

$$V_C = I \times X_C = 3 \times 31.85 = 95.6V$$

$$V_S = \sqrt{V_R^2 + V_C^2} = \sqrt{30^2 + 95.6^2} = 100V$$

Then the resultant voltage triangle for the calculated peak values will be:



Thank you