



Digital Design

Lecture 5: Two and Three Variable K-Maps

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Why to use K-Maps?

Complexity of implementation related to **expression**

Truth table unique but many algebraic forms

Boolean expressions can be **simplified by algebraic means**

Algebraic method **lacks specific rules**

K-map method simple, straight forward and pictorial understanding

K-Maps

A K-map made up of squares, each square represents a minterm/maxterm

Number of squares depend on number of input combinations

Modified form of Truth Tables

Simplified expressions are in two of the standard forms (SOP or POS)

2-Variable K-Map

Truth-table to K-map

	X	Y	F
minterm 0 →	0	0	1
minterm 1 →	0	1	0
minterm 2 →	1	0	1
minterm 3 →	1	1	1

X \ Y	0	1
0	(0,0) 1	(0,1) 0
1	(1,0) 1	(1,1) 1

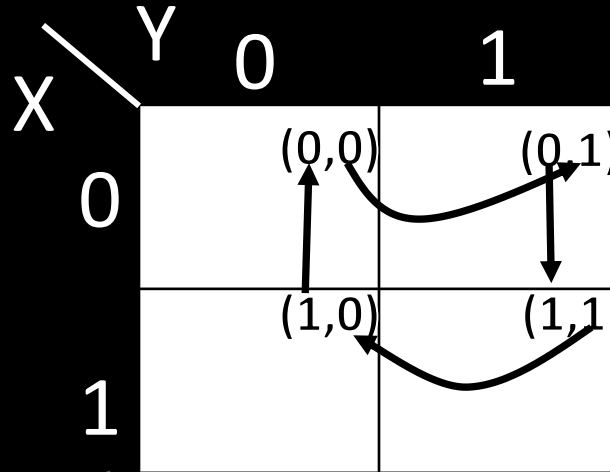
2-Variable K-Map

Representation of cells

		Y	
		0	1
X	0	$m_0 \quad (\bar{x}, \bar{y})$ 1	$m_1 \quad (\bar{x}, y)$ 0
	1	$m_2 \quad (x, \bar{y})$ 1	$m_3 \quad (x, y)$ 1

2-Variable K-Map

One bit variation between adjacent cells



What is the importance of 1-bit variation ??

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	0

		Y	
		0	1
X	0	1	1
	1	0	0

Diagram illustrating the K-map simplification process. The K-map is a 2x2 grid with variables X and Y. The top row (X=0) contains 1s in both columns (Y=0 and Y=1). The bottom row (X=1) contains 0s in both columns. A horizontal arrow points from the two 1s in the top row to the expression $X'Y'$. A vertical arrow points from the two 1s in the top row to the expression $X'Y$. The simplified expression is $X'Y' + X'Y = X'(Y' + Y) = X'$.

$$X'Y' + X'Y = X'(Y' + Y) = X'$$

If there are 1's in adjacent cells then they can be grouped and Minimized functions can be obtained

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	0

		Y	
		0	1
X	0	1	1
	1	0	0

A yellow box highlights the cells where X=0 and Y varies from 0 to 1. An arrow points from the text X' to this box.

$X = 0$ constant Y varies from $0 \rightarrow 1$

2-Variable K-Map

Simplification

X	Y	F
0	0	0
0	1	0
1	0	1
1	1	1

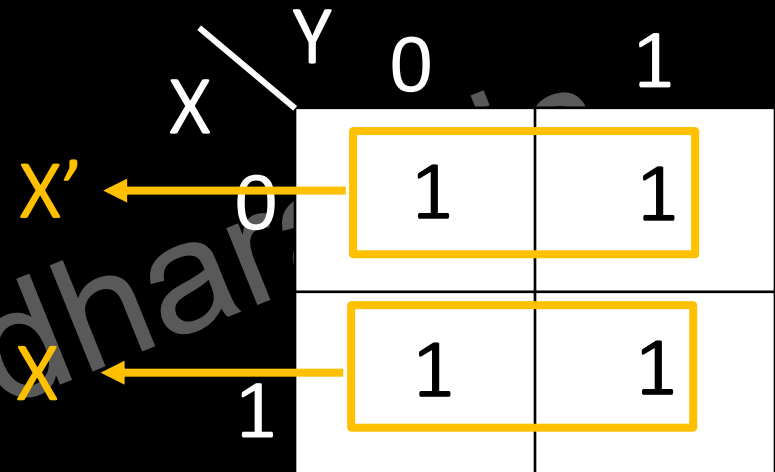
		Y	
		0	1
X	0	0	0
	1	1	1

$X = 1$ constant Y varies from $0 \rightarrow 1$

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	1

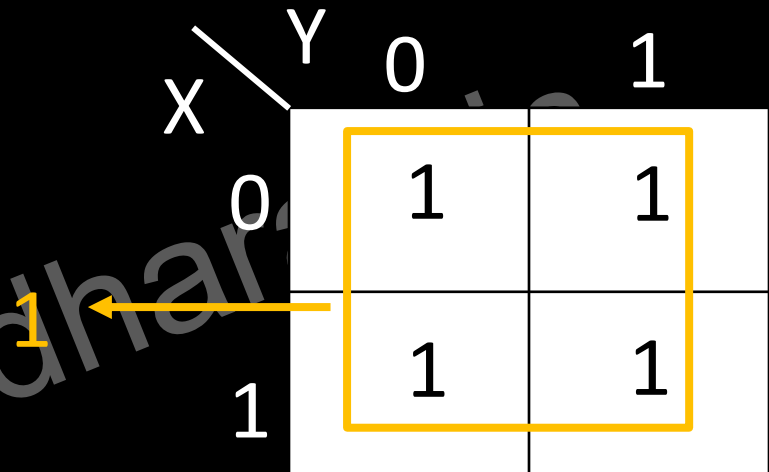


$$X' + X = 1$$

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	1



Adjacent cells can be grouped

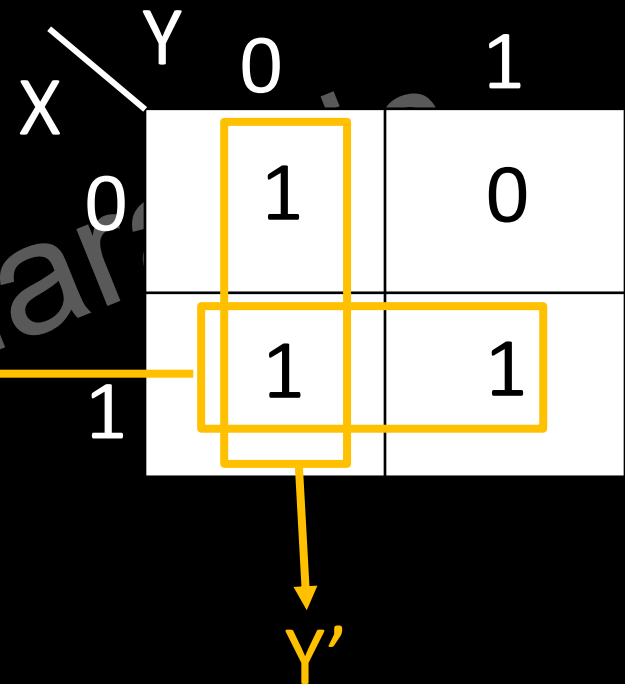
2- cells at a time

4- cells at a time

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1



$$F = X + Y'$$

****** A term can be grouped multiple times if it helps in simplifying the expression

3-Variable K-Map

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

		Y Z			
		00	01	11	10
X	0	(0,0,0) m_0	(0,0,1) m_1	(0,1,1) m_3	(0,1,0) m_2
	1	(1,0,0) m_4	(1,0,1) m_5	(1,1,1) m_7	(1,1,0) m_6

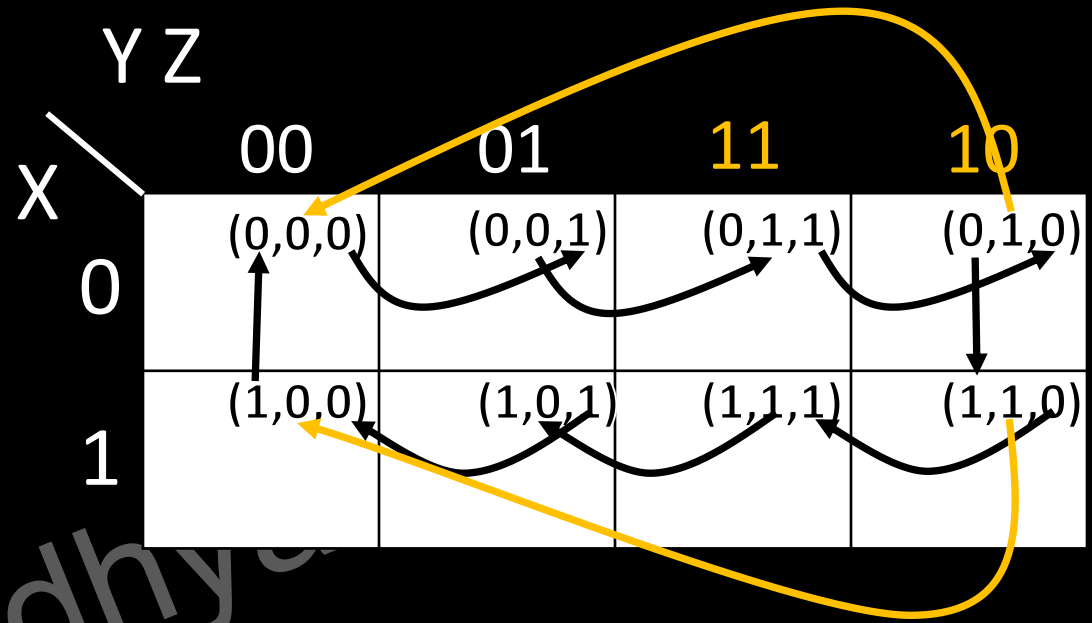
Should the next entry be (0, 1, 0) ?

$(0,0,1) \rightarrow (0,1,0)$

****Two bit variation here, what to do ??**

3-Variable K-Map

Adjacent Cells



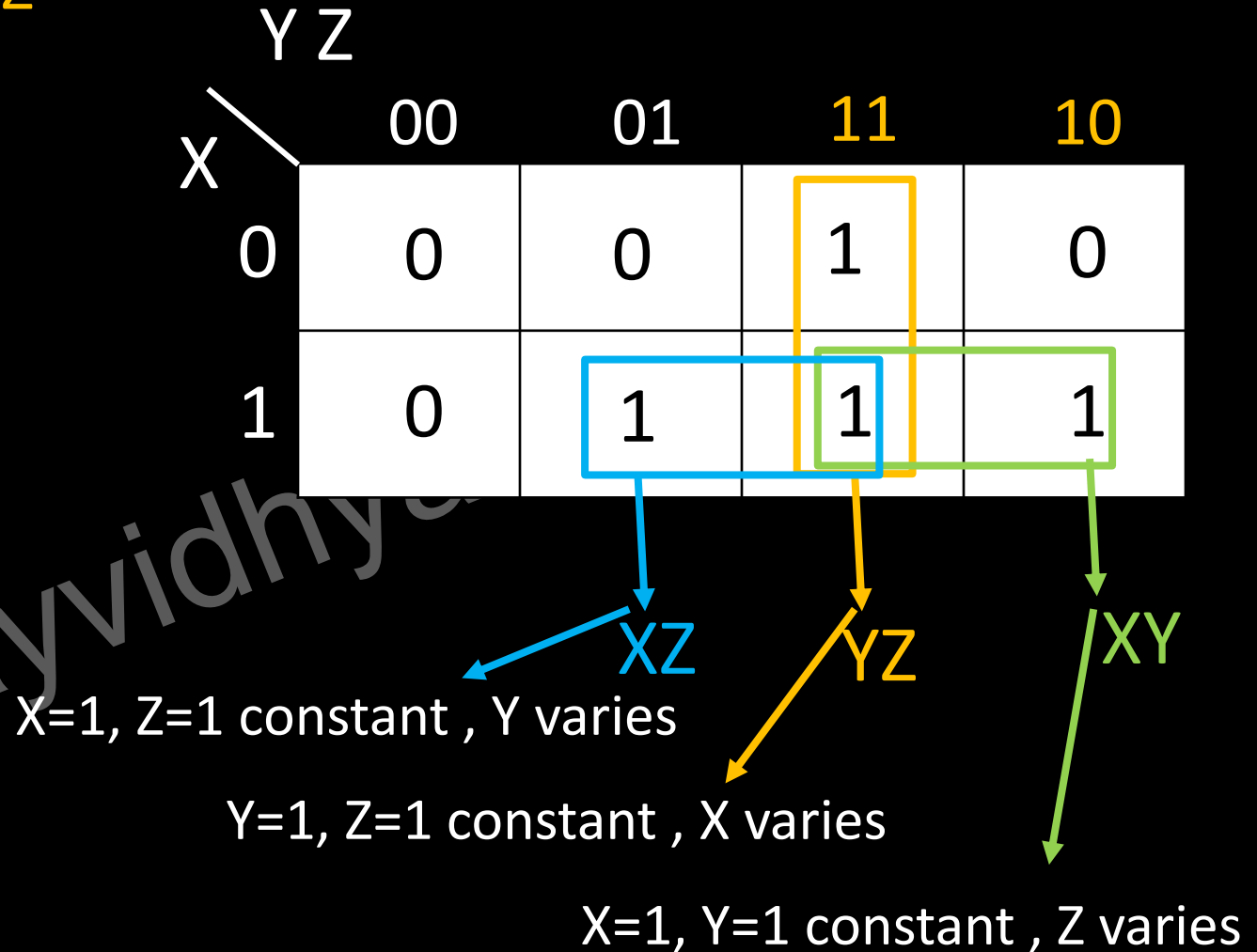
Any more adjacent cells ??

3-Variable K-Map

$$F = XY + YZ + XZ$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

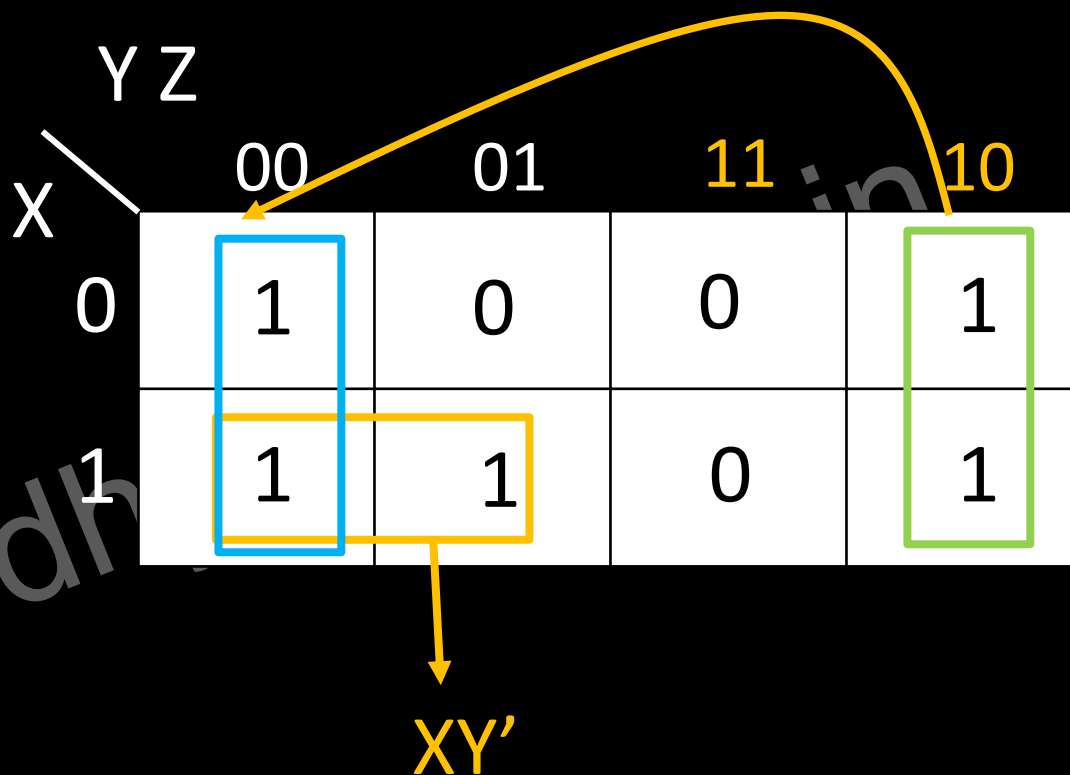


3-Variable K-Map

$$F(X,Y,Z) = \sum (0,2,4,5,6)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

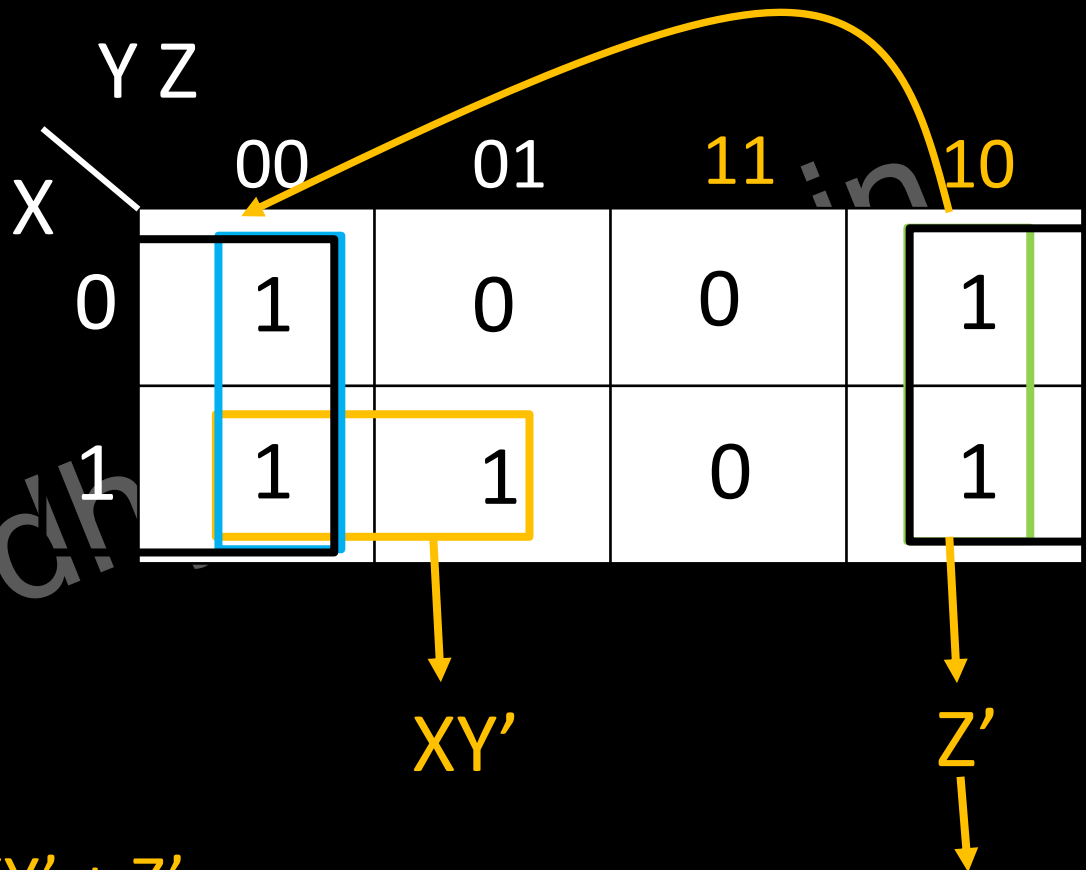


3-Variable K-Map

$$F(X,Y,Z) = \sum (0,2,4,5,6)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



$$F = XY' + Z'$$

Z=0, constant, X, Y Change

3-Variable K-Map

$$F(X,Y,Z) = \sum (2,3,4,5)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

		Y Z			
X		00	01	11	10
	0	0	0	1	1
	1	1	1	0	0

$$F = XY' + X'Y$$

3-Variable K-Map

$$F(X,Y,Z) = \sum (3,4,6,7)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		Y Z			
		00	01	11	10
X	0	0	0	1	0
	1	1	0	1	1

Is the third grouping necessary ?

All ones should get covered atleast once

If all of them are covered then there is no need to group them again

3-Variable K-Map

$$F(X,Y,Z) = \sum (3,4,6,7)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		Y Z			
		00	01	11	10
X	0	0	0	1	0
	1	1	0	1	1

$$F = YZ + XZ'$$

$$F = YZ + XZ' + XZ'$$

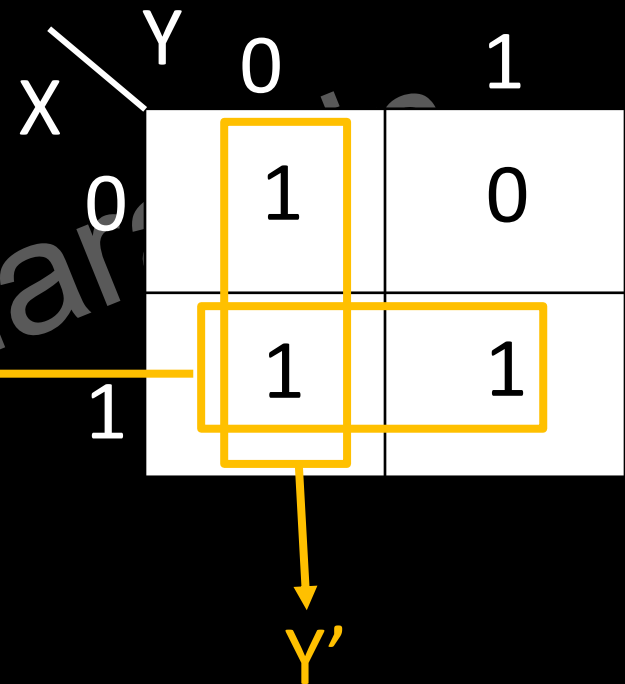
Both expressions will result in same truth table

But only first expression is minimized

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1



$$F = X + Y'$$

A term can be grouped multiple times if it helps in simplified expression

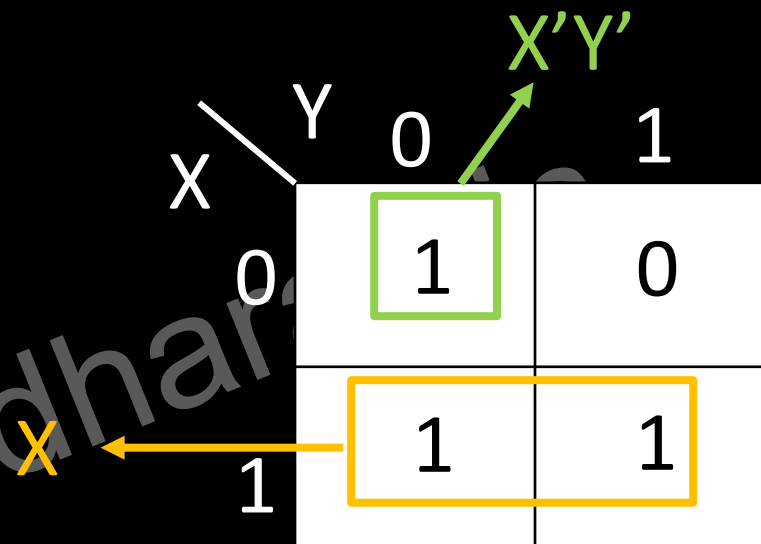
2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1

$$F = X + Y'$$

$$F = X + X'Y'$$



Which of the two is a minimized function ??

3-Variable K-Map

$$F(X,Y,Z) = \pi(0,1,2,5)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		Y Z			
		00	01	11	10
X	0	0	0	1	0
	1	1	0	1	1

Y=0, Z=1 constant, X varies $(Y+Z')$

$(X+Z)$

X=0, Z=0 constant, Y varies

$$F = (Y + Z') (X + Z)$$

3-Variable K-Map

$$F(X,Y,Z) = \pi(2,3,4,5)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

		Y Z			
X		00	01	11	10
	0	0	0	1	1
	1	1	1	0	0

$$F = (X+Y)(X'+Y')$$

Next Class

4/5-variable K-map

K-map with don't care conditions

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