# Digital Design
# First Semester 2020-21
# Tutorial : 02



# Boolean Algebra & Logic Gates

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1. Simplify

$$F = BC + B\bar{C} + BA$$

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1. Simplify

$$F = BC + B\bar{C} + BA$$

- Simplification

$$F = B(C + \bar{C}) + BA$$

$$F = B \cdot 1 + BA$$

$$F = B(1 + A)$$

$$F = B$$

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2. Simplify

$$F = A + \bar{A}B + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}E$$

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2. Simplify

$$F = A + \bar{A}B + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}\bar{C}\bar{D}E$$

- Simplification

$$F = A + \bar{A}(B + \bar{B}C + \bar{B}\bar{C}D + \bar{B}\bar{C}\bar{D}E)$$

$$F = A + B + \bar{B}C + \bar{B}\bar{C}D + \bar{B}\bar{C}\bar{D}E$$

$$F = A + B + \bar{B}(C + \bar{C}D + \bar{C}\bar{D}E)$$

$$F = A + B + C + \bar{C}D + \bar{C}\bar{D}E$$

$$F = A + B + C + \bar{C}(D + \bar{D}E)$$

$$F = A + B + C + D + \bar{D}E$$

$$F = A + B + C + D + E$$

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3. Simplify

$$(X + Y)\,(X + \overline{Y})\,(\overline{X} + Z)$$

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3. Simplify

First simplify $(X + Y)(X + \overline{Y})$

$$(X + Y)(X + \overline{Y}) = XX + X\overline{Y} + YX + Y\overline{Y}$$

$= X + X\overline{Y} + YX + O$, as $XX = X$

as $Y\overline{Y} = 0$

$= X + X(\overline{Y} + Y)$, as $\overline{Y} + Y = 1$

$= X + X \cdot 1$, as $X \cdot 1 = X$

$= X + X$

$= X$

Now $(X + Y)(X + \overline{Y})(\overline{X} + Z)$

$= X(\overline{X} + Z)$

$= X\overline{X} + XZ$, by distributive law

$= 0 + XZ$

$= XZ$

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4. Simplify $XYZ + X\overline{Y}Z + XY\overline{Z}$

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4. Simplify $XYZ + X\overline{Y}Z + XY\overline{Z}$

$XYZ + X\overline{Y}Z + XY\overline{Z}$

$= XZ(Y + \overline{Y}) + XY\overline{Z}$

$= XZ + XY\overline{Z}$, as $Y + \overline{Y} = 1$

$= X(Z + Y\overline{Z})$

$= X[(Z + Y).(Z + \overline{Z})]$, (By Rule 15 dual of distributive

$= X[(Z + Y).1] = X(Z + Y)$

$= X(Y + Z)$, by commutative law.

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5. Show that

$$\overline{A(\bar{B}\bar{C} + BC)} = \bar{A} + (B + C)(\bar{B} + \bar{C})$$

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$$\overline{A(\bar{B}\bar{C} + BC)} = \bar{A} + (B + C)(\bar{B} + \bar{C})$$

- Simplification

$$\overline{A(\bar{B}\bar{C} + BC)} = \bar{A} + \overline{(\bar{B}\bar{C} + BC)}$$

$$= \bar{A} + (\overline{\bar{B}\bar{C}})(\overline{BC})$$

$$= \bar{A} + (B + C)(\bar{B} + \bar{C})$$

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6. Write Minterms of

$$F(A, B, C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C$$

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6. Write Minterms of

$$F(A, B, C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C$$

could instead be expressed as

$$F(A, B, C) = m_0 + m_1 + m_4 + m_5$$

or more compactly

$$F(A, B, C) = \sum m(0, 1, 4, 5) = \textit{one-set}(0, 1, 4, 5)$$

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7. Write maxterms of

$$F(A, B, C) = (A + B + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C})$$

# Digital Design Tutorial : 02

7. Write maxterms of

$$F(A, B, C) = (A + B + \bar{C})(\bar{A} + B + C)(\bar{A} + \bar{B} + \bar{C})$$

could instead be expressed as

$$F(A, B, C) = M_1 \cdot M_4 \cdot M_7$$

or more compactly as

$$F(A, B, C) = \prod M(1, 4, 7) = \textit{zero-set}(1, 4, 7)$$

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8. Write canonical form and Truth Table of

$$F(A, B, C) = AB + \bar{B}(\bar{A} + \bar{C})$$

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8. Write canonical form and Truth Table of

$$
\begin{aligned}
F(A, B, C) &= AB + \bar{B}(\bar{A} + \bar{C}) = AB + \bar{A}\bar{B} + \bar{B}\bar{C} \\
&= AB(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + (A + \bar{A})\bar{B}\bar{C} \\
&= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC \\
&= \sum m(0, 1, 4, 6, 7)
\end{aligned}
$$

| A | B | C | F | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | ← 0 |
| 0 | 0 | 1 | 1 | ← 1 |
| 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 1 | ← 4 |
| 1 | 0 | 1 | 0 | |
| 1 | 1 | 0 | 1 | ← 6 |
| 1 | 1 | 1 | 1 | ← 7 |

Minterms listed as 1s in Truth Table

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8. Write canonical form and Truth Table of

$$F(A, B, C) = AB + \bar{B}(\bar{A} + \bar{C})$$

$$= (A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})$$

$$= \prod M(2, 3, 5)$$

| A | B | C | F | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | |
| 0 | 0 | 1 | 1 | |
| 0 | 1 | 0 | 0 | ← 2 |
| 0 | 1 | 1 | 0 | ← 3 |
| 1 | 0 | 0 | 1 | |
| 1 | 0 | 1 | 0 | ← 5 |
| 1 | 1 | 0 | 1 | |
| 1 | 1 | 1 | 1 | |

Maxterms listed as 0s in Truth Table

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8. Write canonical form and Truth Table of

$$
\begin{aligned}
F(A, B, C) &= AB + \bar{B}(\bar{A} + \bar{C}) = AB + \bar{A}\bar{B} + \bar{B}\bar{C} \\
&= AB(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + (A + \bar{A})\bar{B}\bar{C} \\
&= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC \\
&= \sum m(0, 1, 4, 6, 7)
\end{aligned}
$$

| A | B | C | F | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | ← 0 |
| 0 | 0 | 1 | 1 | ← 1 |
| 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | 0 | |
| 1 | 0 | 0 | 1 | ← 4 |
| 1 | 0 | 1 | 0 | |
| 1 | 1 | 0 | 1 | ← 6 |
| 1 | 1 | 1 | 1 | ← 7 |

Minterms listed as 1s in Truth Table

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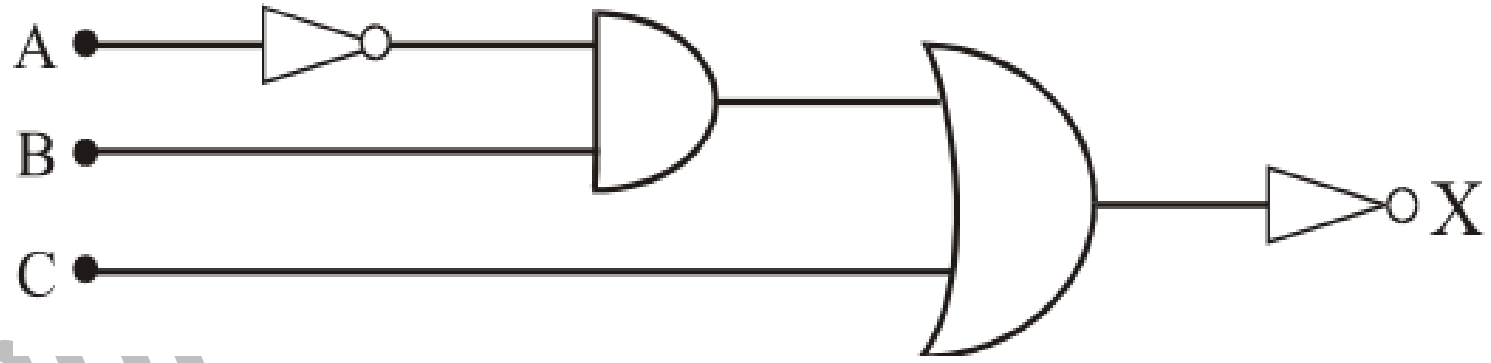
9. : **Write the Boolean expression that describes mathematically the behavior of logic circuit shown in fig.10. Use a truth table to determine what input conditions produce a logic 1 output.**



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9.

**Solution:**

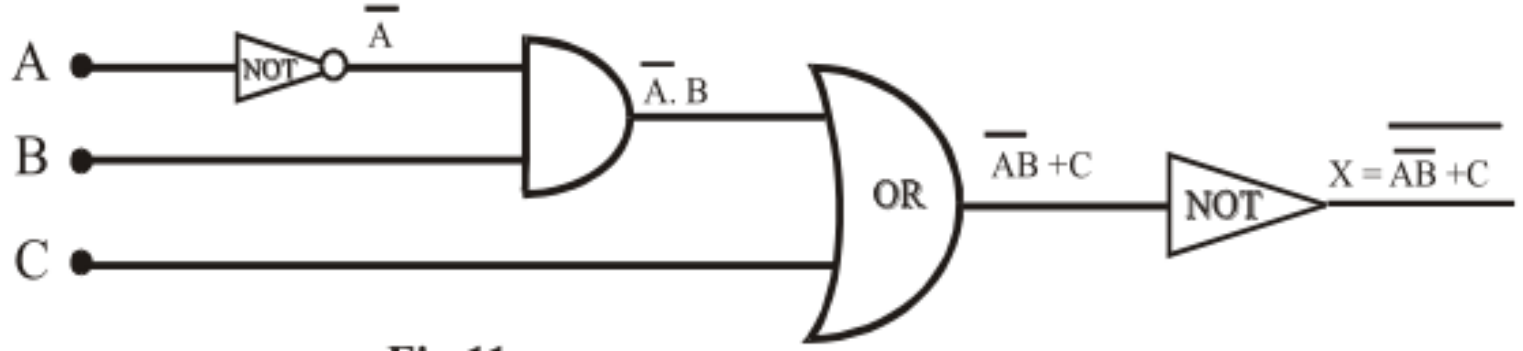


**Fig.11**

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10.

**Given the Boolean expression**

$X = AB + ABC + A\,\overline{B}\,\overline{C} + A\,\overline{C}$

(a) Draw the logic diagram for the expression.

(b) Minimize the expression.

(c) Draw the logic diagram for the reduced expression.

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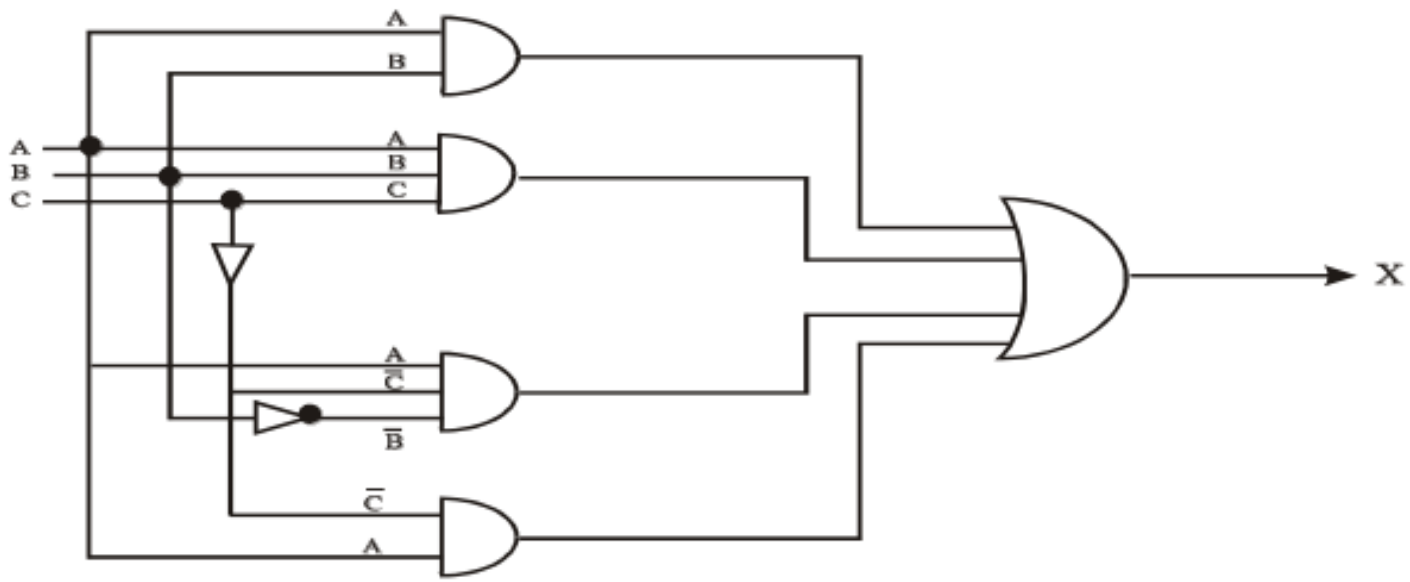
10. Solution



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10. Solution

$$X = AB + ABC + A\,\overline{B}\,\overline{C} + A\,\overline{C}$$

$$= AB\,(1 + C) + A\,\overline{C}\,(\overline{B} + 1)$$

$$= AB.\,1 + A\,\overline{C}\,.1 = AB + A\,\overline{C}$$

$$= A(B + \overline{C})$$