Digital Design
First Semester 2020-21
Tutorial : 02

Boolean Algebra & Logic Gates

1. Simplify

$$F = BC + B\overline{C} + BA$$

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- Simplification

$$F = B(C + \overline{C}) + BA$$

$$F = B \cdot 1 + BA$$

$$F = B(1 + A)$$

$$F = B$$

2. Simplify

$$F = A + \overline{A}B + \overline{A}\overline{B}C + \overline{A}\overline{B}\overline{C}D + \overline{A}\overline{B}\overline{C}\overline{D}E$$

2. Simplify

$$F = A + \overline{A}B + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}E$$

- Simplification

$$F = A + \overline{A}(B + \overline{B}C + \overline{B}\,\overline{C}D + \overline{B}\,\overline{C}\,\overline{D}E)$$

$$F = A + B + \overline{B}C + \overline{B}\,\overline{C}D + \overline{B}\,\overline{C}\,\overline{D}E$$

$$F = A + B + \overline{B}(C + \overline{C}D + \overline{C}\,\overline{D}E)$$

$$F = A + B + C + \overline{C}D + \overline{C}\,\overline{D}E$$

$$F = A + B + C + \overline{C}(D + \overline{D}E)$$

$$F = A + B + C + D + \overline{D}E$$

$$F = A + B + C + D + E$$

3. Simplify

$$(X + Y)(X + \overline{Y})(\overline{X} + Z)$$

3. Simplify

First simplify $(X + Y)(X + \overline{Y})$

$(X + Y)(X + \overline{Y}) = XX + X\overline{Y} + YX + Y\overline{Y}$

$\qquad\qquad = X + X\overline{Y} + YX + O, \quad$ as $XX = X$

$\qquad\qquad\qquad\qquad\qquad\qquad$ as $Y\overline{Y} = 0$

$\qquad\qquad = X + X(\overline{Y} + Y), \qquad$ as $\overline{Y} + Y = 1$

$\qquad\qquad = X + X \cdot 1, \qquad\qquad$ as $X \cdot 1 = X$

$\qquad\qquad = X + X$

$\qquad\qquad = X$

Now $\qquad (X + Y)(X + \overline{Y})(\overline{X} + Z)$

$\qquad\qquad = X(\overline{X} + Z)$

$\qquad\qquad = X\overline{X} + XZ, \quad$ by distributive law

$\qquad\qquad = 0 + XZ$

$\qquad\qquad = XZ$

4. Simplify

$$XYZ + X\overline{Y}Z + XY\overline{Z}$$

4. Simplify

$$XYZ + X \overline{Y} Z + XY \overline{Z}$$

$$XYZ + X \overline{Y} Z + XY \overline{Z}$$

$$= XZ (Y + \overline{Y}) + XY \overline{Z}$$

$$= XZ + XY \overline{Z}, \qquad \text{as} \qquad Y + \overline{Y} = 1$$

$$= X (Z + Y \overline{Z})$$

$$= X[(Z + Y) . (Z + \overline{Z})], \text{ (By Rule 15 dual of distributive}$$

$$= X [(Z + Y) . 1] = X (Z + Y)$$

$$= X (Y + Z), \qquad \text{by commutative law.}$$

5. Show that

$$\overline{A(\overline{B}\,\overline{C} + BC)} = \overline{A} + (B + C)(\overline{B} + \overline{C})$$

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$$\overline{A(\overline{B}\overline{C} + BC)} = \overline{A} + (B + C)(\overline{B} + \overline{C})$$

- Simplification

$$\overline{A(\overline{B}\overline{C} + BC)} = \overline{A} + \overline{(\overline{B}\overline{C} + BC)}$$

$$= \overline{A} + (\overline{\overline{B}\overline{C}})(\overline{BC})$$

$$= \overline{A} + (B + C)(\overline{B} + \overline{C})$$

6. Write Minterms of

$$F(A, B, C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + A\,\overline{B}\,\overline{C} + A\,\overline{B}\,C$$

6. Write Minterms of

$$F(A, B, C) = \overline{A}\overline{B}\overline{C} + \overline{A}\overline{B}C + A\overline{B}\overline{C} + A\overline{B}C$$

could instead be expressed as

$$F(A, B, C) = m_0 + m_1 + m_4 + m_5$$

or more compactly

$$F(A, B, C) = \sum m(0, 1, 4, 5) = \textit{one-set}(0, 1, 4, 5)$$

7. Write maxterms of

$$F(A, B, C) = (A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + \overline{C})$$

7. Write maxterms of

$$F(\mathbf{A}, \mathbf{B}, \mathbf{C}) = (\mathbf{A} + \mathbf{B} + \overline{\mathbf{C}})(\overline{\mathbf{A}} + \mathbf{B} + \mathbf{C})(\overline{\mathbf{A}} + \overline{\mathbf{B}} + \overline{\mathbf{C}})$$

could instead be expressed as

$$F(\mathbf{A}, \mathbf{B}, \mathbf{C}) = M_1 \cdot M_4 \cdot M_7$$

or more compactly as

$$F(\mathbf{A}, \mathbf{B}, \mathbf{C}) = \prod M(1, 4, 7) = \textit{zero-set}(1, 4, 7)$$

8. Write canonical form and Truth Table of

$$F(A, B, C) = AB + \bar{B}(\bar{A} + \bar{C})$$

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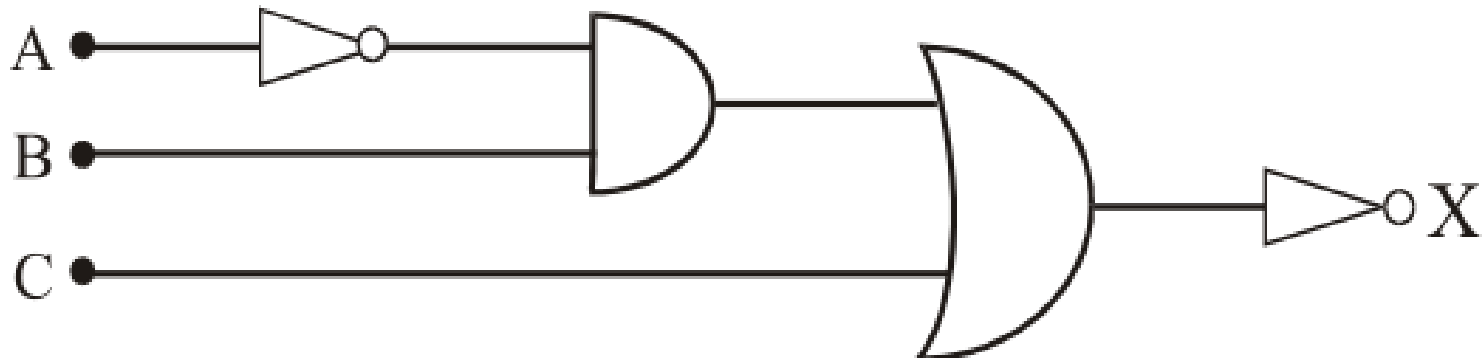
$$F(A, B, C) = AB + \bar{B}(\bar{A} + \bar{C}) = AB + \bar{A}\bar{B} + \bar{B}\bar{C}$$

$$= AB(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + (A + \bar{A})\bar{B}\bar{C}$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

$$= \sum m(0, 1, 4, 6, 7)$$

A	B	C	F	
0	0	0	1	← 0
0	0	1	1	← 1
0	1	0	0	
0	1	1	0	
1	0	0	1	← 4
1	0	1	0	
1	1	0	1	← 6
1	1	1	1	← 7

Minterms listed as
1s in Truth Table

8. Write canonical form and Truth Table of

$$F(A, B, C) = AB + \bar{B}(\bar{A} + \bar{C})$$

$$= (A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})$$

$$= \prod M(2, 3, 5)$$

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	0 ← 2
0	1	1	0 ← 3
1	0	0	1
1	0	1	0 ← 5
1	1	0	1
1	1	1	1

Maxterms listed as 0s in Truth Table

8. Write canonical form and Truth Table of

$$F(A, B, C) = AB + \bar{B}(\bar{A} + \bar{C}) = AB + \bar{A}\bar{B} + \bar{B}\bar{C}$$
$$= AB(C + \bar{C}) + \bar{A}\bar{B}(C + \bar{C}) + (A + \bar{A})\bar{B}\bar{C}$$
$$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AB\bar{C} + ABC$$
$$= \sum m(0, 1, 4, 6, 7)$$

A	B	C	F	
0	0	0	1	← 0
0	0	1	1	← 1
0	1	0	0	
0	1	1	0	
1	0	0	1	← 4
1	0	1	0	
1	1	0	1	← 6
1	1	1	1	← 7

Minterms listed as
1s in Truth Table

9. : Write the Boolean expression that describes mathematically the behavior of logic circuit shown in fig.10. Use a truth table to determine what input conditions produce a logic 1 output.
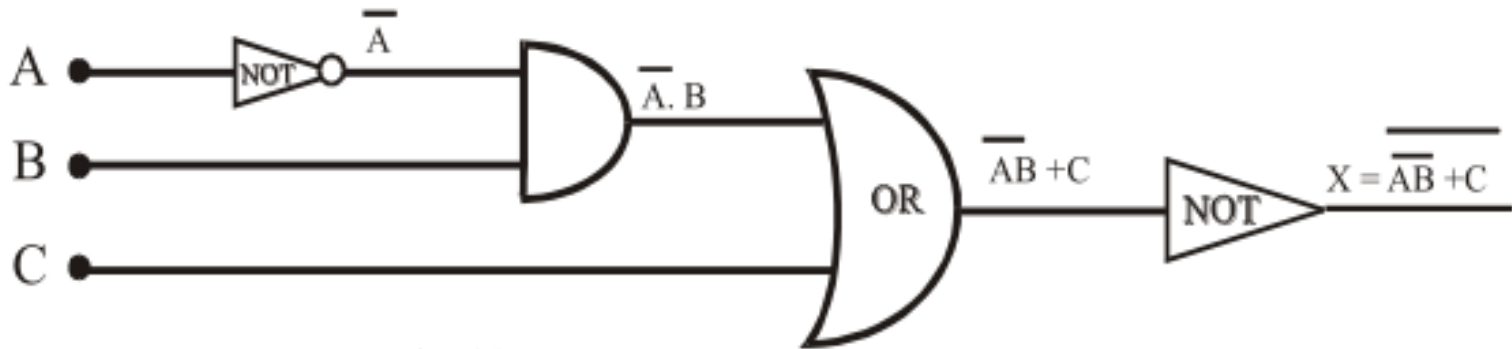
9.

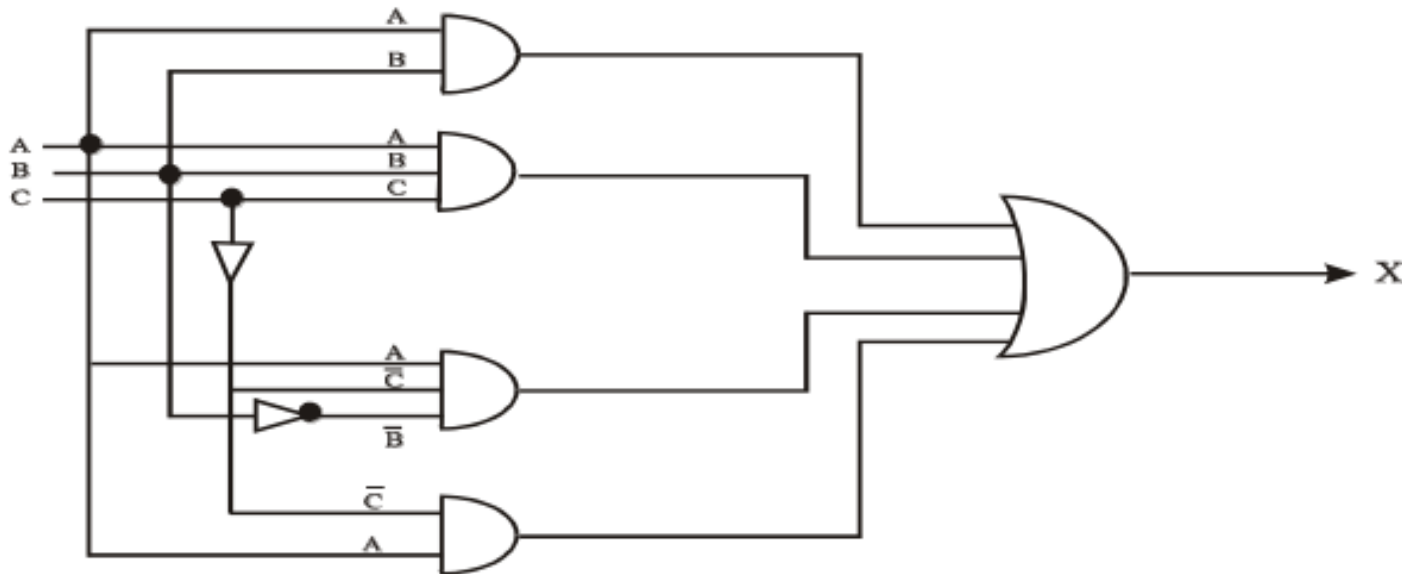**Solution:**

Fig.11

10.

Given the Boolean expression

$$X = AB + ABC + A \overline{B} \ \overline{C} + A \overline{C}$$

(a)   Draw the logic diagram for the expression.

(b)   Minimize the expression.

(c)   Draw the logic diagram for the reduced expression.

10. Solution

10. Solution

$$X = AB + ABC + A\overline{B}\,\overline{C} + A\overline{C}$$

$$= AB(1+C) + A\overline{C}(\overline{B}+1)$$

$$= AB.1 + A\overline{C}.1 = AB + A\overline{C}$$

$$= A(B + \overline{C})$$

$$X = A(B + \overline{C})$$