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Department of Electrical Engineering



Digital Design

First Semester 2020-21

Tutorial : 01

Binary Number System

Digital Design Tutorial : 01

1. Convert the following decimal numbers to their binary equivalents
 - (a) 64
 - (b) 100

Digital Design Tutorial : 01

1. Convert the following decimal numbers to their binary equivalents

(a) 64

(b) 100

Answer

A,	Quotient	Remainder	B	Quotient	Remainder
64/2	32	0	100/2	50	0
32/2	16	0	50/2	25	0
16/2	8	0	25/2	12	1
8/2	4	0	12/2	6	0
4/2	2	0	6/2	3	0
2/2	1	0	3/2	1	1
1/2	0	1	1/2	0	1
$64_{10} = 1000000_2$			$100_{10} = 1100100_2$		

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2. Convert the following decimal numbers to their binary equivalents

(a) 34.75

(b) 25.25

Digital Design Tutorial : 01

2. Convert the following decimal numbers to their binary equivalents

(a) 34.75

(b) 25.25

A, 34.75

the integer part (34) convert to binary format

	Quotient	Remainder
34/2	17	0
17/2	8	1
8/2	4	0
4/2	2	0
2/2	1	0
1/2	0	1

$$34_{10} = 100010_2$$

the fraction part (0.75) convert to binary format

	product	integer part
0.75x2	1.5	1
0.5x2	1.0	1

$$0.75_{10} = 0.11_2$$

$$34.75_{10} = 100010.11_2$$

B, 25.25

the integer part is 25, convert to binary format

	Quotient	Remainder
25/2	12	1
12/2	6	0
6/2	3	0
3/2	1	1
1/2	0	1

$$25_{10} = 11001_2$$

the fraction part is 0.25, convert to binary format

	product	integer part
0.25x2	0.5	0
0.5x2	1.0	1 => 0.01

$$0.25_{10} = 0.01_2$$

$$25.25_{10} = 11001.01_2$$

Digital Design Tutorial : 01

3. Convert the following binary numbers to decimal equivalents
- (a) 001100
 - (b) 000011

Digital Design Tutorial : 01

3. Convert the following binary numbers to decimal equivalents

(a) 001100

(b) 000011

For the binary representation of $y = \{...b_2b_1b_0.b_{-1}b_{-2}b_{-3}...\}$, the value of Y is

$$y = \sum_i b_i \times 2^i$$

$$A, 001100 = 0 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 + 4 = 12$$

$$B, 000011 = 0 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 2 + 1 = 3$$

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4. Convert the following binary numbers to decimal equivalents
- (a) 11100.001
 - (b) 110011.10011

Digital Design Tutorial : 01

4. Convert the following binary numbers to decimal equivalents

(a) 11100.001

(b) 110011.10011

For the binary representation of $y = \{...b_2b_1b_0.b_{-1}b_{-2}b_{-3}...\}$, the value of Y is

$$y = \sum_i b_i \times 2^i$$

A, 11100.001=

$$1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 + 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} = 28 + 0.125 = 28.125$$

B, 110011.10011=

$$1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} + 1 \times 2^{-5} \\ = 51 + 0.5 + 0.0625 + 0.03125 = 51.59375$$

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5. Convert the following hexadecimal number to their decimal equivalents

(a) F117

(b) EBA.C

Digital Design Tutorial : 01

5. Convert the following hexadecimal number to their decimal equivalents

(a) F117

(b) EBA.C

$$F117 = 15 \times 16^3 + 1 \times 16^2 + 1 \times 16^1 + 7 \times 16^0 = 61719$$

$$EBA.C = 14 \times 16^2 + 11 \times 16^1 + 10 \times 16^0 + 12 \times 16^{-1} = 3770.75$$

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6. Convert the following decimal numbers to their hexadecimal equivalents

(a) 80

(b) 204.125

Digital Design Tutorial : 01

6. Convert the following decimal numbers to their hexadecimal equivalents

(a) 80

(b) 204.125

	Quotient	Remainder
80/16	5	0
5/16	0	5
$80_{10} = 50_{16}$		

the integer part 204, convert to hexadecimal format

	Quotient	Remainder
204/16	12	12
12/16	0	12
$204_{10} = CC_{16}$		

the fraction part 0.125, convert to hexadecimal format

	product	integer part
0.125x16	2.0	2
$0.125_{10} = 0.2_{16}$		

$$204.125_{10} = CC.2_{16}$$

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7. Convert

- (a) Binary to Octal : 10001011
- (b) Binary to Hexadecimal : 10001011
- (c) Octal to Binary : 213
- (d) Hexadecimal to Binary : 8B

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7. Answer

From Binary to Hexadecimal

Starting at the binary point and working left, separate the bits into groups of **four** and replace each group with the corresponding **hexadecimal** digit.

$$10001011_2 = 1000 \ 1011 = 8B_{16}$$

From Octal to Binary

Replace each **octal** digit with the corresponding **3-bit** binary string.

$$213_8 = 010 \ 001 \ 011 = 10001011_2$$

From Hexadecimal to Binary

Replace each **hexadecimal** digit with the corresponding **4-bit** binary string.

$$8B_{16} = 1000 \ 1011 = 10001011_2$$

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8. Subtract using 2's Complement (Assume it is a 8-bit Processor)

(a) $26 - 15$

(b) $-31 - 6$

Digital Design Tutorial : 01

8. Subtract using 2's Complement (Assume it is an 8-bit Processor)

(a) $26 - 15$

(b) $-31 - 6$

$26 - 15 = 26 + (-15) = 0001\ 1010 + 1111\ 0001 = 10000\ 1011$, and truncating the leftmost 1 to remain within a register of 8, the answer is $0000\ 1011_2$

$-31 - 6 = (-31) + (-6) = 1110\ 0001 + 1111\ 1010 = 11101\ 1011$, and truncating the leftmost 1 to remain within a register of 8, the answer is $1101\ 1011_2$

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9. Subtract using 2's Complement (Assume it is an 4-bit Processor)

(a) $6+2$

(b) $-4-2$

(c) $-6-3$

Digital Design Tutorial : 01

9. Subtract using 2's Complement (Assume it is an 4-bit Processor)

(a) $6+2$

(b) $-4-2$

(c) $-6-3$

Two positive numbers, sum $\geq 2^{n-1}$

$$\begin{array}{r} + 6 \quad 0110 \\ + 2 \quad 0010 \\ \hline \end{array}$$

Overflow! - too big a number!

- Largest number for $n = 4$ is

- How do we know when overflow occurs?

The 1 in the MSB position indicates a negative number, after adding two +ve numbers.

Digital Design Tutorial : 01

9. Subtract using 2's Complement (Assume it is a 4-bit Processor)

(a) $6+2$

(b) $-4-2$

(c) $-6-3$

Two positive numbers, $\text{sum} \geq 2^{n-1}$

$$\begin{array}{r} +6 \quad 0110 \\ +2 \quad 0010 \\ \hline \end{array}$$

Overflow! - too big a number!

- Largest number for $n = 4$ is

- How do we know when overflow occurs?

The 1 in the MSB position indicates a negative number, after adding two +ve numbers.

Two negative numbers, $|\text{sum}| \leq 2^{n-1}$

$$\begin{array}{r} -4 \quad 1100 \\ -2 \quad 1110 \\ \hline -6 \quad 1010 \end{array}$$

Correct answer. Ignore carry from sign bit. Not an overflow.

Two negative numbers, $|\text{sum}| > 2^{n-1}$

$$\begin{array}{r} -6 \quad 1010 \\ -3 \quad 1101 \\ \hline -9 \quad 0111 \end{array}$$

Wrong answer because of overflow: -9 is too large to be represented in a 4 bit number (including sign).

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10. Subtract using 1's Complement (Assume it is a 4-bit Processor)

(a) $6-2$

(b) $-4-2$

(c) $-6-4$

Digital Design Tutorial : 01

10. Subtract using 1's Complement (Assume it is a 4-bit Processor)

- (a) 6-2 (b) -4-2 (c) -6-4

Positive and negative number (+ve number largest magnitude).

$$\begin{array}{r}
 -2 \\
 +6 \\
 \hline
 +4
 \end{array}
 \quad
 \begin{array}{r}
 0110 \\
 \hline
 \end{array}$$

Correct answer? No, add carry to LSB
No overflow.

Two negative numbers, $|\text{sum}| < 2^{n-1}$

$$\begin{array}{r}
 -4 \\
 -2 \\
 \hline
 -6
 \end{array}
 \quad
 \begin{array}{r}
 1011 \\
 1101 \\
 \hline
 \end{array}$$

Correct answer with end around carry;
no overflow.

Two negative numbers, $|\text{sum}| \geq 2^{n-1}$

$$\begin{array}{r}
 -6 \\
 -4 \\
 \hline
 -10
 \end{array}
 \quad
 \begin{array}{r}
 1001 \\
 1011 \\
 \hline
 \end{array}$$

Wrong answer; overflow!