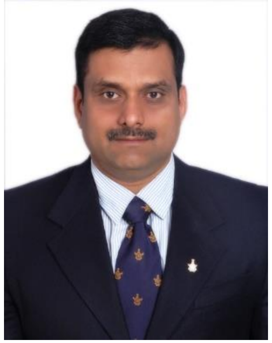
BITS Pilani
Hyderabad Campus
Department of Electrical Engineering

Digital Design
First Semester 2020-21
Tutorial : 01
Binary Number System

By Dr. Sanjay Vidhyadharan



Digital Design Tutorial : 01

1. Convert the following decimal numbers to their binary equivalents
   (a) 64
   (b) 100

Digital Design Tutorial : 01

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   (a) 64
   (b) 100

Answer

A,	Quotient	Remainder
64/2	32	0
32/2	16	0
16/2	8	0
8/2	4	0
4/2	2	0
2/2	1	0
1/2	0	1

$$64_{10} = 1000000_2$$

B	Quotient	Remainder
100/2	50	0
50/2	25	0
25/2	12	1
12/2	6	0
6/2	3	0
3/2	1	1
1/2	0	1

$$100_{10} = 1100100_2$$

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2. Convert the following decimal numbers to their binary equivalents
   (a) 34.75
   (b) 25.25

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   (a) 34.75
   (b) 25.25

A, 34.75

the integer part (34) convert to binary format

	Quotient	Remainder
34/2	17	0
17/2	8	1
8/2	4	0
4/2	2	0
2/2	1	0
1/2	0	1

$34_{10} = 100010_2$

the fraction part (0.75) covert to binary format

	product	integer part
0.75x2	1.5	1
0.5x2	1.0	1

$0.75_{10} = 0.11_2$

$34.75_{10} = 100010.11_2$

B, 25.25

the integer part is 25, convert to binary format

	Quotient	Remainder
25/2	12	1
12/2	6	0
6/2	3	0
3/2	1	1
1/2	0	1

$25_{10} = 11001_2$

the fraction part is 0.25, convert to binary format

	product	integer part
0.25x2	0.5	0
0.5x2	1.0	1 => 0.01

$0.25_{10} = 0.01_2$

$25.25_{10} = 11001.01_2$

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3. Convert the following binary numbers to decimal equivalents
   (a) 001100
   (b) 000011

Digital Design Tutorial : 01

3. Convert the following binary numbers to decimal equivalents
   (a) 001100
   (b) 000011

For the binary representation of $y = \{\ldots b_2 b_1 b_0 . b_{-1} b_{-2} b_{-3} \ldots\}$, the value of Y is

$$y = \sum_i b_i \times 2^i$$

A, $001100 = 0\times 2^5 + 0\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0 = 8 + 4 = 12$

B, $000011 = 0\times 2^5 + 0\times 2^4 + 0\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 = 2 + 1 = 3$

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4. Convert the following binary numbers to decimal equivalents
   (a) 11100.001
   (b) 110011.10011

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4. Convert the following binary numbers to decimal equivalents
   (a) 11100.001
   (b) 110011.10011

For the binary representation of $y = \{\ldots b_2 b_1 b_0 . b_{-1} b_{-2} b_{-3} \ldots\}$, the value of Y is

$$y = \underset{i}{} b_i \times 2^i$$

A, 11100.001=

$$1\times 2^4 + 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 0\times 2^0 + 0\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} = 28 + 0.125 = 28.125$$

B, 110011.10011=

$$1\times 2^5 + 1\times 2^4 + 0\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 + 1\times 2^{-1} + 0\times 2^{-2} + 0\times 2^{-3} + 1\times 2^{-4} + 1\times 2^{-5}$$

$$= 51 + 0.5 + 0.0625 + 0.03125 = 51.59375$$

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5. Convert the following hexadecimal number to their decimal equivalents

(a) F117

(b) EBA.C

Digital Design Tutorial : 01

5. Convert the following hexadecimal number to their decimal equivalents

(a) F117

(b) EBA.C

$$F117 = 15\times16^3 + 1\times16^2 + 1\times16^1 + 7\times16^0 = 61719$$

$$EBA.C\ 14\times16^2 + 11\times16^1 + 10\times16^0 + 12\times16^{-1} = 3770.75$$

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6. Convert the following decimal numbers to their hexadecimal equivalents

(a) 80

(b) 204.125

Digital Design Tutorial : 01

6. Convert the following decimal numbers to their hexadecimal equivalents

(a) 80

(b) 204.125

	Quotient	Remainder
80/16	5	0
5/16	0	5

$$80_{10} = 50_{16}$$

the integer part 204, convert to hexadecimal format

	Quotient	Remainder
204/16	12	12
12/16	0	12

$$204_{10} = CC_{16}$$

the fraction part 0.125, convert to hexadecimal format

	product	integer part
0.125x16	2.0	2

$$0.125_{10} = 0.2_{16}$$

$$204.125_{10} = CC.2_{16}$$

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7. Convert

(a) Binary to Octal : 10001011

(b) Binary to Hexadecimal : 10001011

(c) Octal to Binary : 213

(d) Hexadecimal to Binary : 8B

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7. Answer

From Binary to Hexadecimal

Starting at the binary point and working left, separate the bits into groups of **four** and replace each group with the corresponding **hexadecimal** digit.

$$10001011_2 = 1000 \ \ 1011 = 8B_{16}$$

From Octal to Binary

Replace each **octal** digit with the corresponding **3-bit** binary string.

$$213_8 = 010 \ \ 001 \ \ 011 = 10001011_2$$

From Hexadecimal to Binary

Replace each **hexadecimal** digit with the corresponding **4-bit** binary string.

$$8B_{16} = 1000 \ \ 1011 = 10001011_2$$

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8. Subtract using 2's Complement (Assume it is a 8-bit Processor)

(a) 26 –15

(b) -31 –6

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8. Subtract using 2's Complement (Assume it is an 8-bit Processor)

(a) 26 –15

(b) -31 –6

26 – 15 = 26 + (-15) = 0001 1010 + 1111 0001 = 10000 1011, and truncating the leftmost 1 to remain within a register of 8, the answer is $0000\ 1011_2$

–31 – 6 = (–31) + (–6) = 1110 0001 + 1111 1010 = 11101 1011, and truncating the leftmost 1 to remain within a register of 8, the answer is $1101\ 1011_2$

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9. Subtract using 2's Complement (Assume it is an 4-bit Processor)
   (a) 6+2
   (b) -4-2
   (c) -6-3

Digital Design Tutorial : 01

9. Subtract using 2’s Complement (Assume it is an 4-bit Processor)

(a) 6+2

(b) -4-2

(c) -6-3

Two positive numbers, sum ≥ 2^{n-1}

+ 6	0 1 1 0
+ 2	0 0 1 0

Overflow! - too big a number!

- Largest number for n = 4 is

- How do we know when overflow occurs?

The 1 in the MSB position indicates a negative number, after adding two +ve numbers.

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9. Subtract using 2's Complement (Assume it is a 4-bit Processor)

(a) 6+2 (b) -4-2 (c) -6-3

Two positive numbers, sum $\geq 2^{n-1}$

+ 6	0 1 1 0
+ 2	0 0 1 0

Overflow! - too big a number!

- Largest number for n = 4 is

- How do we know when overflow occurs?

The 1 in the MSB position indicates a negative number, after adding two +ve numbers.

Two negative numbers, $|sum| \leq 2^{n-1}$

- 4	1 1 0 0
- 2	1 1 1 0
- 6	

Correct answer. Ignore carry from sign bit. Not an overflow.

Two negative numbers, $|sum| > 2^{n-1}$

- 6	1 0 1 0
- 3	1 1 0 1
- 9	

Wrong answer because of overflow: -9 is too large to be represented in a 4 bit number (including sign).

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10. Subtract using 1's Complement (Assume it is a 4-bit Processor)
(a) 6-2 (b) -4-2 (c) -6-4

Digital Design Tutorial : 01

10. Subtract using 1's Complement (Assume it is a 4-bit Processor)

(a) 6-2 (b) -4-2 (c) -6-4

Positive and negative number (+ve number largest magnitude).

- 2		
+ 6	0 1 1 0	
+ 4		Correct answer? No, add carry to LSB. No overflow.

Two negative numbers, |sum| < 2^{n-1}

- 4	1 0 1 1	
- 2	1 1 0 1	
- 6		Correct answer with end around carry; no overflow.

Two negative numbers, |sum| ≥ 2^{n-1}

- 6	1 0 0 1	
- 4	1 0 1 1	
-10		Wrong answer; overflow!