

Digital Design

Lecture 7: Quine-McCluskey (QM) Technique



Birla Institute of Technology & Science, Pilani
Hyderabad Campus

Innovate

achieve

lead



Quine-McCluskey (QM) Technique

QM Method effective for increased number of input variables

Optimal for implementing on a Computer

K-map is a **graphical method** whereas QM is a **Tabular Method**



Quine-McCluskey (QM) Technique

1. Find **Prime Implicants** of the function
2. Find **Essential Prime implicants** of the function.
3. Include Essential prime implicants in partial solution and delete them from the **prime implicant table**
4. Determine and delete **dominated rows and dominating columns**. Find the (secondary) prime implicants.
5. **Repeat steps 3 and 4** as many times as they are applicable until a minimal cover of the function is found



Example 1

$$f(w, x, y, z) = \sum(0,1,2,5,6,7,8,9,10,14)$$

1. Find all the prime Implicants

group 0	<u>0 0000</u>
group 1	1 0001
	2 0010
	<u>8 1000</u>
group 2	5 0101
	6 0110
	9 1001
	<u>10 1010</u>
group 3	7 0111
	<u>14 1110</u>

Group the Minterms according to the **number of 1's** in the Minterm



Column I

Column II

group 0	0	0000
group 1	1	0001
	2	0010
	8	1000
group 2	5	0101
	6	0110
	9	1001
	10	1010
group 3	7	0111
	14	1110

Combining
group 0 and
group 1:



Combining group 0 and group 1:

Column I

Column II

group 0	<u>0 0000</u> ✓	→	0,1 000-
group 1	<u>1 0001</u> ✓	→	
	2 0010		
	8 1000		
group 2	<u>5 0101</u>		
	6 0110		
	9 1001		
	<u>10 1010</u>		
group 3	<u>7 0111</u>		
	14 1110		

Find matched pair with only one variable change

$$w'x'y'z' + w'x'y'z = w'x'y'(z+z') = w'x'y'$$

' - ' indicates absence of literal



Combining group 0 and group 1:

	Column I	Column II
group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	<u>2 0010</u> ✓	
	8 1000	
group 2	<u>5 0101</u>	
	6 0110	
	9 1001	
	<u>10 1010</u>	
group 3	7 0111	
	14 1110	



Combining group 0 and group 1:

	Column I	Column II
group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	<u>8 1000</u> ✓	
group 2	5 0101	
	6 0110	
	9 1001	
	<u>10 1010</u>	
group 3	7 0111	
	14 1110	



Combining group 1 and group 2:

	Column I	Column II
group 0	0 0000✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2	5 0101✓	
	6 0110	
	9 1001	
	10 1010	
group 3	7 0111	
	14 1110	



Combining group 1 and group 2:

Column I

Column II

group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	<hr/>
group 2	5 0101✓	1,5 0-01
	6 0110	
	9 1001	
	<u>10 1010</u>	
group 3	7 0111	
	14 1110	



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	Column I	Column II
group 0	<u>0 0000</u> ✓	0,1 000-
group 1	<u>1 0001</u> ✓	0,2 00-0
	2 0010 ✓	0,8 -000
	<u>8 1000</u> ✓	<u>1,5 0-01</u>
group 2	<u>5 0101</u> ✓	1,9 -001
	6 0110	
	<u>9 1001</u> ✓	
group 3	<u>10 1010</u>	
	7 0111	
	14 1110	



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Column I

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group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	<u>8 1000</u> ✓	1,5 0-01
group 2	5 0101✓	1,9 -001
	6 0110	
	9 1001✓	
	<u>10 1010</u>	
group 3	7 0111	
	14 1110	



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Column I

Column II

group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010 ✓	0,8 -000
	<u>8 1000</u> ✓	<hr/>
group 2	5 0101 ✓	1,5 0-01
	6 0110	1,9 -001
	9 1001 ✓	
	<u>10 1010</u>	
group 3	7 0111	
	14 1110	



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	Column I	Column II
group 0	0 0000 ✓	0,1 000-
group 1	1 0001 ✓	0,2 00-0
	2 0010 ✓	0,8 -000
	8 1000 ✓	1,5 0-01
group 2	5 0101 ✓	1,9 -001
	6 0110 ✓	2,6 0-10
	9 1001 ✓	
group 3	10 1010	
	7 0111	
	14 1110	



Combining group 1 and group 2:

Column I

Column II

group 0	0 0000 ✓	0,1 000-
group 1	1 0001 ✓	0,2 00-0
	2 0010 ✓	0,8 -000
	8 1000 ✓	1,5 0-01
group 2	5 0101 ✓	1,9 -001
	6 0110 ✓	2,6 0-10
	9 1001 ✓	
group 3	10 1010	
	7 0111	
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Combining group 1 and group 2:

Column I	Column II	
group 0 <u>0 0000</u> ✓	0,1 000-	
group 1 {	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2 {	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
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group 3 {	7 0111	
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Column I

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group 0	0 0000✓	0,1 000-
group 1 {	1 0001✓	0,2 00-0
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	6 0110✓	2,6 0-10
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group 3 {	7 0111	
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group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	<u>8 1000</u> ✓	<hr/>
group 2	5 0101✓	1,5 0-01
	<u>6 0110</u> ✓	1,9 -001
	9 1001✓	2,6 0-10
	<u>10 1010</u> ✓	2,10 -010
group 3	7 0111	
	14 1110	



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	Column I	Column II
group 0	0 0000 ✓	0,1 000-
group 1	1 0001 ✓	0,2 00-0
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group 2	5 0101 ✓	1,9 -001
	6 0110 ✓	2,6 0-10
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	10 1010 ✓	8,9 100-
group 3	7 0111	
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Column I	Column II	
group 0 <u>0 0000</u> ✓	0,1 000-	
group 1 {	1 0001 ✓	0,2 00-0
	2 0010 ✓	0,8 -000
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group 2 {	<u>5 0101</u> ✓	1,9 -001
	6 0110 ✓	2,6 0-10
	9 1001 ✓	2,10 -010
	10 1010 ✓	8,9 100-
group 3 {	7 0111	8,10 10-0
	14 1110	



Combining group 2 and group 3:

Column I	Column II	
group 0 <u>0 0000</u> ✓	0,1 000-	
group 1 {	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	8 1000✓	1,5 0-01
group 2 {	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	10 1010✓	8,9 100-
group 3 {	7 0111✓	8,10 10-0
	14 1110	5,7 01-1



Combining group 2 and group 3:

Column I

Column II

group 0	<u>0 0000</u> ✓	0,1 000-
group 1	1 0001✓	0,2 00-0
	2 0010✓	0,8 -000
	<u>8 1000</u> ✓	1,5 0-01
group 2	5 0101 ✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	<u>10 1010</u> ✓	8,9 100-
group 3	7 0111✓	<u>8,10 10-0</u>
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group 2	5 0101✓	1,9 -001
	6 0110✓	2,6 0-10
	9 1001✓	2,10 -010
	<u>10 1010</u> ✓	8,9 100-
group 3	7 0111✓	<u>8,10 10-0</u>
	14 1110	5,7 01-1
		6,7 011-



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	<u>14 1110</u> ✓	<u>8,10 10-0</u>
		5,7 01-1
		6,7 011-
		6,14 -110
		10,14 1-10



Combining group (0,1) and group (1,2):

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-	
group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓		
group 2	5 0101✓	1,5 0-01	
	6 0110✓	1,9 -001	
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group 3	7 0111✓	8,9 100-	
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		5,7 01-1	
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group 3	7 0111✓	8,9 100-	
	14 1110✓	8,10 10-0	
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	6 0110✓	1,9 -001	
	9 1001✓	2,6 0-10	
	10 1010✓	2,10 -010	
group 3	7 0111✓	8,9 100-✓	
	14 1110✓	8,10 10-0	
		5,7 01-1	
	6,7 011-		
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		5,7 01-1	
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	2 0010✓	0,8 -000	
	8 1000✓		
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	9 1001✓	2,6 0-10	
	10 1010✓	2,10 -010	
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group 3	7 0111✓	8,9 100-✓	
	14 1110✓	8,10 10-0	
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group 1	1 0001✓	0,2 00-0	
	2 0010✓	0,8 -000	
	8 1000✓		
group 2	<u>5 0101</u> ✓	1,5 0-01	
	6 0110✓	1,9 -001	
	9 1001✓	2,6 0-10	
	10 1010✓	2,10 -010	
group 3	<u>7 0111</u> ✓	8,9 100-✓	
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		5,7 01-1	
		6,7 011-	
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group 1	1 0001✓	0,2 00-0 ✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000	
	8 1000✓		
group 2	<u>5 0101</u> ✓	1,5 0-01	
	6 0110✓	1,9 -001	
	9 1001✓	2,6 0-10	
	<u>10 1010</u> ✓	2,10 -010	
group 3	7 0111✓	8,9 100-✓	
	14 1110 ✓	8,10 10-0 ✓	
		5,7 01-1	
	6,7 011-		
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	Column I	Column II	Column III
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group 1	1 0001✓	0,2 00-0✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000	
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group 2	5 0101✓	1,9 -001	
	6 0110✓	2,6 0-10	
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	10 1010✓	8,9 100-✓	
group 3	7 0111✓	8,10 10-0✓	
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group 1	1 0001✓	0,2 00-0✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000✓	0,8,1,9 -00-
	8 1000✓		
group 2	5 0101✓	1,5 0-01	
	6 0110✓	1,9 -001✓	
	9 1001✓	2,6 0-10	
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group 3	7 0111✓	8,9 100-✓	
	14 1110✓	8,10 10-0✓	
		5,7 01-1	
		6,7 011-	
		6,14 -110	
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Need not repeat if already Covered

Same as



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	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000✓	
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group 2	5 0101✓	1,5 0-01	
	6 0110✓	1,9 -001✓	
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	14 1110✓	8,10 10-0✓	
		5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	



Combining group (0,1) and group (1,2):

	Column I	Column II	Column III
group 0	0 0000 ✓	0,1 000- ✓	0,1,8,9 -00-
group 1	1 0001 ✓	0,2 00-0 ✓	0,2,8,10 -0-0 Already Covered
	2 0010 ✓	0,8 -000 ✓	
	8 1000 ✓		
group 2	5 0101 ✓	1,5 0-01	
	6 0110 ✓	1,9 -001 ✓	
	9 1001 ✓	2,6 0-10	
	10 1010 ✓	2,10 -010 ✓	
group 3	7 0111 ✓	8,9 100- ✓	
	14 1110 ✓	8,10 10-0 ✓	
		5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	



Combining group (0,1) and group (1,2):

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000✓	
	8 1000✓		
group 2	5 0101✓	1,5 0-01	
	6 0110✓	1,9 -001✓	
	9 1001✓	2,6 0-10	
	10 1010✓	2,10 -010✓	
group 3	7 0111✓	8,9 100-✓	
	14 1110✓	8,10 10-0✓	
		5,7 01-1	
	6,7 011-		
	6,14 -110		
	10,14 1-10		



Combining group (0,1) and group (1,2):

	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000 ✓	
	8 1000✓	1,5 0-01	
group 2	5 0101✓	1,9 -001✓	
	6 0110✓	2,6 0-10	
	9 1001✓	2,10 -010✓	
	10 1010✓	8,9 100-✓	
group 3	7 0111✓	8,10 10-0 ✓	
	14 1110✓	5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10	



Combining group (1,2) and group (2,3):



	Column I	Column II	Column III
group 0	0 0000✓	0,1 000-✓	0,1,8,9 -00-
group 1	1 0001✓	0,2 00-0✓	0,2,8,10 -0-0
	2 0010✓	0,8 -000✓	2,6,10,14 --10
	8 1000✓		
group 2	5 0101✓	1,5 0-01	
	6 0110✓	1,9 -001✓	
	9 1001✓	2,6 0-10✓	
	10 1010✓	2,10 -010✓	
group 3	7 0111✓	8,9 100-✓	
	14 1110✓	8,10 10-0✓	
		5,7 01-1	
		6,7 011-	
		6,14 -110	
		10,14 1-10✓	



Combining group (1,2) and group (2,3):

	Column I	Column II	Column III	
group 0	0 0000✓	0,1 000-✓	0,1,8,9 -00-	
group 1	1 0001✓	0,2 00-0✓	0,2,8,10 -0-0	
	2 0010✓	0,8 -000✓	<u>2,6,10,14 --10</u>	Already Covered
	8 1000✓			
group 2	5 0101✓	1,5 0-01		
	6 0110✓	1,9 -001✓		
	9 1001✓	2,6 0-10✓		
	10 1010✓	2,10 -010 ✓		
group 3	7 0111✓	8,9 100-✓		
	14 1110✓	<u>8,10 10-0</u> ✓		
		5,7 01-1		
	6,7 011-			
	6,14 -110 ✓			
	10,14 1-10✓			

Any more groupings possible ??



Prime Implicants

Column I

Column II

Column III

group 0	<u>0 0000</u> ✓
group 1	1 0001✓
	2 0010✓
	8 1000✓
group 2	<u>5 0101</u> ✓
	6 0110✓
	9 1001✓
	<u>10 1010</u> ✓
group 3	7 0111✓
	14 1110✓

0,1	000-	✓
0,2	00-0	✓
0,8	-000	✓
<hr/>		
1,5	0-01	← A = w'y'z
1,9	-001	✓
2,6	0-10	✓
2,10	-010	✓
8,9	100-	✓
8,10	10-0	✓
5,7	01-1	← B = w'xz
6,7	011-	← C = w'xy
6,14	-110	✓
10,14	1-10	✓

0,1,8,9	-00-	← D = x'y'
0,2,8,10	-0-0	← E = x'z'
<u>2,6,10,14</u>	--10	← F = yz'

A, B, C, D, E, F are Prime Implicants



Essential Prime Implicants

2. Find Essential Prime Implicants

The Minterms 9 and 14 are covered by single terms

		Minterms									
		0	1	2	5	6	7	8	9	10	14
Prime Implicants	(1,5) $A=w'y'z$		X		X						
	(5,7) $B=w'xz$				X		X				
	(6,7) $C=w'xy$					X	X				
	(0,1,8,9) $D=x'y'$	X	X					X	X		
	(0,2,8,10) $E=x'z'$	X		X				X		X	
	(2,6,10,14) $F=yz'$			X		X				X	X

$$f(w, x, y, z) = \sum(0,1,2,5,6,7,8,9,10,14)$$

yz' and $x'y'$

Essential Prime Implicants



Essential Prime Implicants

Partial Solution: $D+F + \text{-----}$

		0	1	2	5	6	7	8	9	10	14
(1,5)	$A=w'y'z$		X		X						
(5,7)	$B=w'xz$				X		X				
(6,7)	$C=w'xy$					X	X				
(0,1,8,9)	$D=x'y'$	X	X					X	X		
(0,2,8,10)	$E=x'z'$	X		X				X		X	
(2,6,10,14)	$F=yz'$			X		X				X	X

Once the Essential Prime Implicants are part of solution, The columns covered by them can be removed

Columns 0, 1, 8, 9, 2, 6, 10, 14 can be removed



Partial Solution: $D+F + \text{-----}$

		5	7
(1,5)	$A=w'y'z$	X	
(5,7)	$B=w'xz$	X	X
(6,7)	$C=w'xy$		X
(0,2,8,10)	$E=x'z'$		

↓
Redundant

		5	7
(1,5)	$A=w'y'z$	X	
(5,7)	$B=w'xz$	X	X
(6,7)	$C=w'xy$		X

Row corresponding to B is dominating the other Rows

Including B in the final solution both columns get covered

Final Solution: $D + F + B = x'y' + yz' + w'xz$



Row Dominance

In Prime Implicant table **dominated rows can be deleted**

Row B is dominating the **rows A and C**

		5	7
(1,5)	$A=w'y'z$	X	
(5,7)	$B=w'xz$	X	X
(6,7)	$C=w'xy$		X

By making **B part of minimal function** both columns 5 and 7 get covered

Dominated rows A and C can be deleted (if A and C have higher cost than B)



Column Dominance

In Prime Implicant table **dominating columns can be deleted**

Column corresponding to **9** dominates column corresponding to **8**

		8	9
(8,9,--, --)	A	X	X
(--,9,--,--)	B		X
(--,8,--,9)	C	X	X

Any set of **rows that covers dominated** column must **also cover dominating** column

If there is 8 there will always be 9 with it, **covering 8 is sufficient**

Dominating column 9 can be deleted



Special Cases

$$F(a,b,c) = \sum m(0, 1, 2, 5, 6, 7)$$

<u>0</u> 000✓	0,1 00-
1 001✓	<u>0,2</u> 0-0
<u>2</u> 010✓	1,5 -01
5 101✓	<u>2,6</u> -10
<u>6</u> 110✓	5,7 1-1
7 111✓	6,7 11-

		0	1	2	5	6	7
(0,1)	a'b'	X	X				
(0,2)	a'c'	X		X			
(1,5)	b'c		X		X		
(2,6)	bc'			X		X	
(5,7)	ac				X		X
(6,7)	ab					X	X

Essential Prime Implicants ??

Row or Column Dominance??



Special Cases

$$F(a,b,c) = \sum m(0, 1, 2, 5, 6, 7)$$

<u>0</u> 000✓	0,1 00-
1 001✓	<u>0,2</u> 0-0
<u>2</u> 010✓	1,5 -01
5 101✓	<u>2,6</u> -10
<u>6</u> 110✓	5,7 1-1
7 111✓	6,7 11-

		0	1	2	5	6	7
(0,1)	a'b'	X	X				
(0,2)	a'c'	X		X			
(1,5)	b'c		X		X		
(2,6)	bc'			X		X	
(5,7)	ac				X		X
(6,7)	ab					X	X

Low cost terms can be included, but all have same cost ??

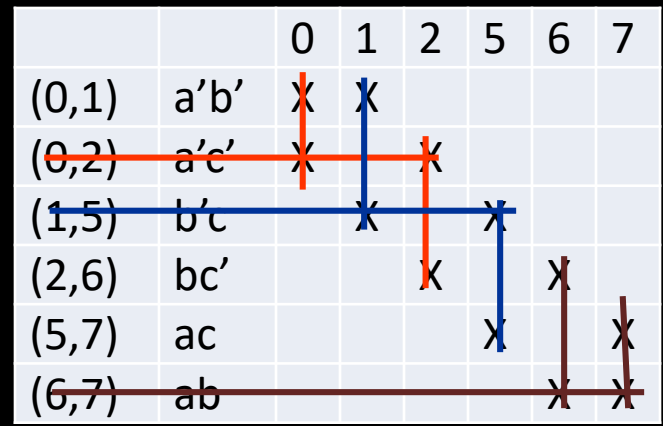
Solution 1 : $a'b' + bc' + ac$



Special Cases

$$F(a,b,c) = \sum m(0, 1, 2, 5, 6, 7)$$

<u>0</u> 000✓	0,1 00-
1 001✓	<u>0,2</u> 0-0
<u>2</u> 010✓	1,5 -01
5 101✓	<u>2,6</u> -10
<u>6</u> 110✓	5,7 1-1
7 111✓	6,7 11-



Solution 2 : $a'c' + b'c + ab$



So Trial and Error ???

Should we throw Dice for such cases ??

Petrick's Method (Interested can look it up)

The Gray Code

Imp. Features:

1. Only one bit ever changes between two successive numbers in the sequence.

→ unit distance code

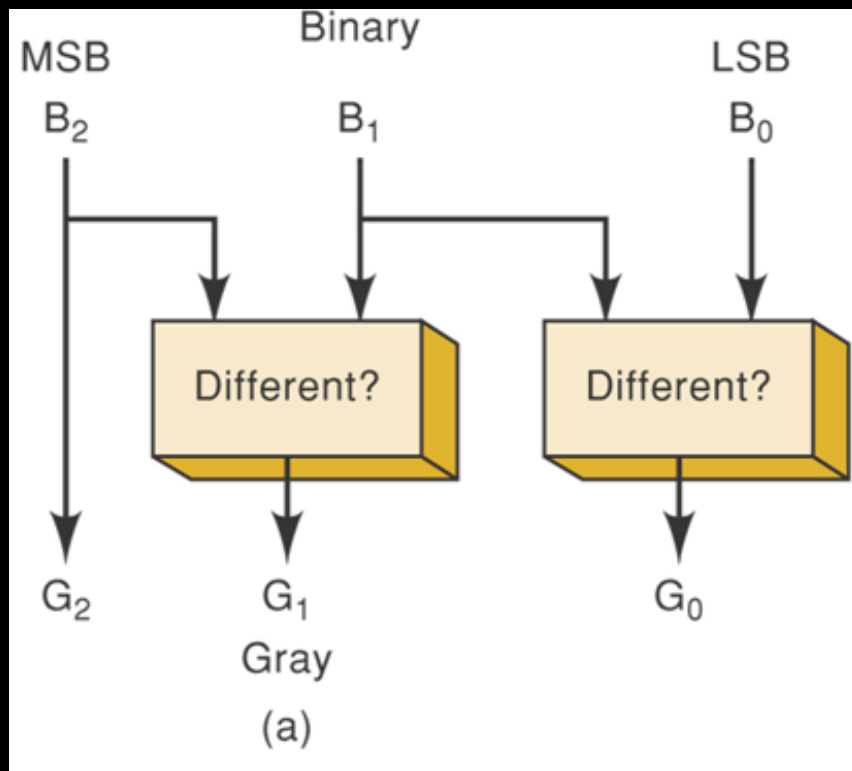
2. It is a non-weighted code.

→ not suitable for arithmetic operations.

3. Gray code is a reflective code.

i.e., the $n-1$ least significant bits for 2^{n-1} through $2^n - 1$ are the mirror images of those for 0 through $2^{n-1} - 1$.

Binary to Gray conversion:



$$G_n = B_n$$

$$G_{n-1} = B_n \oplus B_{n-1}$$

$$G_{n-2} = B_{n-1} \oplus B_{n-2}$$

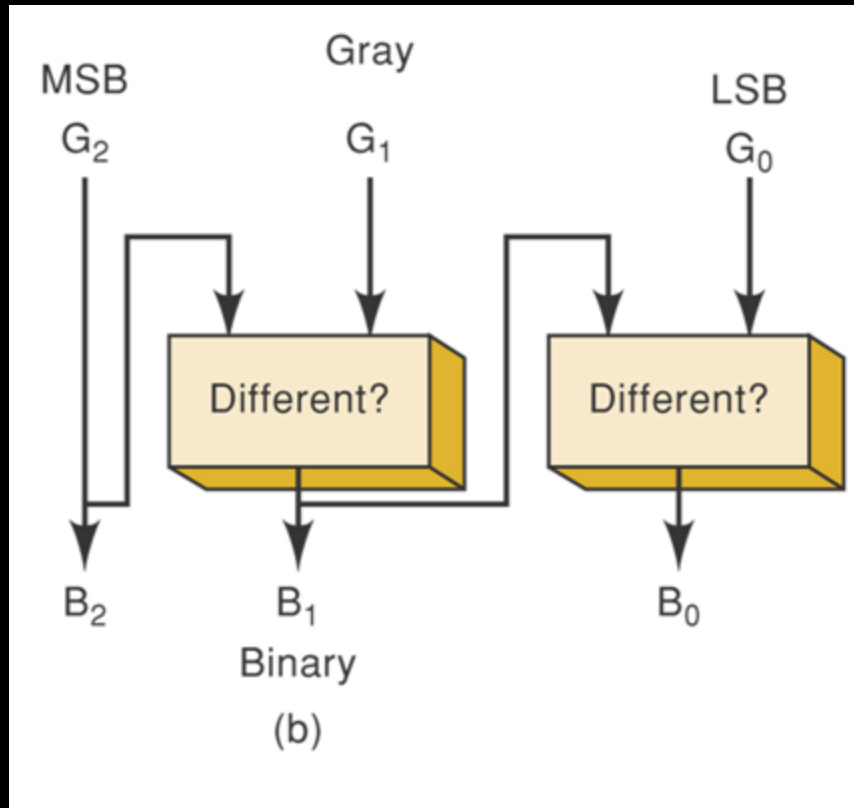
.

.

.

$$G_1 = B_2 \oplus B_1$$

Gray to binary conversion:



$$B_n = G_n$$

$$B_{n-1} = B_n \oplus G_{n-1}$$

$$B_{n-2} = B_{n-1} \oplus G_{n-2}$$

.

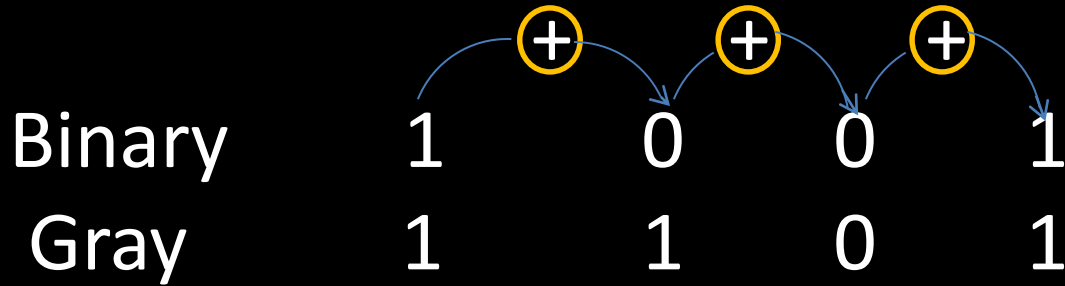
.

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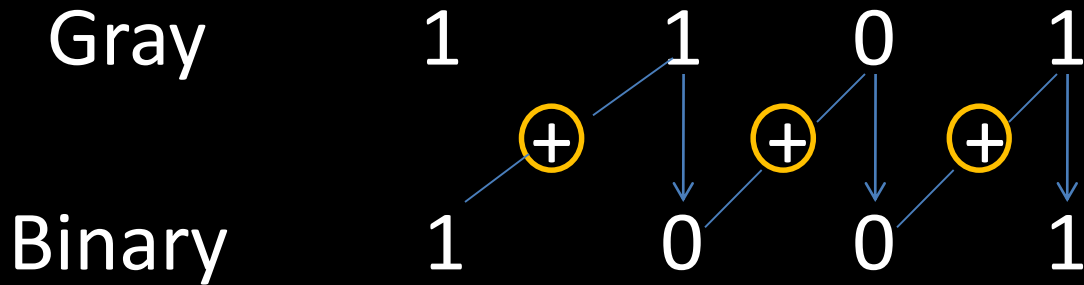
$$B_1 = B_2 \oplus G_1$$

Example:

code.



Gray to Binary conversion



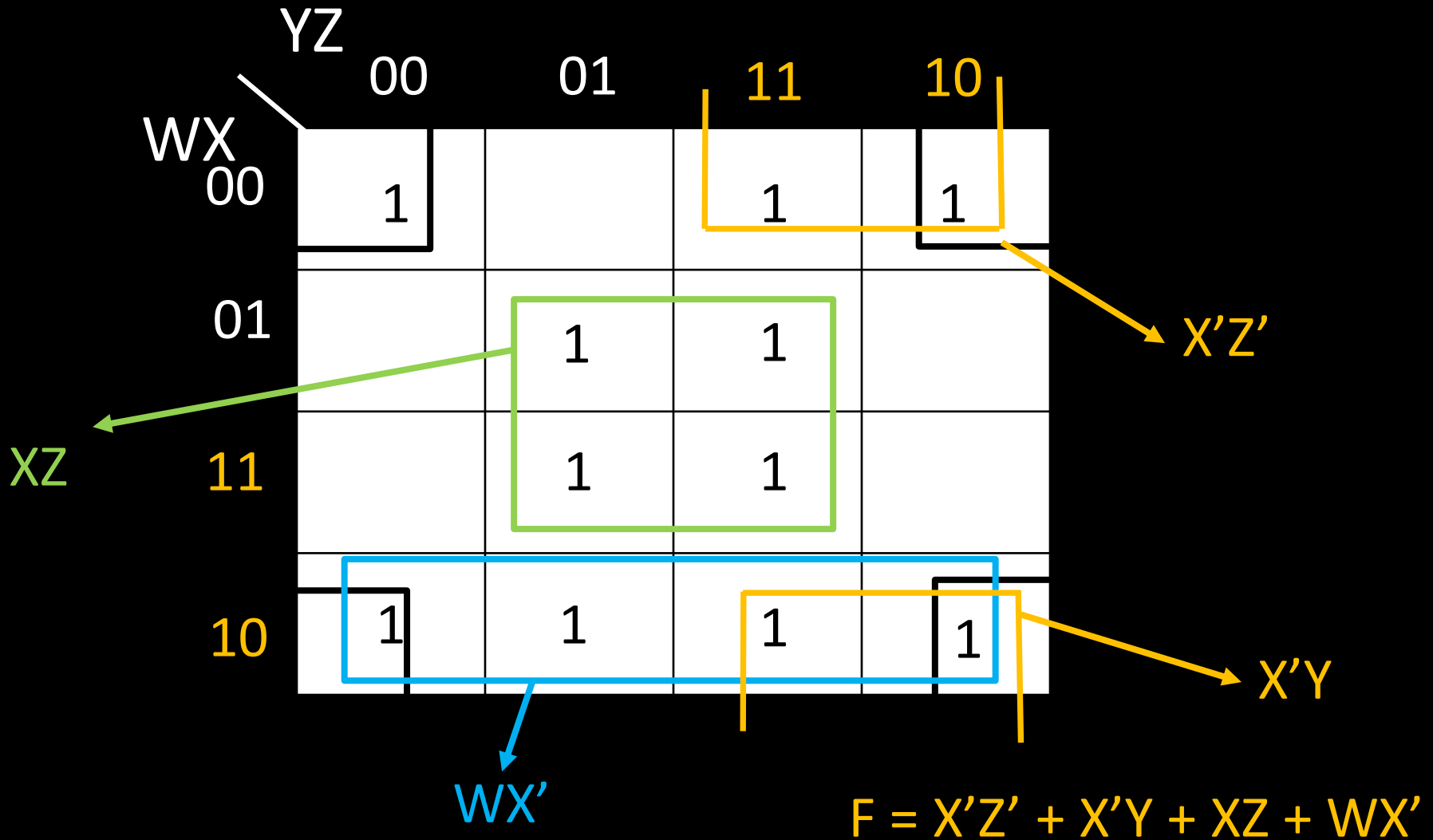
Gray code → Reflective - code

Gray Code				Decimal	Binary
1- bit	2-bit	3-bit	4-bit		4-bit
0	00	000	0000	0	0000
1	01	001	0001	1	0001
	11	011	0011	2	0010
	10	010	0010	3	0011
		110	0110	4	0100
		111	0111	5	0101
		101	0101	6	0110
		100	0100	7	0111
			1100	8	1000
			1101	9	1001
			1111	10	1010
			1110	11	1011
			1010	12	1100
			1011	13	1101
			1001	14	1110
			1000	15	1111



Using QM method solve 4-Variable K-Map

$$F(w,x,y,z) = \sum(1,3,4,5,6,7,11,14,15)$$





Thank you