Digital Design

Lecture 6: K-Maps

Last class

2/3- Variable K-maps

Basic concepts involved in K-map to reduce Boolean expressions

Solve Using K-Map

$F(A,B,C) = A'C + AB'C + BC + AB' + A'BC'$

A \ BC	00	01	11	10
0		1	1	1
1	1	1	1	

$F = C + A'B + AB'$

3-Variable K-Map

F (X,Y,Z) = π (0,1,2,5)

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

X \ Y Z	00	01	11	10
0	0	0	1	0
1	1	0	1	1

Y=0, Z=1 constant , X varies (Y+Z’)

(X+Z)

X=0, Z=0 constant , Y varies

F = (Y + Z’) (X + Z)

3-Variable K-Map

$F(X,Y,Z) = \pi(2,3,4,5)$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

X \ Y Z	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$F = (X+Y)(X'+Y')$

Don’t care conditions

$F(A,B,C) = \sum(3,4,6,7)$

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

A \ BC	00	01	11	10
0	0	0	1	0
1	1	0	1	1

$F = BC + AC'$

Don’t care conditions

F (A,B,C) = ∑ (3,4,6,7)

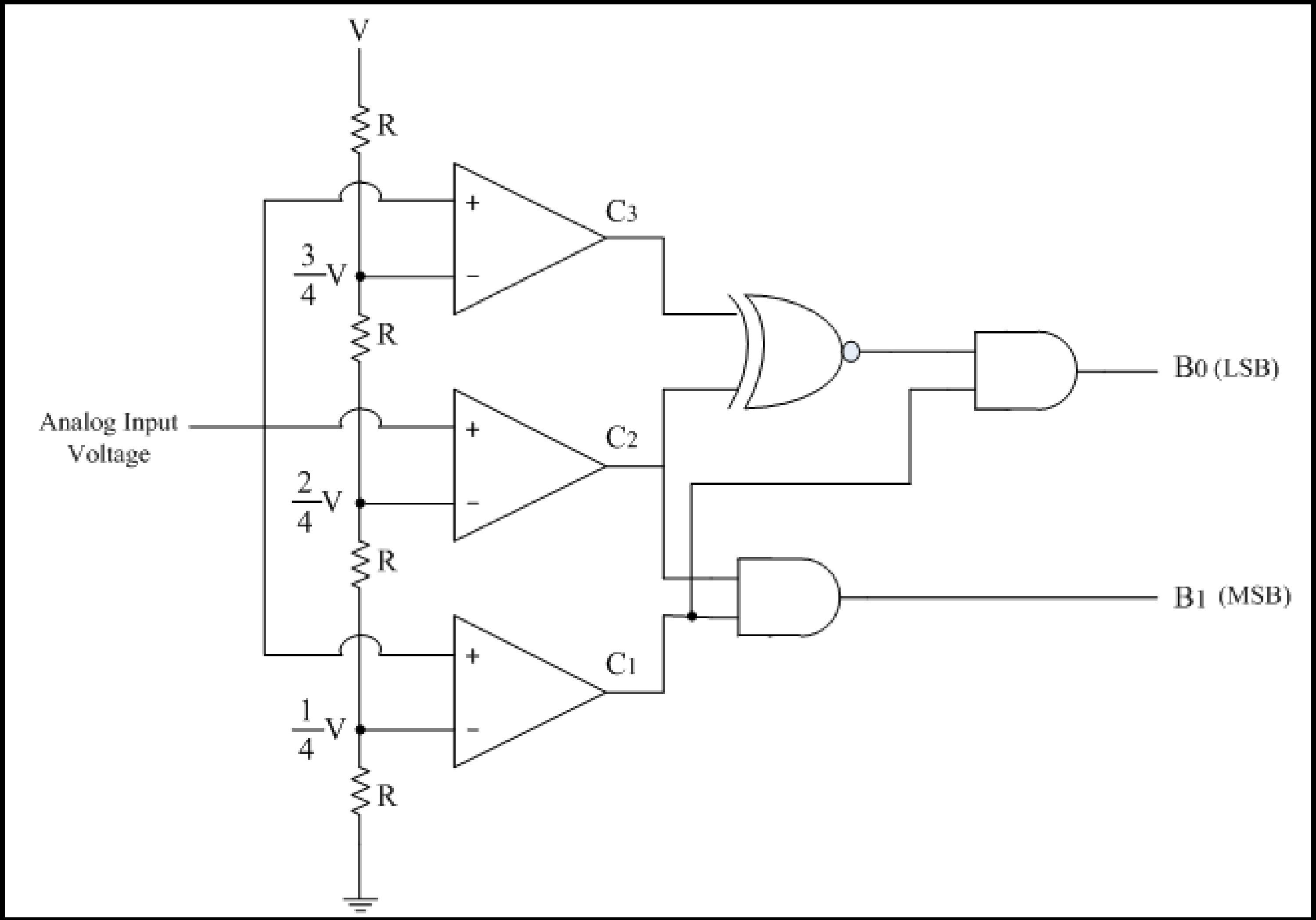
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	X
1	1	0	1
1	1	1	1

A \ BC	00	01	11	10
0	0	0	1	0
1	1	X	1	1

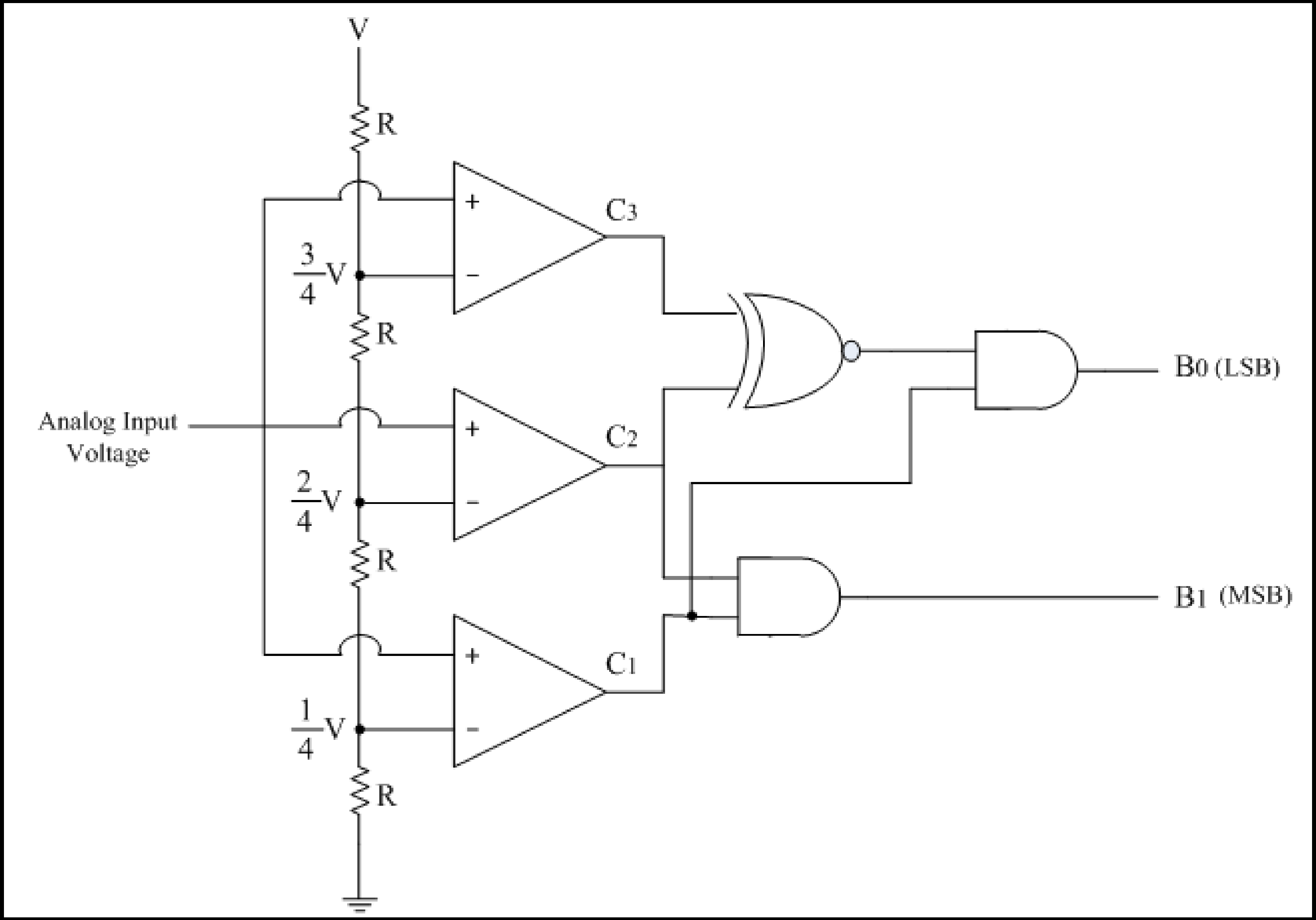
F = BC + A

Earlier expression F = BC + AC’

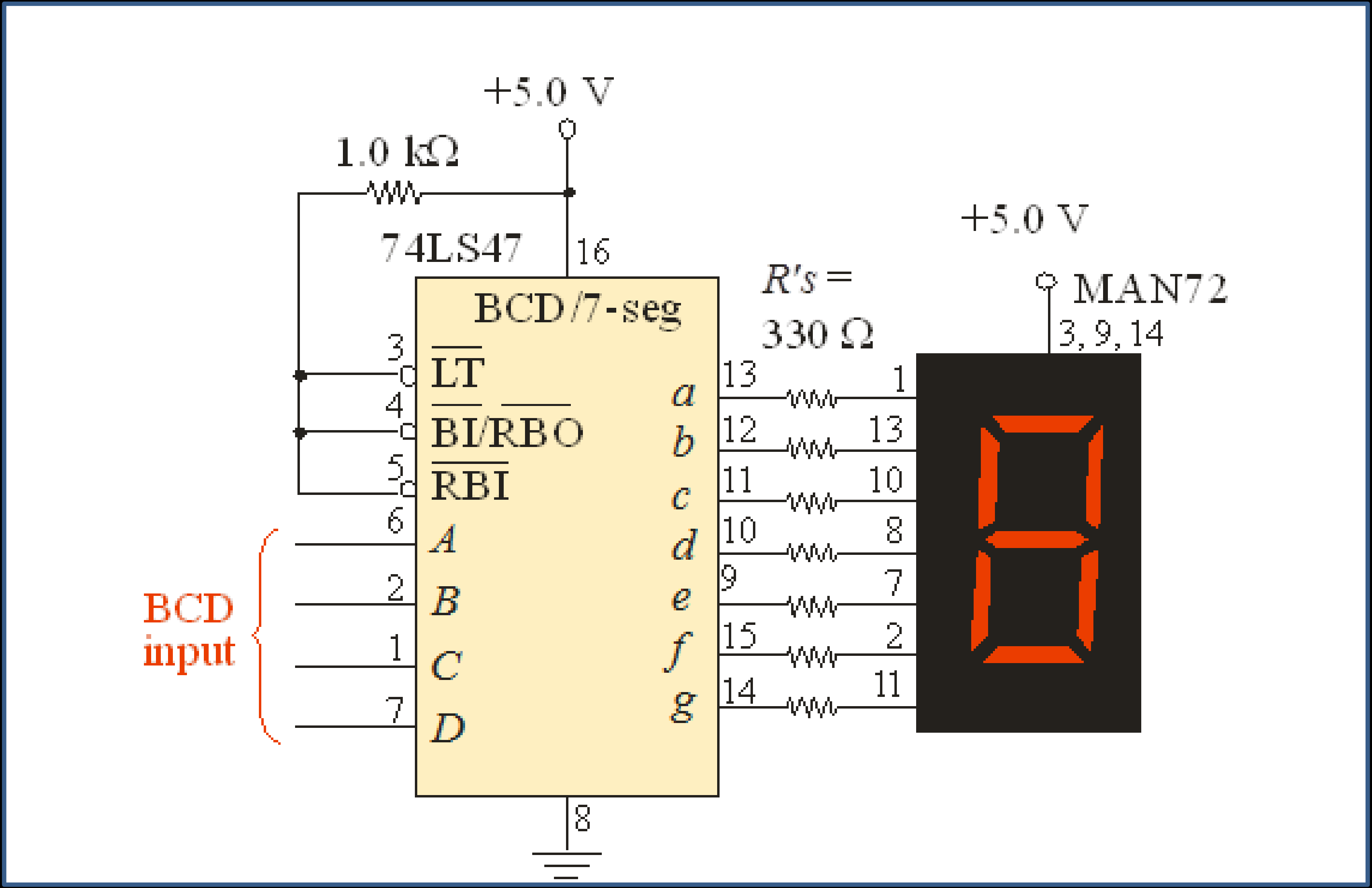
DAC



DAC



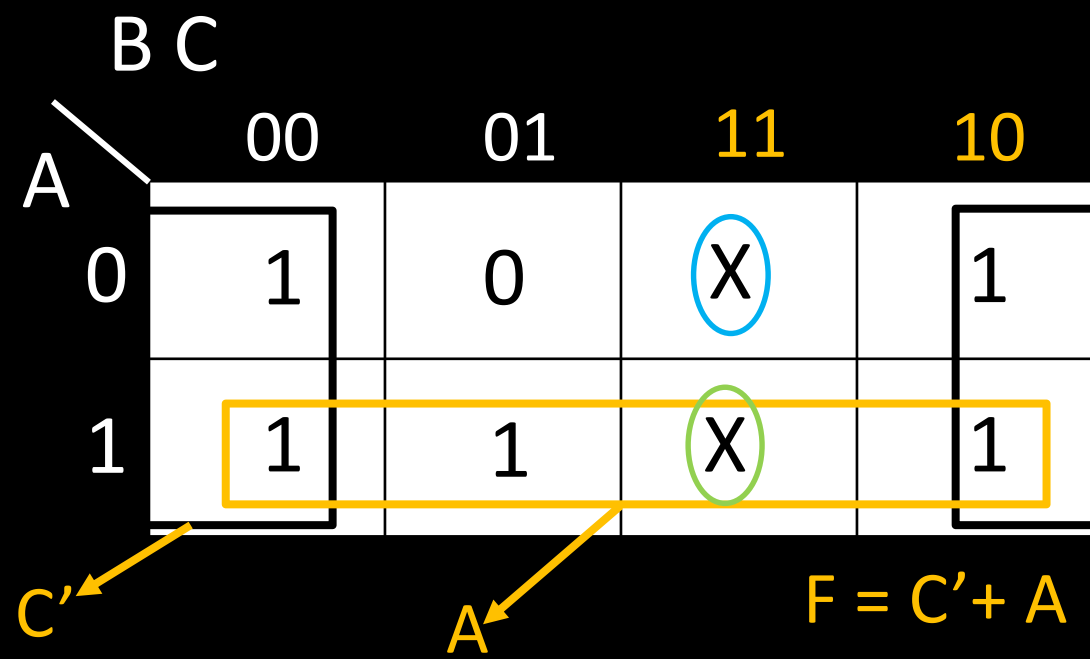
7 Segment Display Driver



Don’t care conditions

F (A,B,C) = $\sum$ (0,2,4,5,6) + d (3,7)

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	X
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	X



X Considered as 1 if it helps in reducing the terms or literals

X taken as 0 and neglected if it does not reduce terms or literals

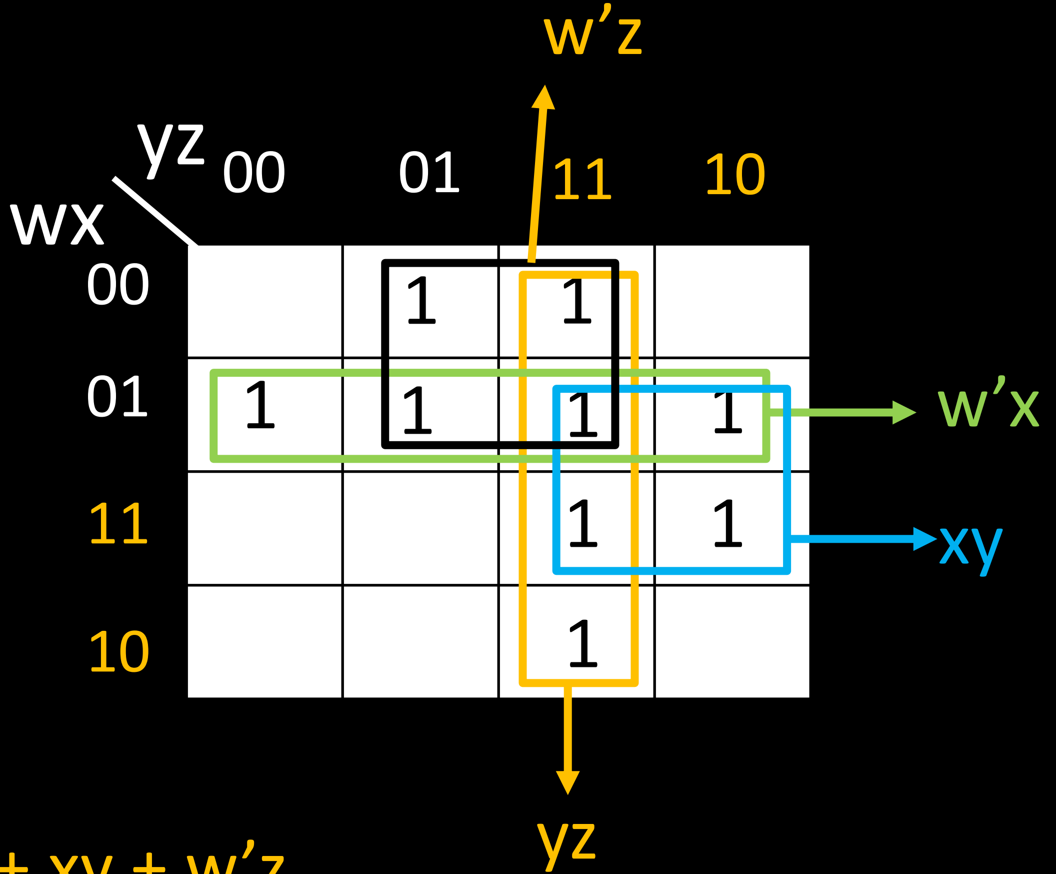
4-Variable K-Map

WX \ YZ	00	01	11	10
00	(0,0,0,0) m_0	(0,0,0,1) m_1	(0,0,1,1) m_3	(0,0,1,0) m_2
01	(0,1,0,0) m_4	(0,1,0,1) m_5	(0,1,1,1) m_7	(0,1,1,0) m_6
11	(1,1,0,0) m_{12}	(1,1,0,1) m_{13}	(1,1,1,1) m_{15}	(1,1,1,0) m_{14}
10	(1,0,0,0) m_8	(1,0,0,1) m_9	(1,0,1,1) m_{11}	(1,0,1,0) m_{10}

Verify 1-bit variations between adjacent cells

4-Variable K-Map

$F(w,x,y,z) = \sum(1,3,4,5,6,7,11,14,15)$



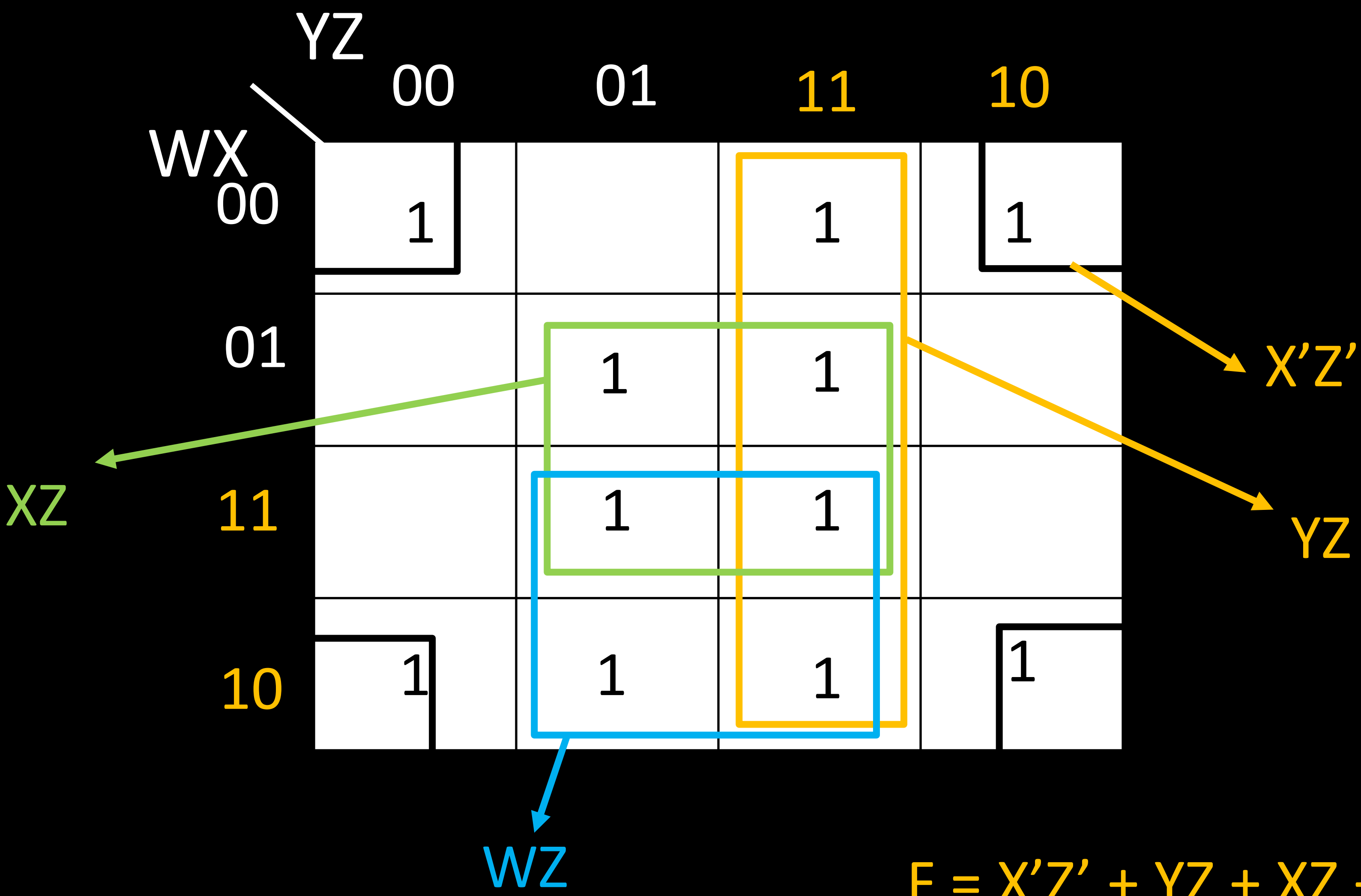
F = w'x + yz + xy + w'z

4-Variable K-Map

WX \ YZ	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

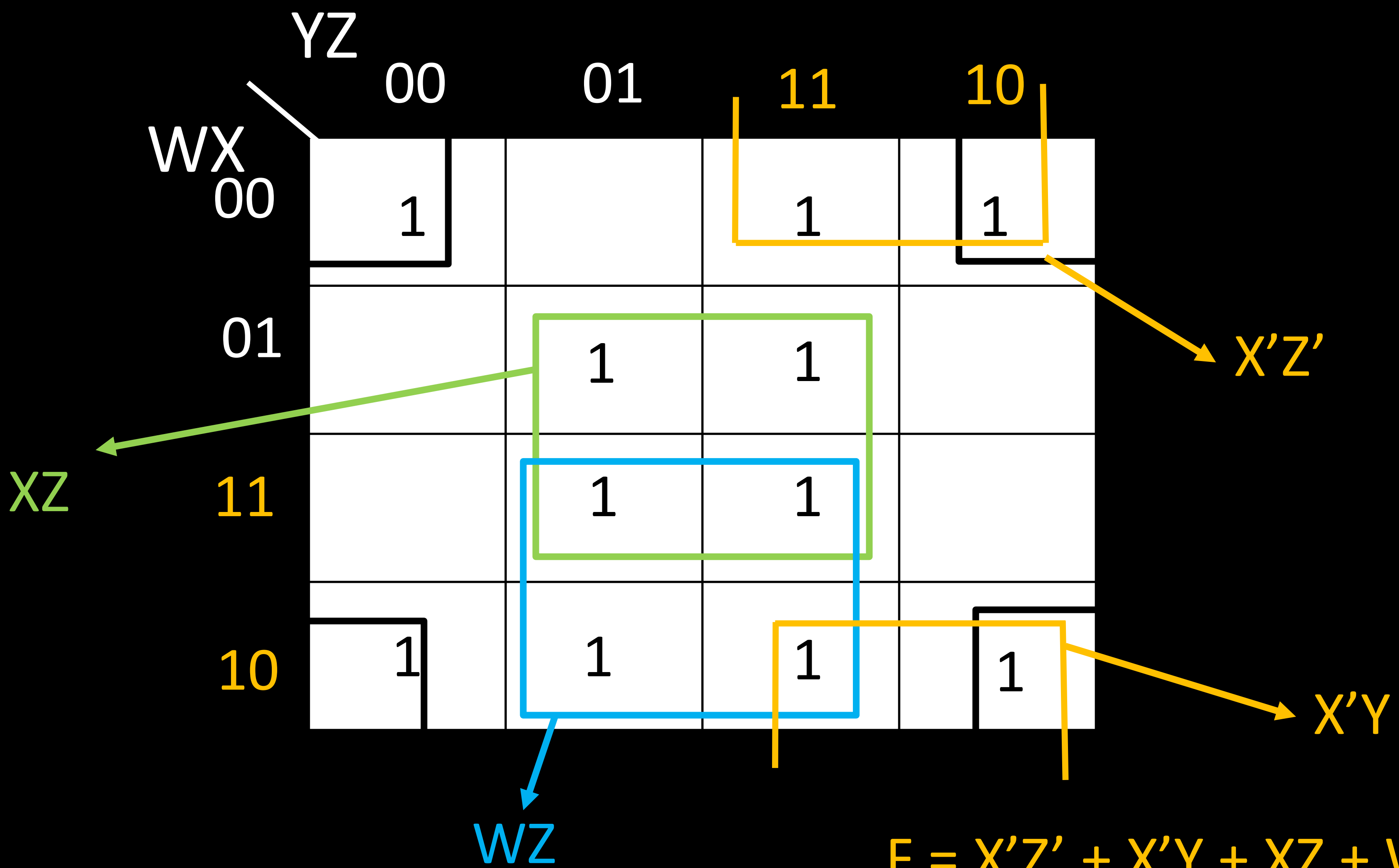
$\sum (0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$

4-Variable K-Map



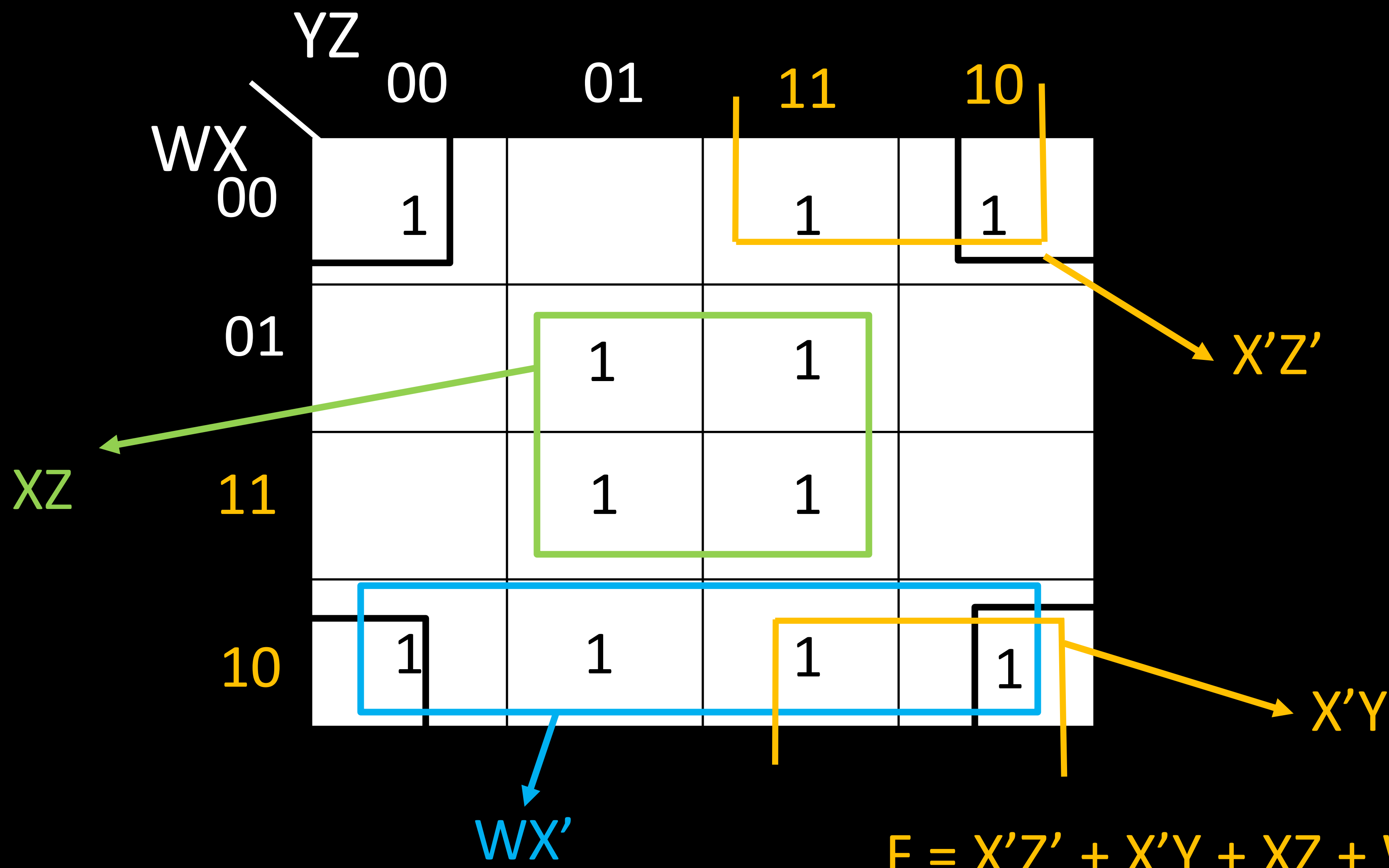
F = X'Z' + YZ + XZ + WZ

4-Variable K-Map



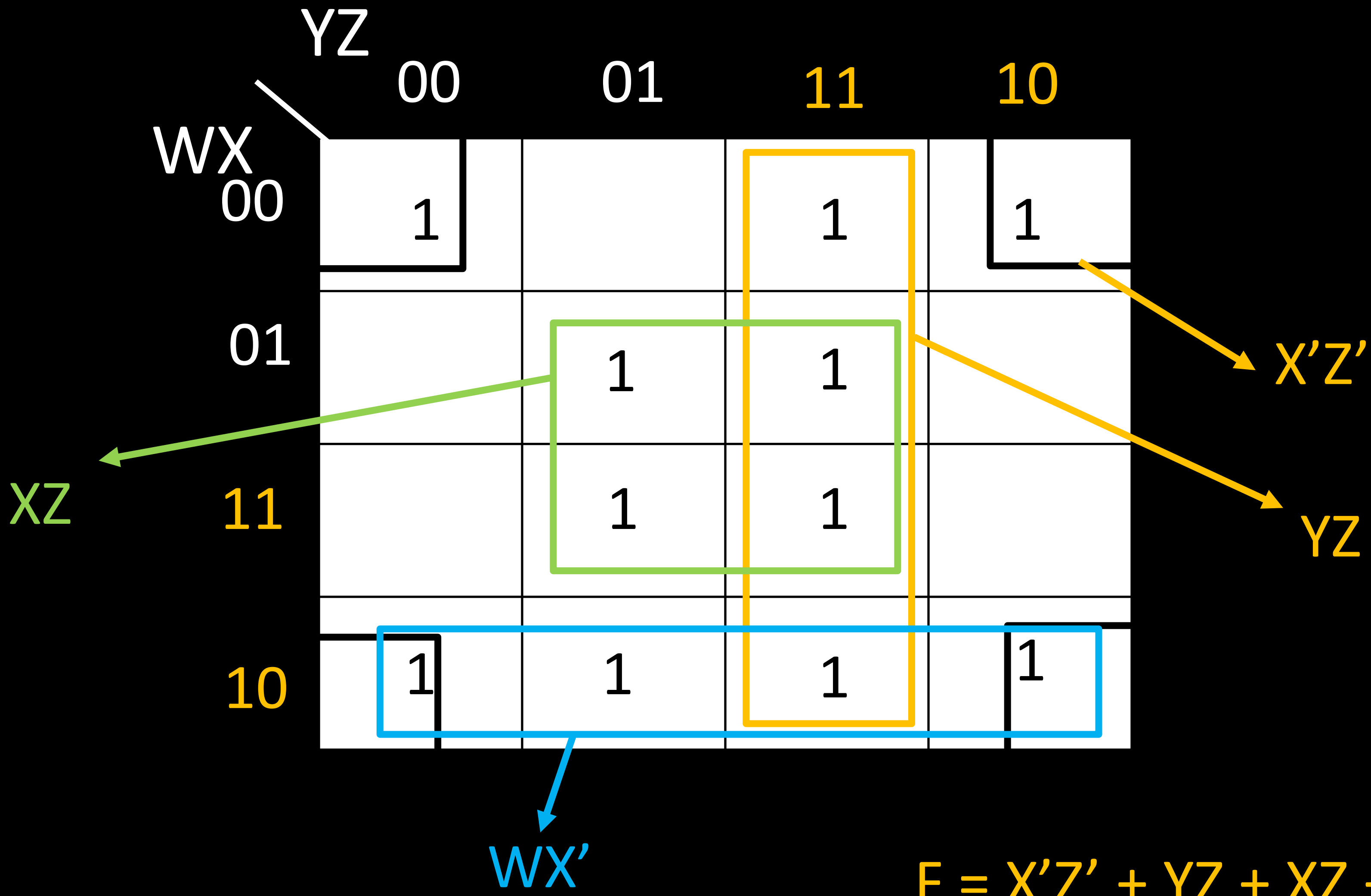
F = X'Z' + X'Y + XZ + WZ

4-Variable K-Map



F = X'Z' + X'Y + XZ + WX'

4-Variable K-Map



F = X'Z' + YZ + XZ + WX'

4-Variable K-Map

F = X'Z' + YZ + XZ + WZ

F = X'Z' + X'Y + XZ + WZ

F = X'Z' + X'Y + XZ + WX'

F = X'Z' + YZ + XZ + WX'

X'Z', YZ, XZ, WZ, X'Y, WX' are Prime Implicants

All are minimized and all are correct solutions

What are the terms common here ??

X'Z' and XZ appear in all the solutions

X'Z' and XZ are called Essential Prime Implicants

1. Implicant of Functions

Implicant is a product/minterm term in Sum of Products (SOP) or sum/maxterm term in Product of Sums (POS) of a Boolean function.

E.g.,

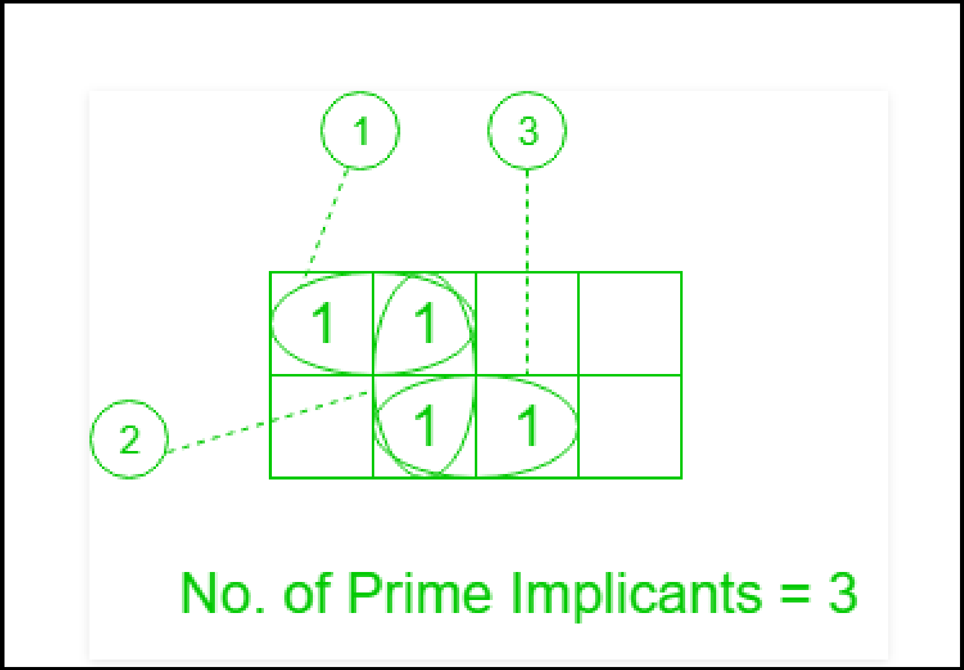
Consider a boolean function, F = AB + ABC + BC.

Implicants are AB, ABC and BC.

Implicant: For a Boolean function F expressed in SOP form, a product term p is an implicant of the Function F if and only if F=1 for every combination of the input values to the variables of the product term p, for which p =1.

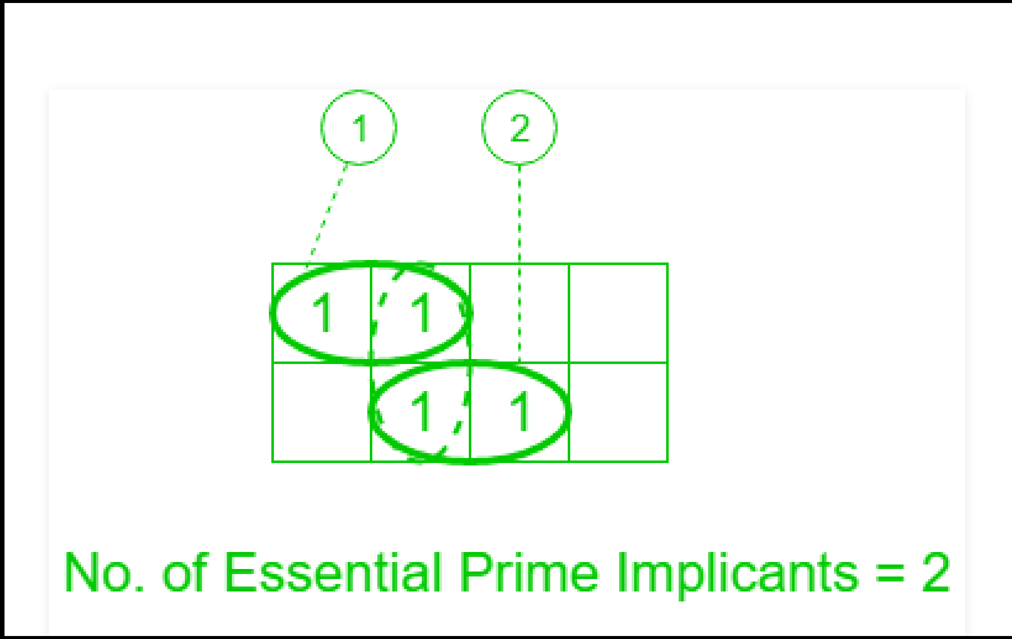
2. Prime Implicant of Functions

A group of square or rectangle made up of bunch of adjacent minterms which is allowed by definition of K-Map are called **prime implicants(PI)** i.e. all possible groups formed in K-Map.



3. Essential Implicant of Functions

These are those subcubes(groups) which cover atleast one minterm that can't be covered by any other prime implicant. Essential prime implicants(EPI) are those prime implicants which always appear in final solution..



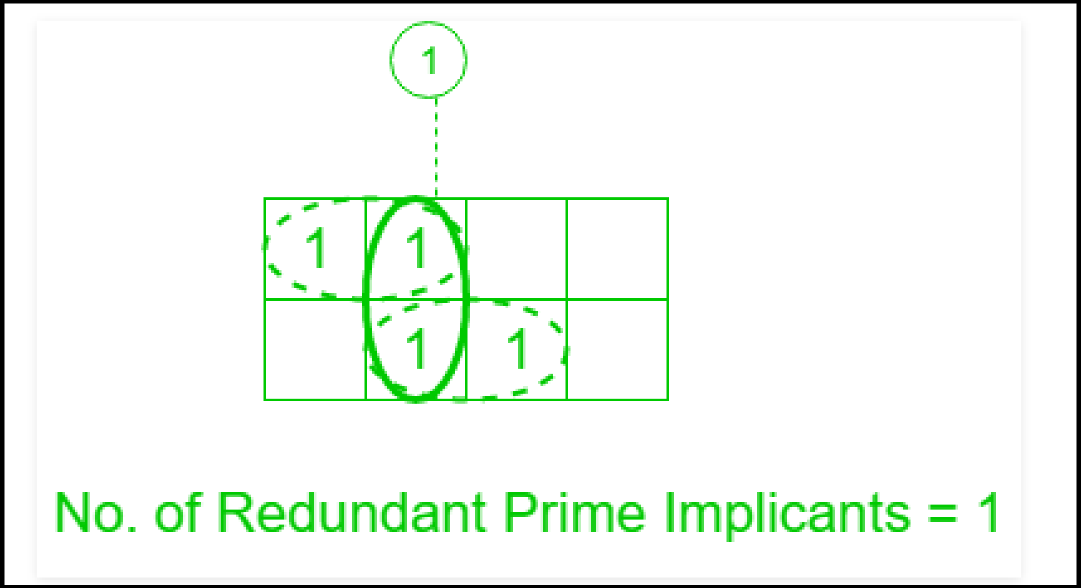
Implicant: For a Boolean function F expressed in SOP form, a product term p is an implicant of the Function F if and only if F=1 for every combination of the input values to the variables of the product term p, for which p =1.

Prime Implicant: An implicant of a function F is a prime implicant of F if it is not completely enclosed in bigger valid group of minterms (or maxterms) in the K-map.

Essential Prime Implicant: A prime implicant of a function F is an essential prime implicant of F if the loop for prime implicant contains at least one minterm (maxterm) box that is not contained in any other prime implicant loop of F.

4. Redundant Implicant of Functions

The prime implicants for which each of its minterm is covered by some essential prime implicant are redundant prime implicants(RPI). This prime implicant never appears in final solution..



4-Variable K-Map

WX \ YZ	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

How to find essential prime Implicants ?

Check how many ways a minterm can be covered ?

Select all minterms that are covered in only one way

Minimized expressions corresponding to minterms that are covered in only one way are essential prime implicants

4-Variable K-Map

WX \ YZ	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

m_0 can be covered in only one way

Corresponding expression X’Z’

Is there any way that m_0 can be covered ??

4-Variable K-Map

WX \ YZ	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

m_3 can be covered in multiple ways

4-Variable K-Map

WX \ YZ	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

m_2 can be covered in multiple ways

4-Variable K-Map

WX \ YZ	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

m_5 can be covered in only one way

Corresponding expression is XZ

4-Variable K-Map

WX \ YZ	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

Similarly it can be analyzed that m_7, m_8,m_9,m_{11},m_{13},m_{15} can be covered in multiple ways

4-Variable K-Map

WX \ YZ	00	01	11	10
00	1		1	1
01		1	1	
11		1	1	
10	1	1	1	1

Only m_0 and m_5 can be covered in only one way

Hence the corresponding expressions XZ and X'Z' are called Essential prime Implicants

Thank You