

Digital Design

Lecture 4: K-Map [For simplification of Boolean expressions]



Birla Institute of Technology & Science, Pilani
Hyderabad Campus

2020

Innovate

achieve

1

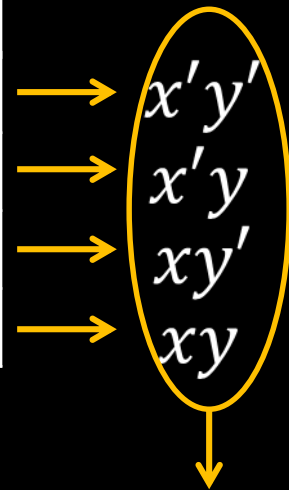
lead



Boolean Functions

Each input entry will be represented by a term

x	y
0	0
0	1
1	0
1	1



Minterms

Standard Product

Designation

m_0

m_1

m_2

m_3

$F3$
1
0
1
1

$$F3 = x'y' + xy' + xy$$

$$F3 = m_0 + m_2 + m_3$$

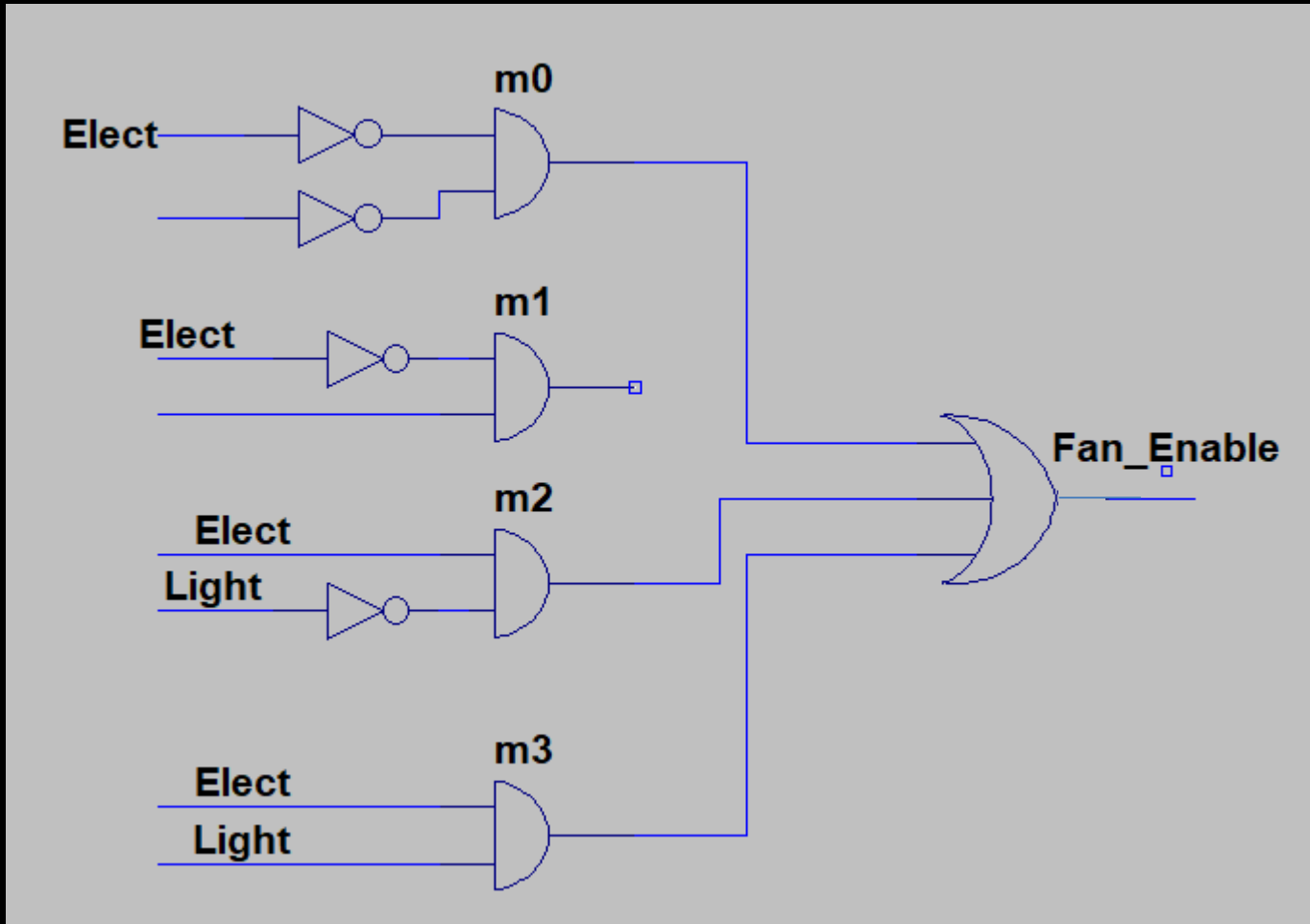
$$F3 = \sum(0,2,3)$$

Sum of Products

(Canonical form-Type1)



Boolean Functions





Boolean Functions

Each input entry will be represented by a term

x	y		Designation		$F3$
0	0	→	$x + y$	M_0	0
0	1	→	$x + y'$	M_1	1
1	0	→	$x' + y$	M_2	0
1	1	→	$x' + y'$	M_3	0

Maxterms

Standard Sums

$$F3 = (x + y). (x' + y). (x' + y')$$

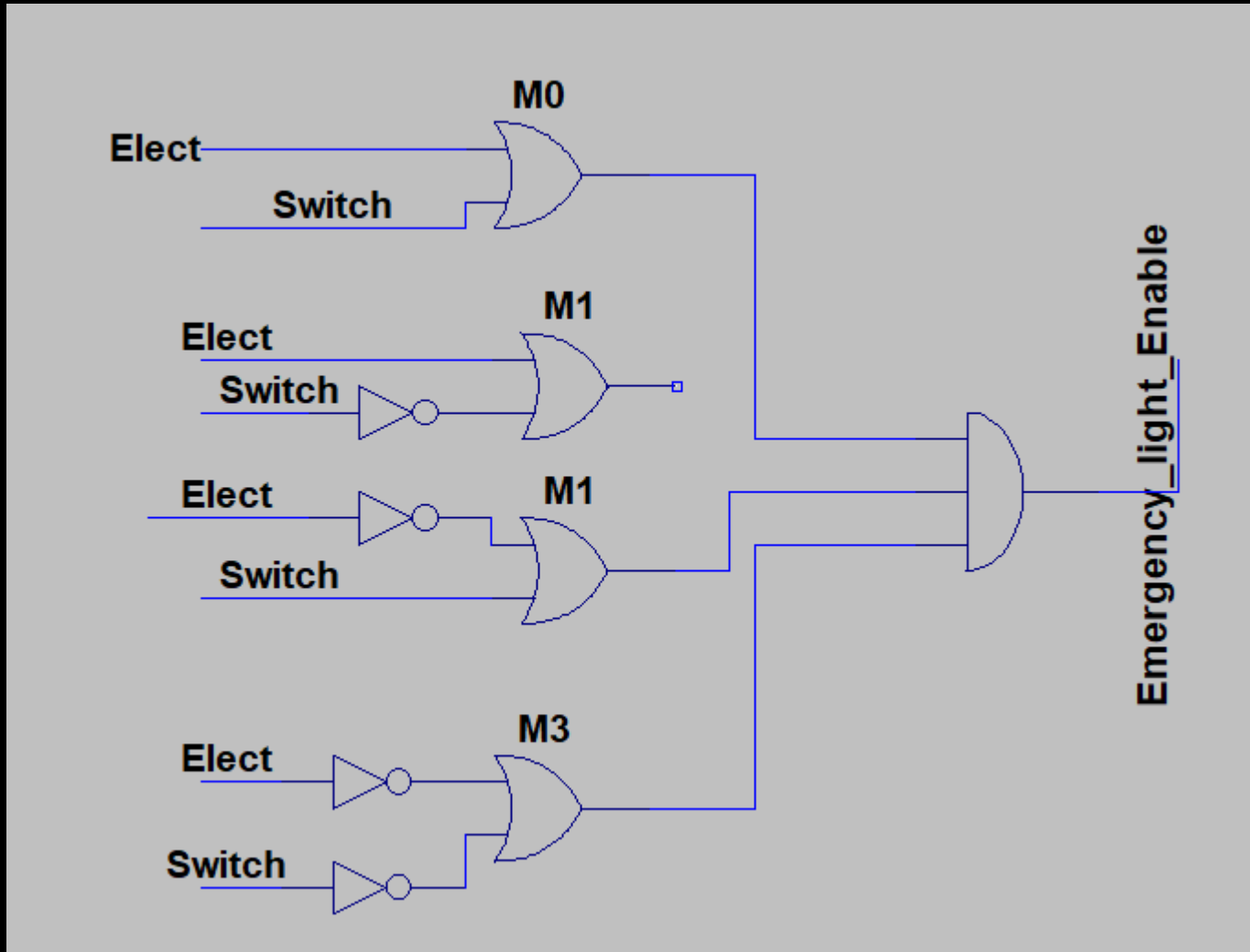
$$F3 = M_0. M_2. M_3$$

$$F3 = \pi(0,2,3)$$

Product of Sums
(Canonical form-Type2)



Boolean Functions





Why to use K-Maps?

Canonical Expression can be **simplified by algebraic means**

Algebraic method **lacks specific rules**

K-map method simple, straight forward and pictorial understanding and gives both SOP and POS forms



2-Variable K-Map

Truth-table to K-map

	X	Y	F
minterm 0 →	0	0	1
minterm 1 →	0	1	0
minterm 2 →	1	0	1
minterm 3 →	1	1	1

X \ Y	0	1
0	(0,0) 1	(0,1) 0
1	(1,0) 1	(1,1) 1



2-Variable K-Map

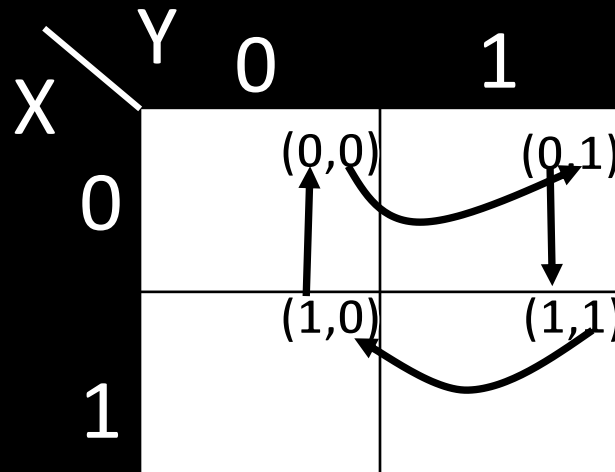
Representation of cells

		Y	
		0	1
X	0	$m_0 \quad (\bar{x}\bar{y})$ 1	$m_1 \quad (\bar{x}y)$ 0
	1	$m_2 \quad (x\bar{y})$ 1	$m_3 \quad (xy)$ 1



2-Variable K-Map

One bit variation between adjacent cells



What is the importance of 1-bit variation ??



2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	0

		Y	
		0	1
X	0	1	1
	1	0	0

$X'Y'$ ← (grouping arrow from (0,0) to (0,1))
+
 $X'Y$ ← (grouping arrow from (0,1) to (1,1))

$$X'Y' + X'Y = X'(Y' + Y) = X'$$

If there are 1's in adjacent cells then they can be grouped and Minimized functions can be obtained



2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	0

		Y	
		0	1
X	0	1	1
	1	0	0

A yellow box highlights the two cells in the row where X=0 (both Y=0 and Y=1). A yellow arrow points from the text 'X'' to the X=0 row label.

$X = 0$ constant Y varies from $0 \rightarrow 1$



2-Variable K-Map

Simplification

X	Y	F
0	0	0
0	1	0
1	0	1
1	1	1

		Y	
		0	1
X	0	0	0
	1	1	1

A yellow box highlights the two cells in the row where X=1 (the bottom row), which contain the values 1 and 1. A yellow arrow points from the text 'X' to the left side of this row, indicating that the function is constant at 1 for X=1.

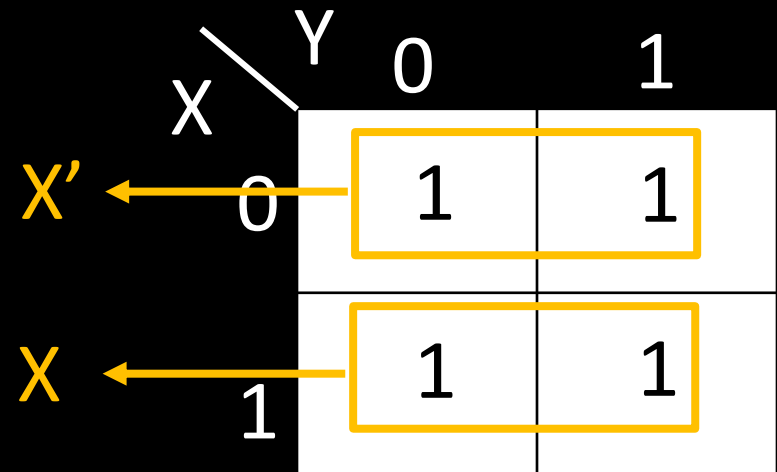
$X = 1$ constant Y varies from $0 \rightarrow 1$



2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	1



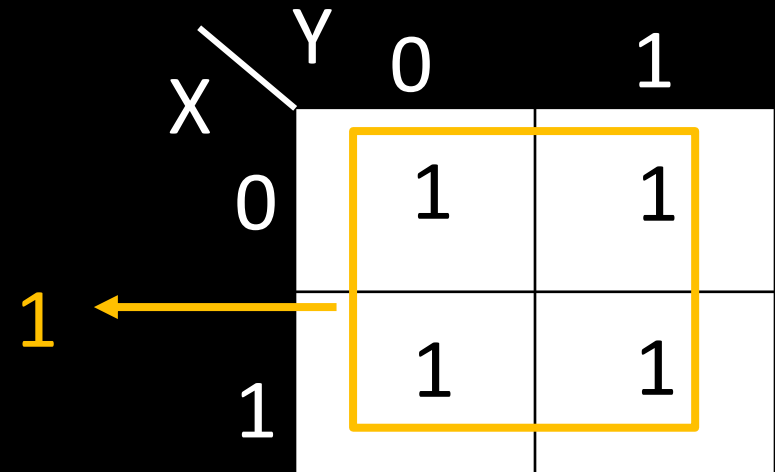
$$X' + X = 1$$



2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	1



Adjacent cells can be grouped

2- cells at a time

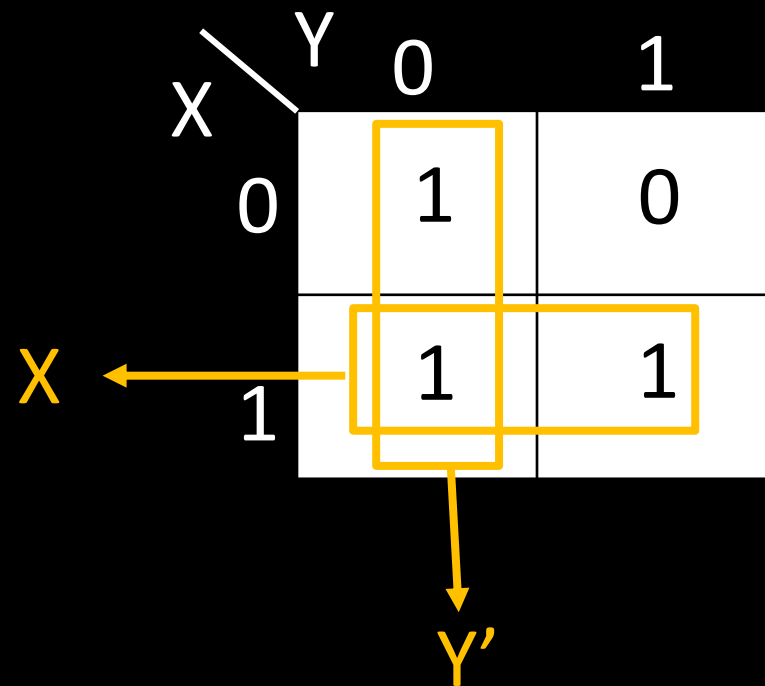
4- cells at a time



2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1



$$F = X + Y'$$

******A term can be grouped multiple times if it helps in simplifying the expression



3-Variable K-Map

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

		YZ			
		00	01	11	10
X	0	(0,0,0) m_0	(0,0,1) m_1	(0,1,1) m_3	(0,1,0) m_2
	1	(1,0,0) m_4	(1,0,1) m_5	(1,1,1) m_7	(1,1,0) m_6

Should the next entry be (0, 1, 0) ?

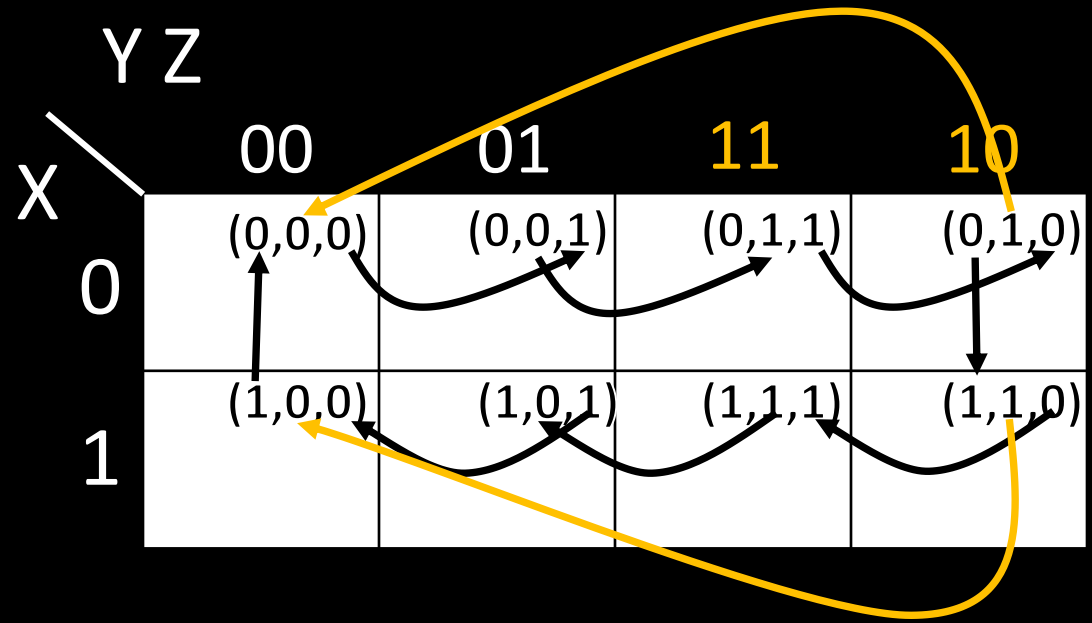
(0,0,1) → (0,1,0)

**Two bit variation here, what to do ??



3-Variable K-Map

Adjacent Cells



Any more adjacent cells ??

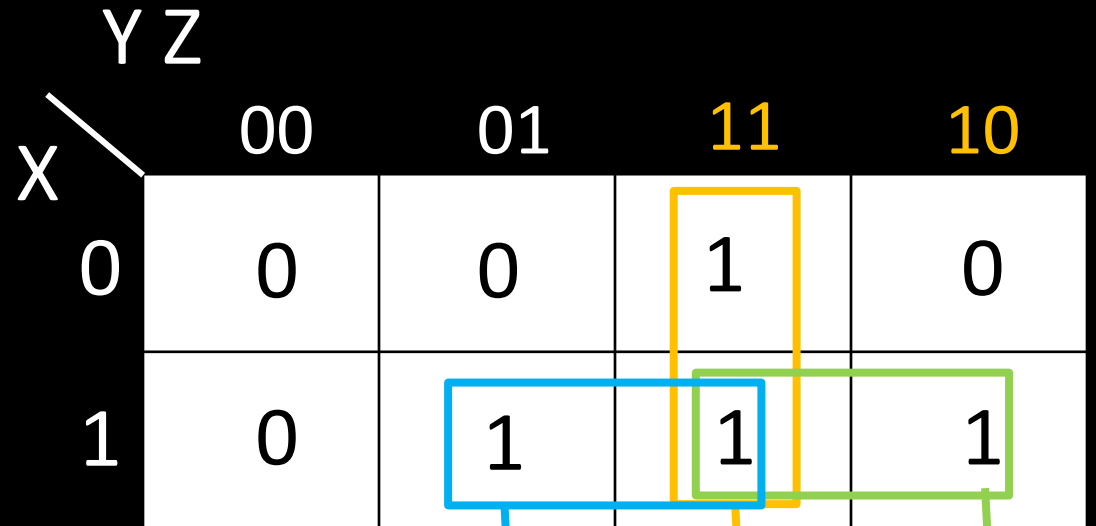


3-Variable K-Map

$$F = XY + YZ + XZ$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1



X=1, Z=1 constant, Y varies

Y=1, Z=1 constant, X varies

X=1, Y=1 constant, Z varies

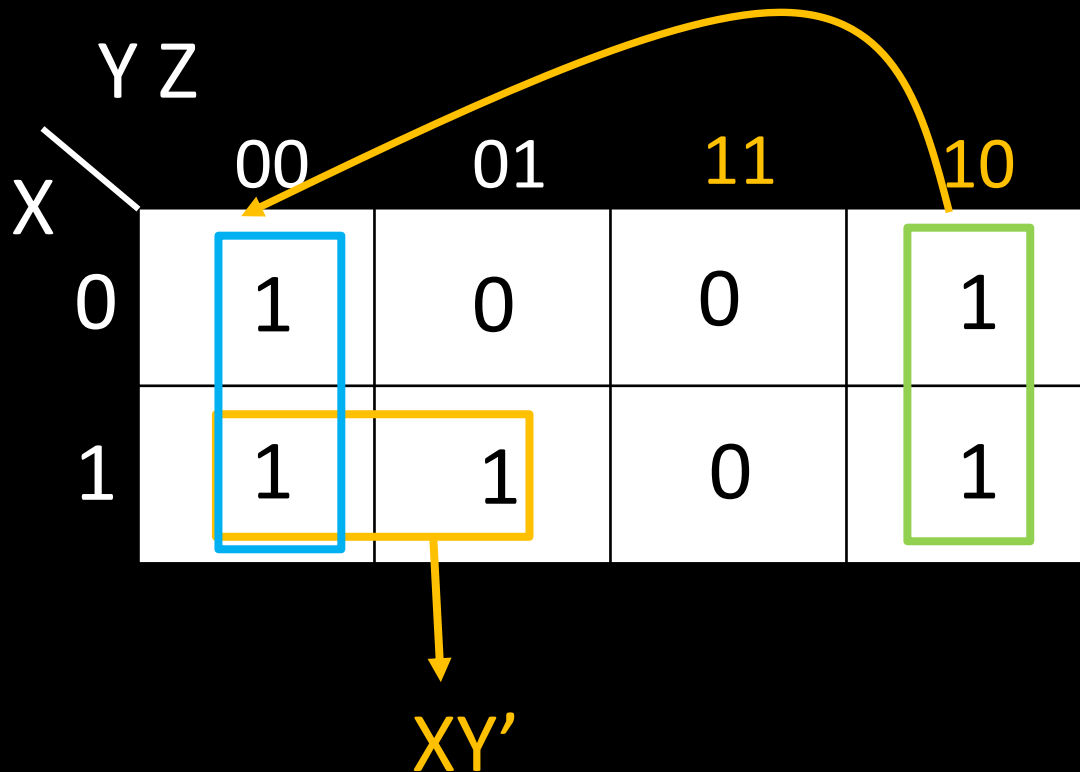


3-Variable K-Map

$$F(X,Y,Z) = \sum (0,2,4,5,6)$$

2-cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



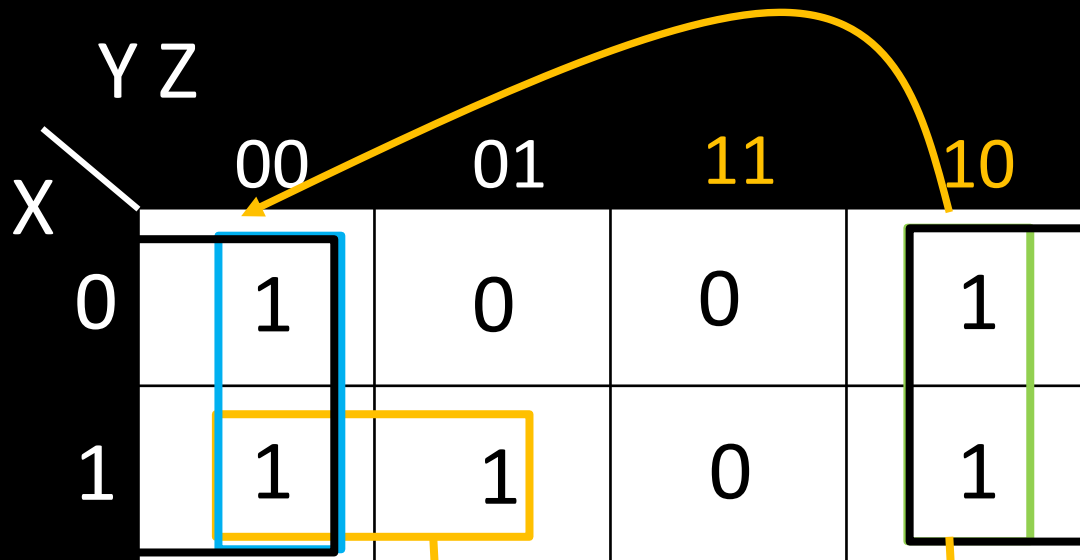


3-Variable K-Map

$$F(X,Y,Z) = \sum (0,2,4,5,6)$$

2-cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



$$F = XY' + Z'$$

Z=0, constant, X, Y Change



3-Variable K-Map

$$F(X,Y,Z) = \sum (2,3,4,5)$$

2-cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

		YZ			
		00	01	11	10
X	0	0	0	1	1
	1	1	1	0	0

$$F = XY' + X'Y$$



3-Variable K-Map

$$F(X,Y,Z) = \sum (3,4,6,7)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		YZ			
		00	01	11	10
X	0	0	0	1	0
	1	1	0	1	1

The K-map shows three groupings: a vertical group of two cells (0,1) in the 11 column (yellow box), a horizontal group of three cells (1,0,1) in the 11 row (green box), and a vertical group of two cells (0,1) in the 10 column (black box).

Is the third grouping necessary ?

All ones should get covered atleast once

If all of them are covered then there is no need to group them again



3-Variable K-Map

$$F(X,Y,Z) = \sum (3,4,6,7)$$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		YZ			
		00	01	11	10
X	0	0	0	1	0
	1	1	0	1	1

$$F = YZ + XZ'$$

$$F = YZ + XZ' + XZ'$$

Both expressions will result in same truth table

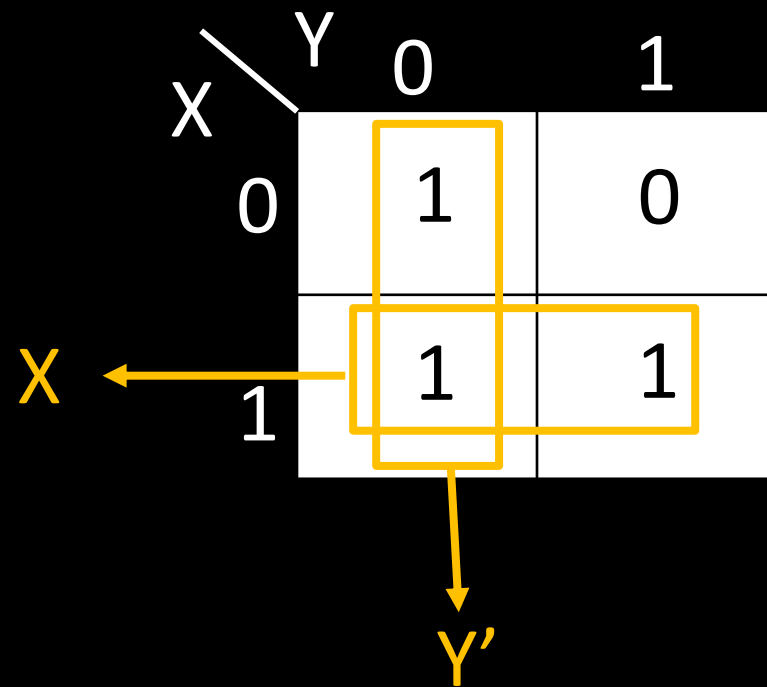
But only first expression is minimized



2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1



$$F = X + Y'$$

A term can be grouped multiple times if it helps in simplified expression



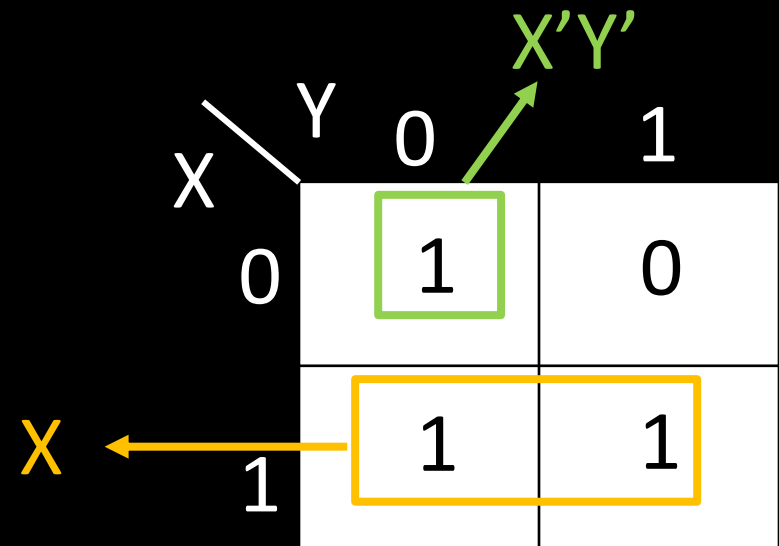
2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1

$$F = X + Y'$$

$$F = X + X'Y'$$



Which of the two is a minimized function ??



3-Variable K-Map

$$F(X,Y,Z) = \pi(0,1,2,5)$$

2-cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

		YZ			
		00	01	11	10
X	0	0	0	1	0
	1	1	0	1	1

Y=0, Z=1 constant, X varies $(Y+Z')$

$(X+Z)$

X=0, Z=0 constant, Y varies

$$F = (Y + Z') (X + Z)$$



3-Variable K-Map

$$F(X,Y,Z) = \pi(2,3,4,5)$$

2-cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

		YZ			
		00	01	11	10
X	0	0	0	1	1
	1	1	1	0	0

$$F = (X+Y)(X'+Y')$$



Next Class

4/5-variable K-map

K-map with don't care conditions