Digital Design

Lecture 4: K-Map

[For simplification of Boolean expressions]

Boolean Functions

Each input entry will be represented by a term

x	y
0	0
0	1
1	0
1	1

Designation

$x'y'$ m_0

$x'y$ m_1

xy' m_2

xy m_3

Minterms

Standard Product

F3
1
0
1
1

$$F3 = x'y' + xy' + xy$$

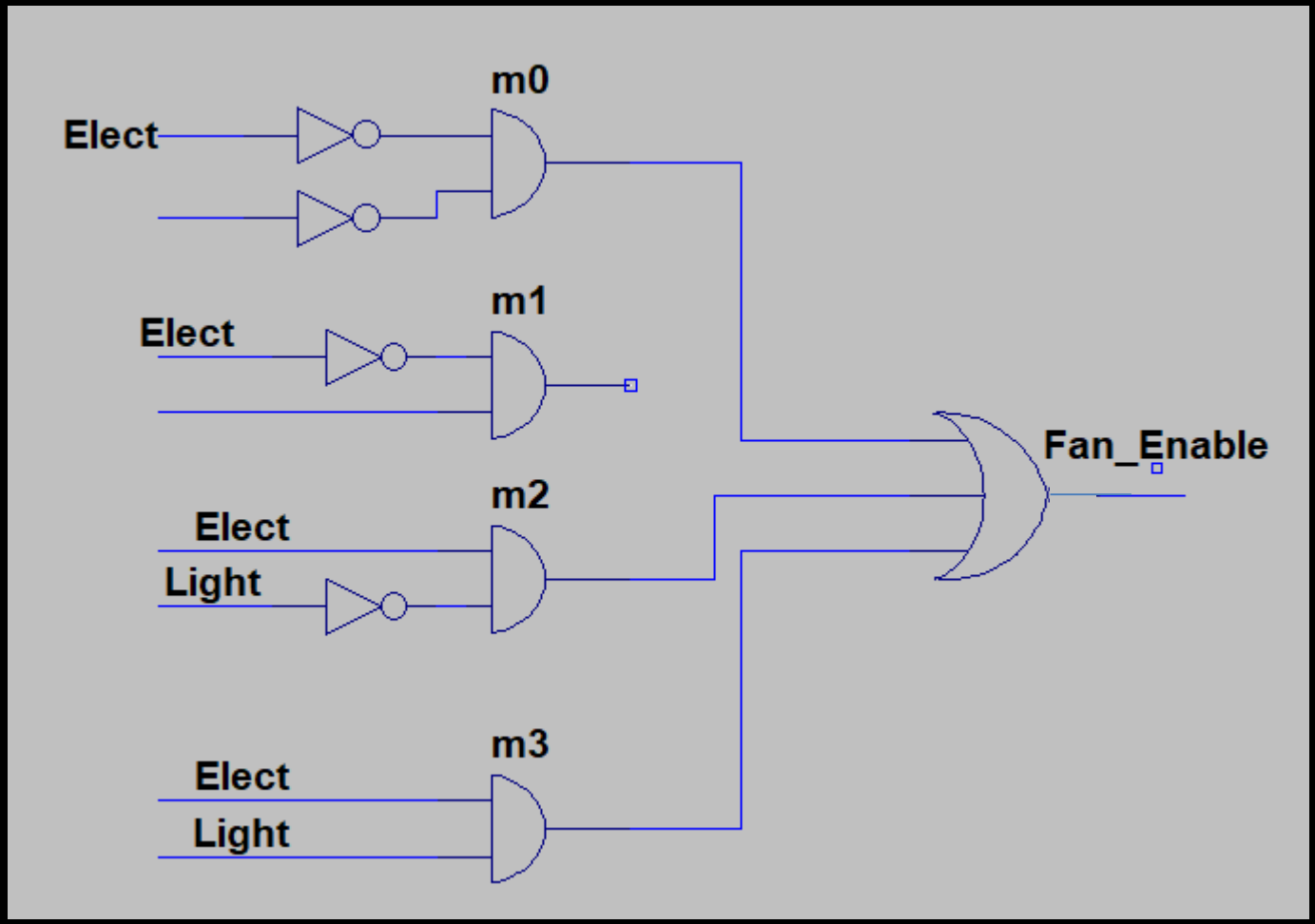
$$F3 = m_0 + m_2 + m_3$$

$$F3 = \sum(0,2,3)$$

Sum of Products

(Canonical form-Type1)

Boolean Functions



Boolean Functions

Each input entry will be represented by a term

x	y
0	0
0	1
1	0
1	1

Designation

$x + y$	M_0
$x + y'$	M_1
$x' + y$	M_2
$x' + y'$	M_3

F3
0
1
0
0

Maxterms

Standard Sums

$$F3 = (x + y).(x' + y).(x' + y')$$

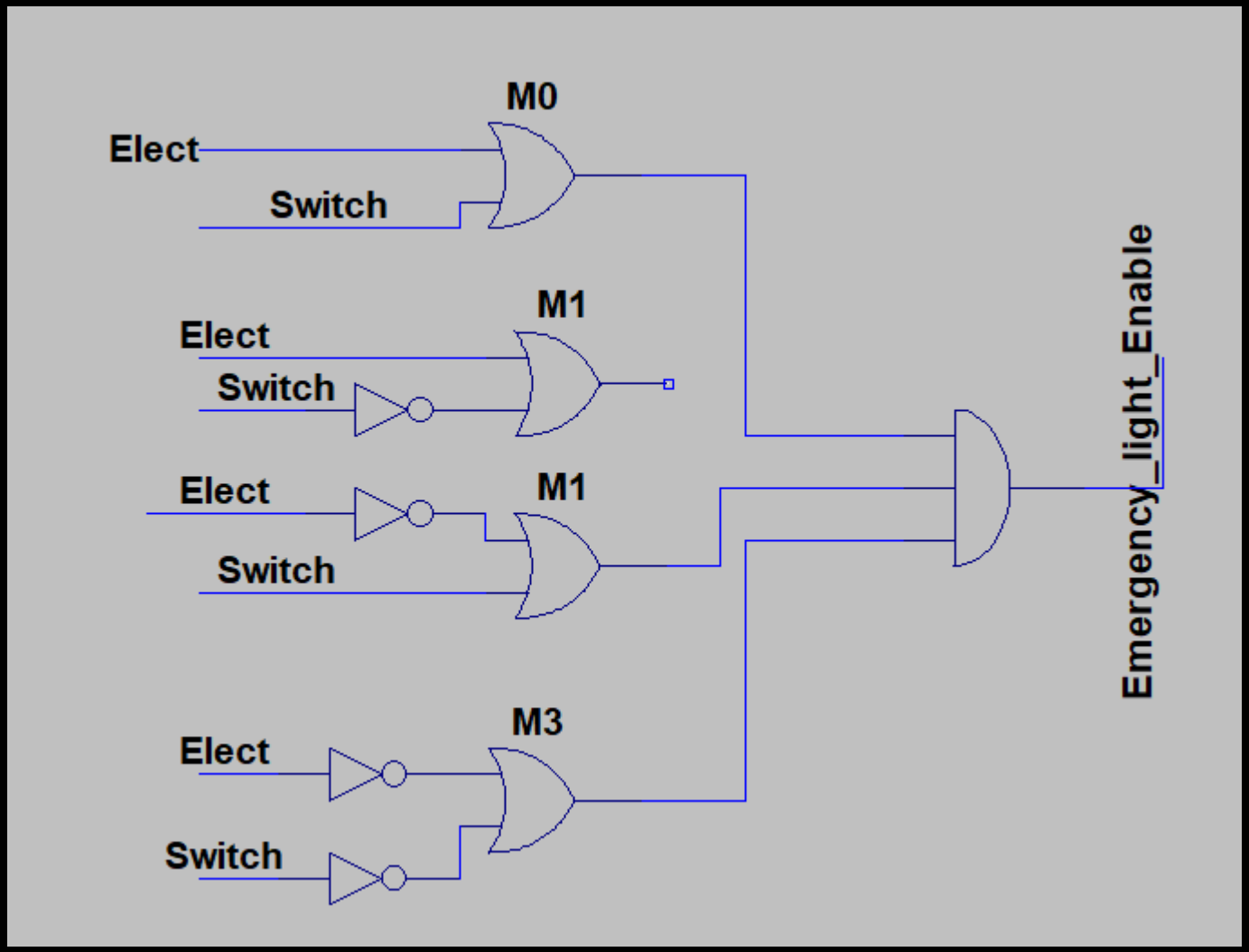
$$F3 = M_0.M_2.M_3$$

$$F3 = \pi(0,2,3)$$

Product of Sums

(Canonical form-Type2)

Boolean Functions



Why to use K-Maps?

Canonical Expression can be simplified by algebraic means

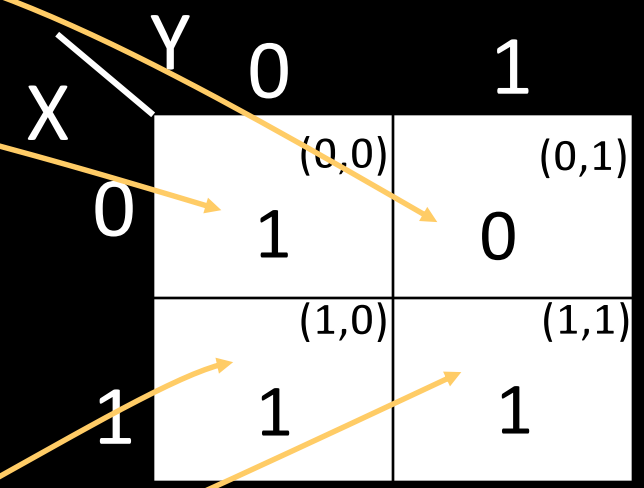
Algebraic method lacks specific rules

K-map method simple, straight forward and pictorial understanding and gives both SOP and POS forms

2-Variable K-Map

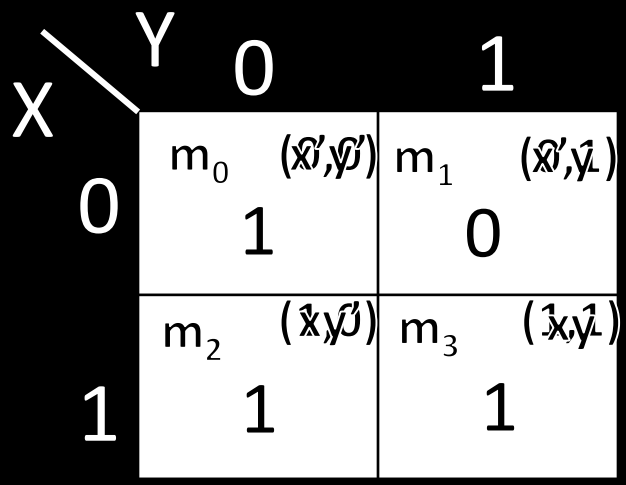
Truth-table to K-map

	X	Y	F
minterm 0 →	0	0	1
minterm 1 →	0	1	0
minterm 2 →	1	0	1
minterm 3 →	1	1	1



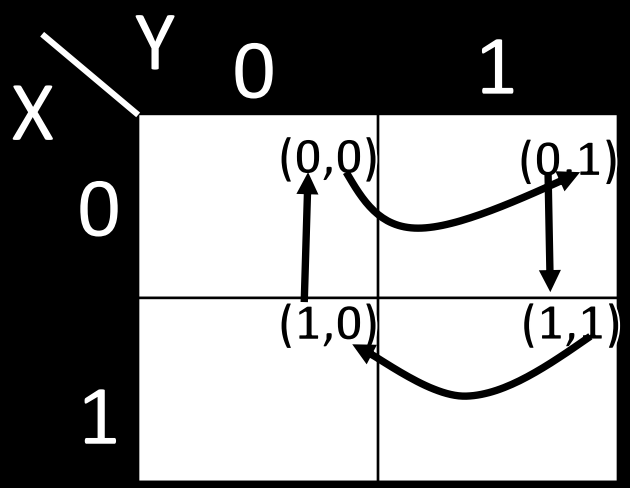
2-Variable K-Map

Representation of cells



2-Variable K-Map

One bit variation between adjacent cells

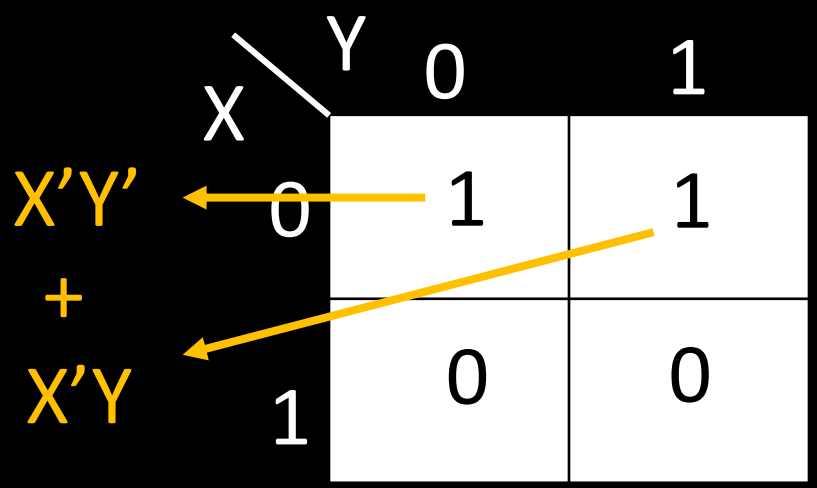


What is the importance of 1-bit variation ??

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	0



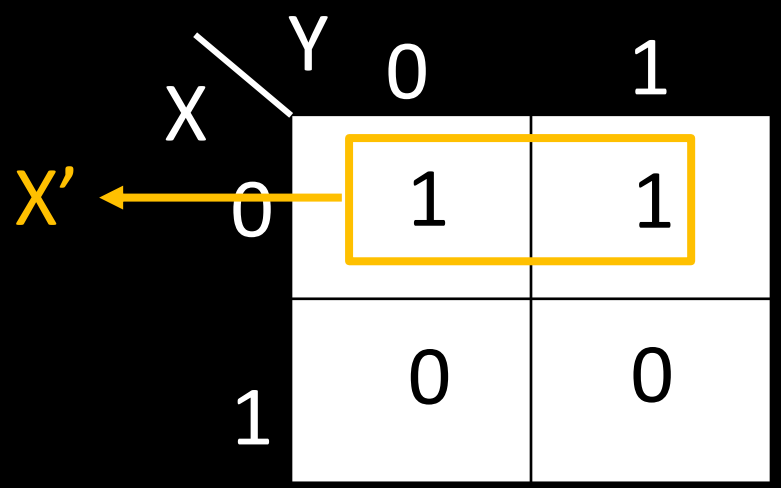
$$X'Y' + X'Y = X'(Y' + Y) = X'$$

If there are 1's in adjacent cells then they can be grouped and Minimized functions can be obtained

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	0
1	1	0

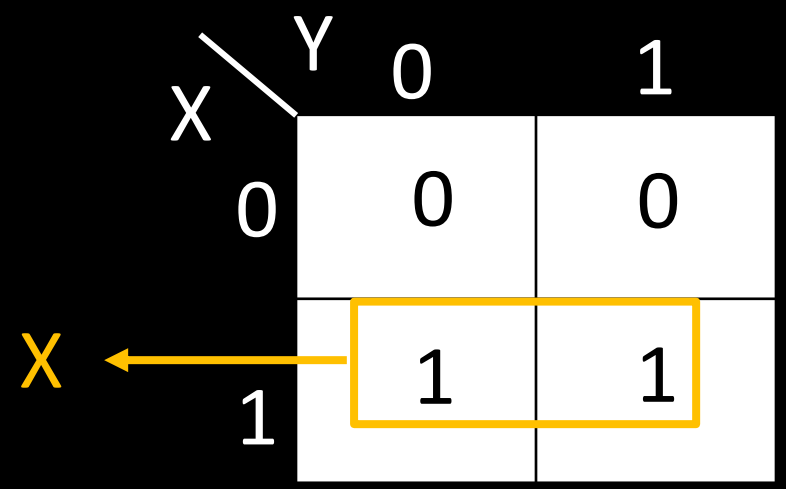


X = 0 constant Y varies from 0 → 1

2-Variable K-Map

Simplification

X	Y	F
0	0	0
0	1	0
1	0	1
1	1	1

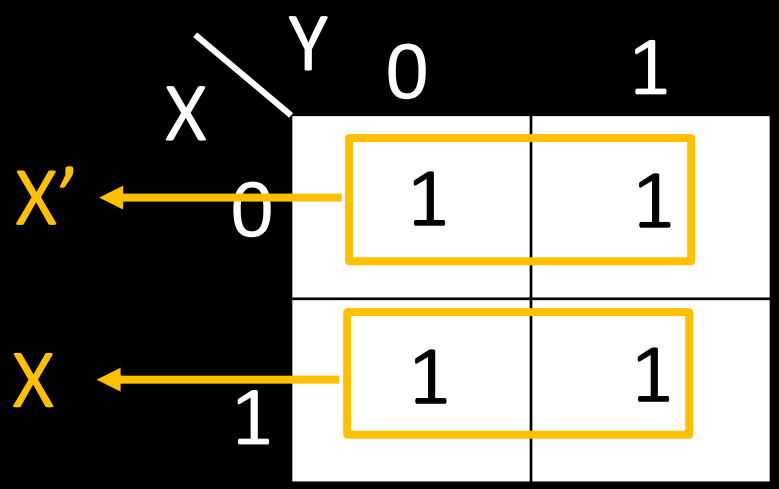


X = 1 constant Y varies from 0 → 1

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	1

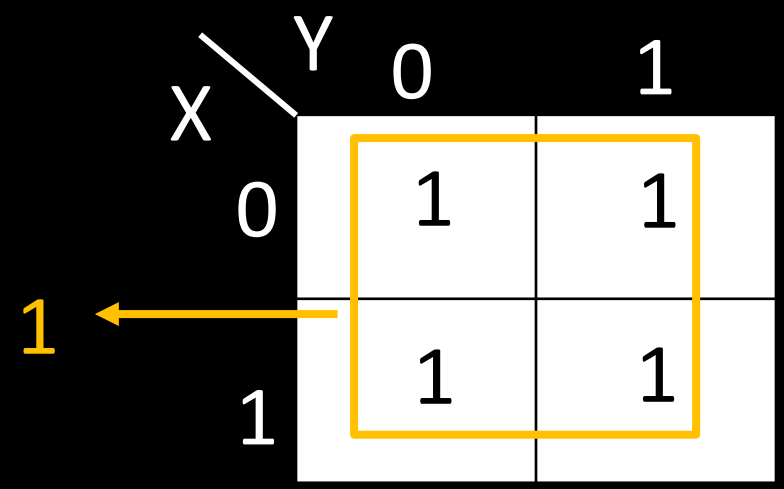


$X' + X = 1$

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	1
1	0	1
1	1	1



Adjacent cells can be grouped

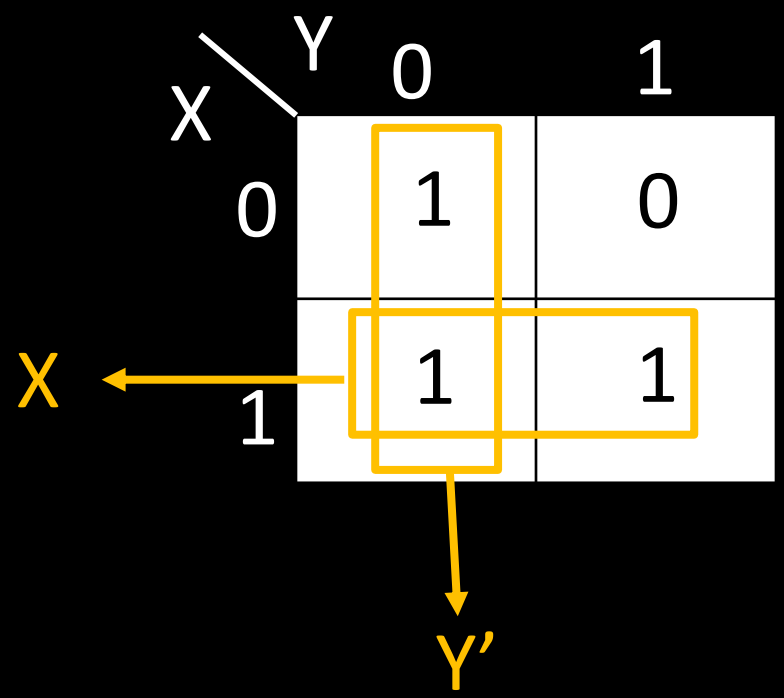
2- cells at a time

4- cells at a time

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1



F = X + Y'

**A term can be grouped multiple times if it helps in simplifying the expression

3-Variable K-Map

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

X \ YZ	00	01	11	10
0	(0,0,0) m_0	(0,0,1) m_1	(0,1,1) m_3	(0,1,0) m_2
1	(1,0,0) m_4	(1,0,1) m_5	(1,1,1) m_7	(1,1,0) m_6

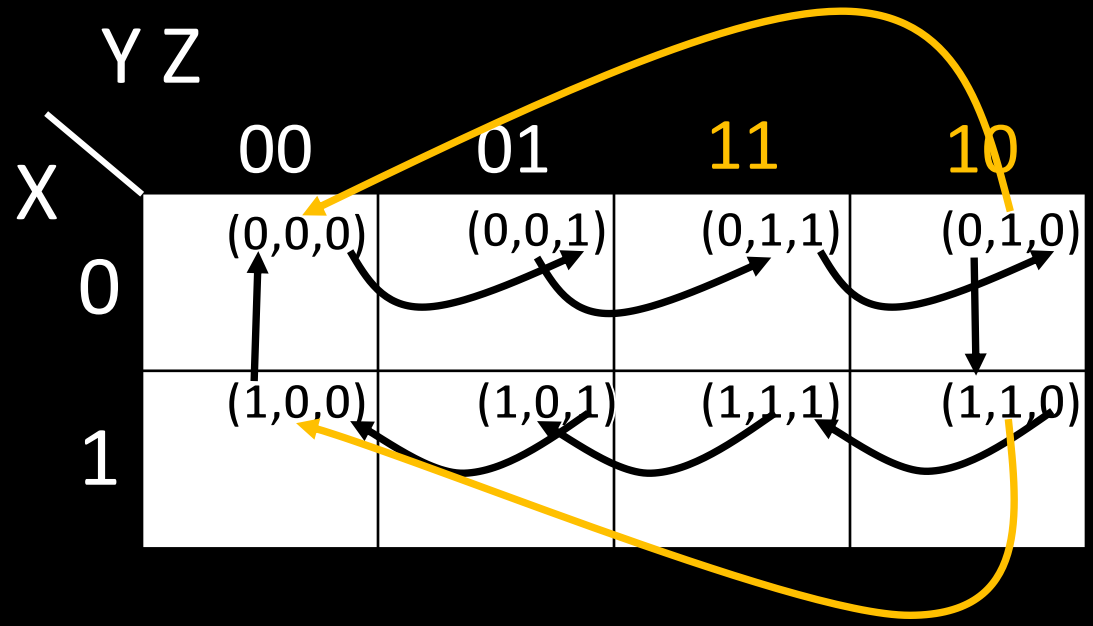
Should the next entry be (0, 1, 0) ?

(0,0,1) → (0,1,0)

**Two bit variation here, what to do ??

3-Variable K-Map

Adjacent Cells



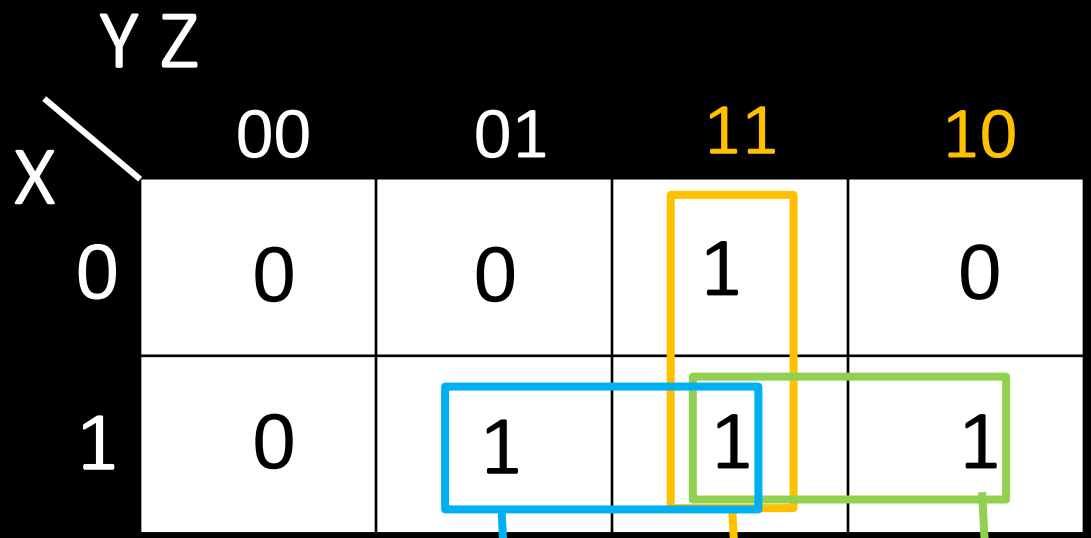
Any more adjacent cells ??

3-Variable K-Map

$F = XY + YZ + XZ$

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

2- cells or 4-cells or 8-cells at a time



XZ

X=1, Z=1 constant , Y varies

YZ

Y=1, Z=1 constant , X varies

XY

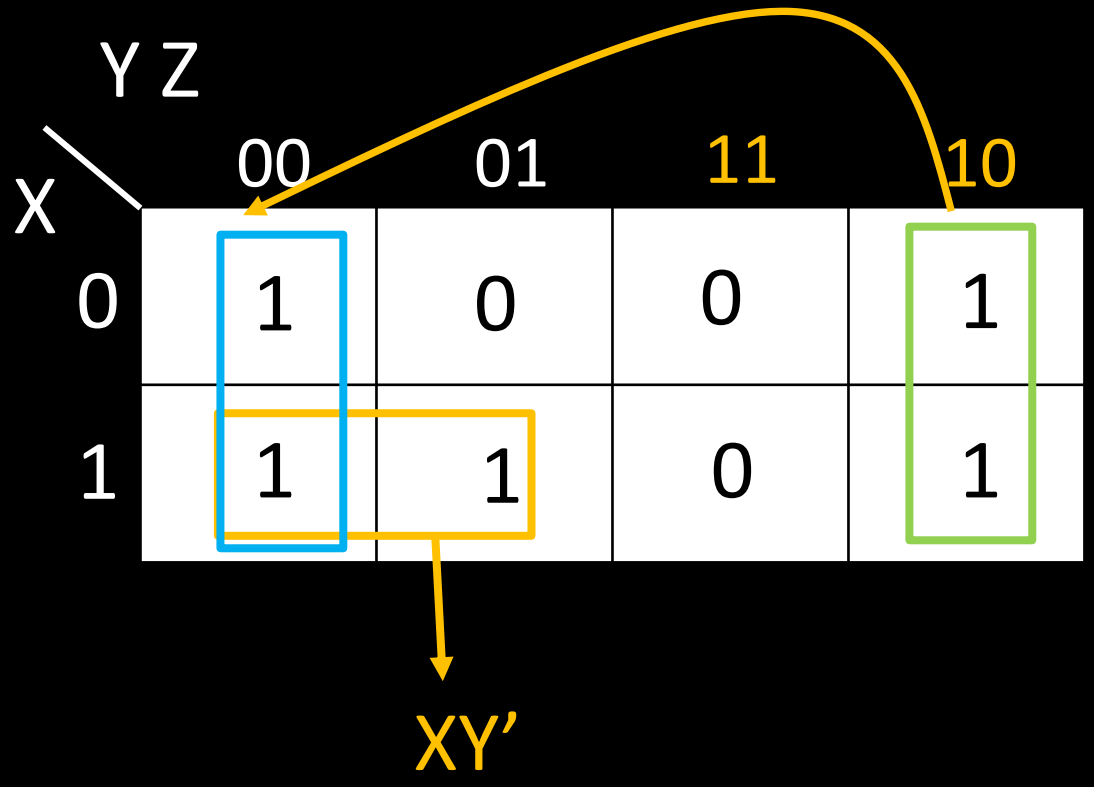
X=1, Y=1 constant , Z varies

3-Variable K-Map

$F(X,Y,Z) = \sum (0,2,4,5,6)$

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

2- cells or 4-cells or 8-cells at a time

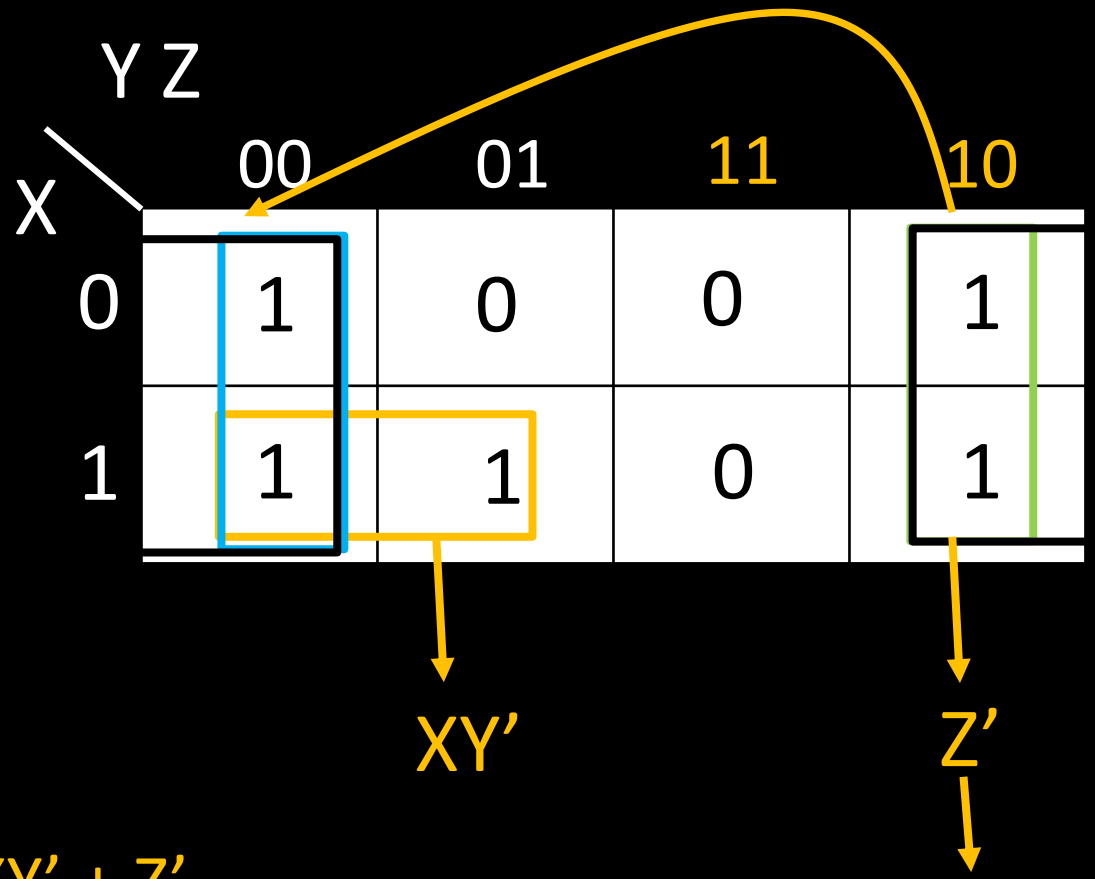


3-Variable K-Map

$F(X,Y,Z) = \sum (0,2,4,5,6)$

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

2- cells or 4-cells or 8-cells at a time



$F = XY' + Z'$

Z=0, constant , X , Y Change

3-Variable K-Map

$F(X,Y,Z) = \sum (2,3,4,5)$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

X \ YZ	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$F = XY' + X'Y$

3-Variable K-Map

$F(X,Y,Z) = \sum (3,4,6,7)$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

X \ YZ	00	01	11	10
0	0	0	1	0
1	1	0	1	1

Is the third grouping necessary ?

All ones should get covered atleast once

If all of them are covered then there is no need to group them again

3-Variable K-Map

$F(X,Y,Z) = \sum (3,4,6,7)$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

X \ YZ	00	01	11	10
0	0	0	1	0
1	1	0	1	1

$F = YZ + XZ'$

$F = YZ + XZ' + XZ'$

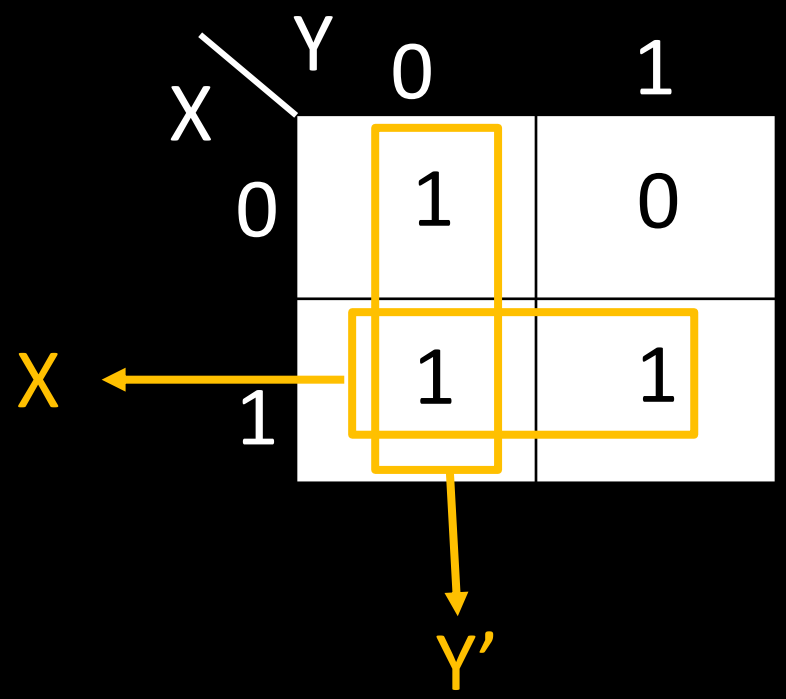
Both expressions will result in same truth table

But only first expression is minimized

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1



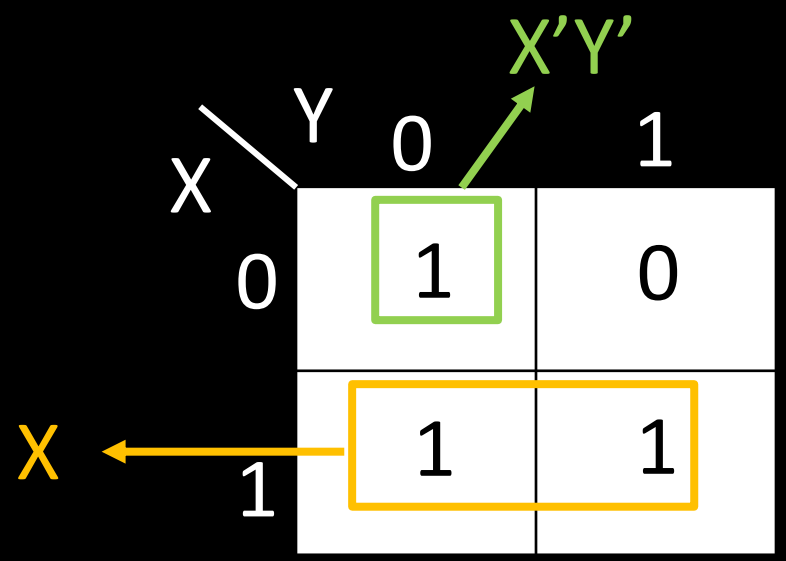
F = X + Y'

A term can be grouped multiple times if it helps in simplified expression

2-Variable K-Map

Simplification

X	Y	F
0	0	1
0	1	0
1	0	1
1	1	1



F = X + Y'

F = X + X'Y'

Which of the two is a minimized function ??

3-Variable K-Map

F (X,Y,Z) = π (0,1,2,5)

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

X \ Y Z	00	01	11	10
0	0	0	1	0
1	1	0	1	1

Y=0, Z=1 constant , X varies (Y+Z')

(X+Z)

X=0, Z=0 constant , Y varies

F = (Y + Z') (X + Z)

3-Variable K-Map

$F(X,Y,Z) = \pi(2,3,4,5)$

2- cells or 4-cells or 8-cells at a time

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

X \ YZ	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$F = (X+Y)(X'+Y')$

Next Class

4/5-variable K-map

K-map with don’t care conditions