Digital Design

CS/EEE /ECE/INSTR F215

Lecture 2: Number systems

Number systems

General use Decimal numbers : 8956

Digits used are 0 - 9

$8956 = 8 \times 10^3 + 9 \times 10^2 + 5 \times 10^1 + 6 \times 10^0$

Can be generalized to any decimal number

$$a_3a_2a_1a_0 . a_{-1}a_{-2}$$

$$= a_3 \times 10^3 + a_2 \times 10^2 + a_1 \times 10^1 + a_0 \times 10^0 + a_{-1} \times 10^{-1} + a_{-2} \times 10^{-2}$$

Number systems

Decimal number system: Base is 10 Numbers used : 0-9

Base also called radix

Binary number system : Base is 2 Numbers used : 0-1

For example: 101.11

$= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2}$

$= 4 + 0 + 1 + 0.5 + 0.25$

$= 5.75$

Number systems

For Base - r system $(a_n a_{n-1} \ldots a_1 a_0 . a_{-1} a_{-2} .. a_{-m})_r$

$a_n \times r^n + a_{n-1} \times r^{n-1} \ldots\ldots a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + \ldots\ldots a_{-m} \times r^{-m}$

Find the decimal equivalent of

$(123.4)_8$ [Octal] $= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 + 4 \times 8^{-1} = 83.5$

$(B2.4)_{16}$ [Hexa decimal] $= 11 \times 16^1 + 2 \times 16^0 + 4 \times 16^{-1} = ??$

$(110101)_2$ [Binary]

$= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = ??$

Number Base conversions

Typical conversions

Base-10 to Base-r

Convert $(49)_{10}$ to $()_2$

Divisor	Dividend / Quotient		Remainder	
2	49			
2	24	→	1	LSB
2	12	→	0	
2	6	→	0	
2	3	→	0	
2	1	→	1	
	0	→	1	MSB

(Remainders are read upward, from MSB to LSB.)

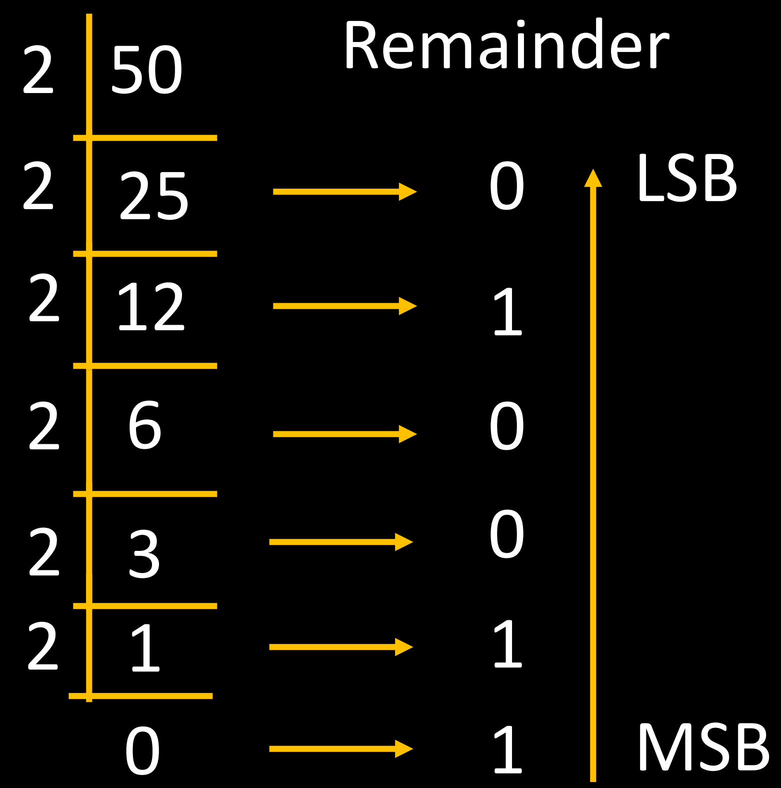
$(110001)_2$

Number Base conversions

Typical conversions

Base-10 to Base-r

Convert $(50)_{10}$ to $()_2$



$(110010)_2$

Number Base conversions

Typical conversions

Base-10 to Base-r (fraction)

Convert $(0.125)_{10}$ to $()_2$

	Integer
0.125x 2 = 0.25	0
0.25 x 2 = 0.5	0
0.5 x 2 = 1.0	1

$(0.125)_{10} = (0.001)_2$

Limited to required number of digits

Number Base conversions

Typical conversions

Base-10 to Base-r (fraction)

Convert $(0.49)_{10}$ to $()_2$

	Integer
$0.49 \times 2 = 0.98$	0
$0.98 \times 2 = 1.96$	1
$0.96 \times 2 = 1.92$	1
$0.92 \times 2 = 1.84$	1

$(0.49)_{10} = (0.011111\ldots)_2$ Limited to required number of digits

Number Base conversions

Typical conversions

Base-r to Base-10

Convert $(110110)_2$ to $(\ )_{10}$

$=1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$=1 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 2^0$

$=1 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 2^0$

$=54$

Number systems

$(B65F)_{16}$

Express the following numbers in decimal

$(10110.0101)_2$

$(1010.1010)_2$

$(26.24)_8$

$(16.5)_{16}$

$(FAFA)_{16}$

Problems

(1) Use binary expansion to convert binary fractions into decimals

(i) $(101.1101)_2$ (ii) $(1101.0111)_2$
(iii) $(111.111)_2$ (iv) $(101.01011)_2$

(2) Convert
(i) $(13.6875)_{10}$ (ii) $(32.45)_{10}$
(iii) $(28.555)_{10}$ (iv) $(7.0202)_{10}$ into binary fraction

(3) Convert the following numbers with the indicated base to decimal :

(i) $(4310)_5$ (ii) $(198)_{12}$

(iii) $(735)_8$ (iv) $(525)_6$

Number conversions

Other conversions

Binary to octal   Octal: base 8 digits used 0 - 7

1110001010101   **For Octal- 2^3, 8bit: 2^3**

001	110	001	010	101
↓	↓	↓	↓	↓
1	6	1	2	5

$(1110001010101)_2 \longrightarrow (16125)_8$

Number conversions

Other conversions

octal to binary

Octal: base 8 digits used 0-7

6373

6	3	7	3
↓	↓	↓	↓
110	011	111	011

$(6373)_8 \longrightarrow (110011111011)_2$

Number conversions

Other conversions

Binary to Hexadecimal   Hexa: base 16 digits used 0-F

1110001010101   4 bit, then Hexa decimal-2^4

0001   1100   0101   0101

1   C   5   5

$(1110001010101)_2 \longrightarrow (1C55)_{16}$

Representation of Negative Numbers

- Signed Magnitude
- Diminished radix complement
- Radix complement

Representation of Negative Numbers

- Signed Magnitude

Limitations

1. Two Zeros

2. Add +2 & -1

0 1 0
1 0 1
1 1 1

3-bit numbers	Signed magnitude
0 0 0	+0
0 0 1	+1
0 1 0	+2
0 1 1	+3
1 0 0	-0
1 0 1	-1
1 1 0	-2
1 1 1	-3

MSB indicates Sign : 0 indicates positive, 1 indicates negative

Complements

Diminished radix complement

Given a number N in base r having n digits (r-1)'s complement is defined as $(r^n - 1 - N)$

In case of decimal it is called 9's complement

9's complement of 865 is $10^3 - 1 - 865 = 999 - 865 = 134$

In case of binary it is called 1's complement for 1011

1's complement of 1011 is $2^4 - 1 - 1011 = 1111 - 1011 = 0100$
(or you can simply use the complement ~ 1 for 0 and 0 for 1)

Complements

Decimal	S.M.	1's comp.
7	0111	0111
6	0110	0110
5	0101	0101
4	0100	0100
3	0011	0011
2	0010	0010
1	0001	0001
0	0000	0000
−0	1000	1111
−1	1001	1110
−2	1010	1101
−3	1011	1100
−4	1100	1011
−5	1101	1010
−6	1110	1001
−7	1111	1000
−8	—	—

- **Two Zeros**
- **End-around-carry-bit addition**

Add 4 & -7

0100
<u>1000</u>
1100

Add 4 & -3

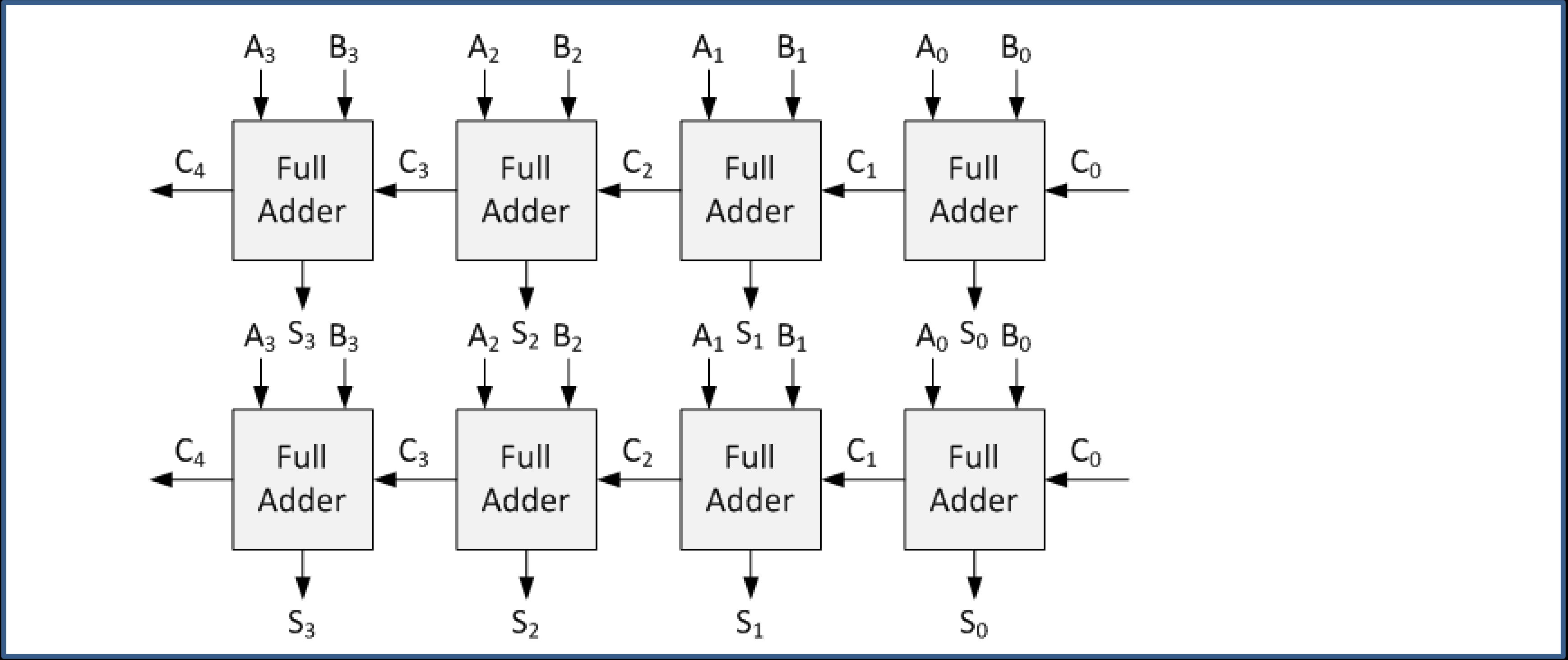
0100
<u>1100</u>
1 0000
<u>1</u>
0001

Complements

Add 4 & -3

```
  0100
  1100
------
1 0000
     1
------
  0001
```

- **Two Zeros**
- **End-around-carry-bit addition**



Complements

Radix complement

Given a number N in base r having n digits r's complement is defined as (r^n-N)

In case of decimal it is called 10's complement

10's complement of 865 is $10^3 - 865 = 1000 - 865 = 135$

10's complement = 9's complement + 1

In case of binary it is called 2's complement

2's complement of 1011 is $2^4 - 1011 = 10000 - 1011 = 0101$

2's complement = 1's complement + 1

Complements

Decimal	S.M.	1's comp.	2's comp.
7	0111	0111	0111
6	0110	0110	0110
5	0101	0101	0101
4	0100	0100	0100
3	0011	0011	0011
2	0010	0010	0010
1	0001	0001	0001
0	0000	0000	0000
−0	1000	1111	—
−1	1001	1110	1111
−2	1010	1101	1110
−3	1011	1100	1101
−4	1100	1011	1100
−5	1101	1010	1011
−6	1110	1001	1010
−7	1111	1000	1001
−8	—	—	1000

- **Two Zeros**
- **No End-around-carry-bit addition**

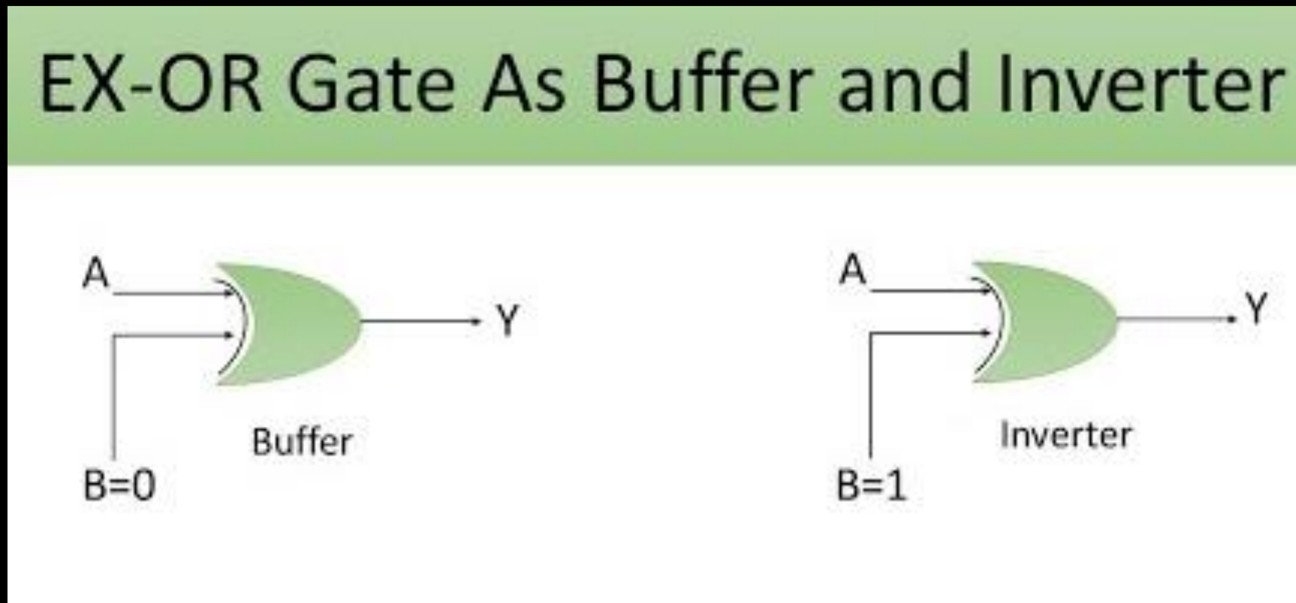
Add 4 & -7

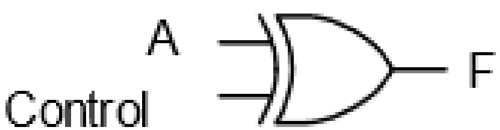
0100
1001
1101

Add 4 & -3

0100
1101
1 0001

Complements

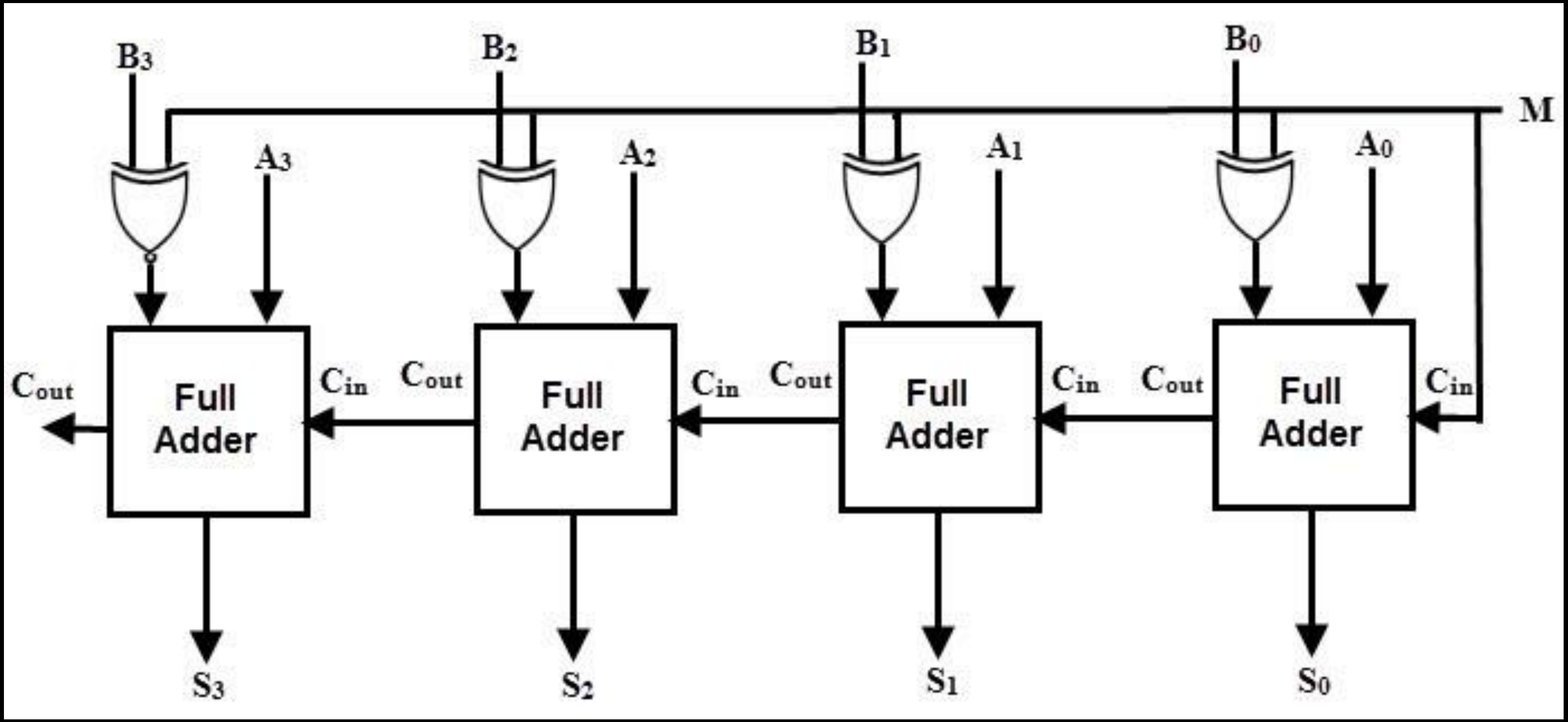




Control	A	F	
0	0	0	Pass
0	1	1	
1	0	1	Invert
1	1	0	

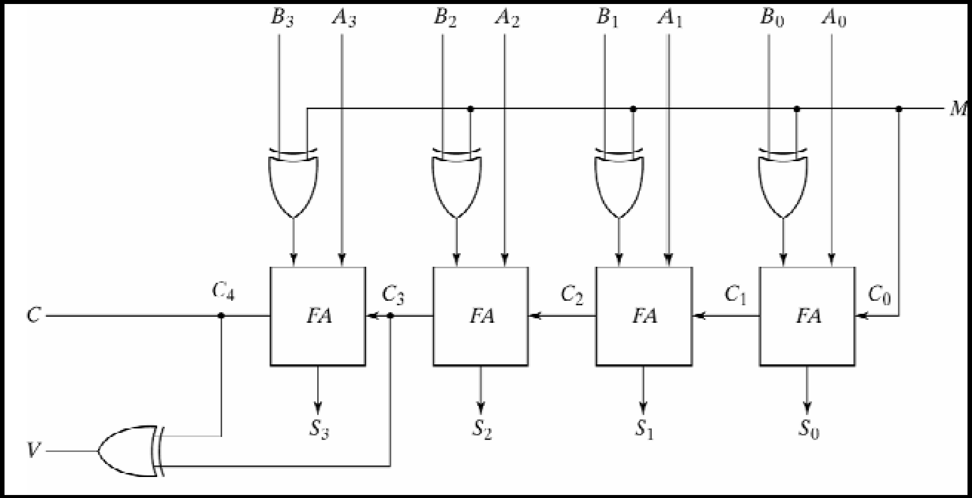
Complements

- **Easy Implementation: Adder Subtractor M=0 adder, M=1 Subtractor**



Complements

Add 4 & -3	Add -4 & -5	Add -8 & 4	Add 4 & 4
0100	1100	1000	0100
1101	1011	0100	0100
1 0001	1 0111	1100	1000



Binary Codes - BCD

0 0 0 0	0
0 0 0 1	1
0 0 1 0	2
0 0 1 1	3
0 1 0 0	4
0 1 0 1	5
0 1 1 0	6
0 1 1 1	7
1 0 0 0	8
1 0 0 1	9

Consider example 7698

7 6 9 8

BCD code

BCD and Binary comparison

$(185)_{10}$

BCD = (0001 1000 0101)

Binary = $(10111001)_2$

BCD = 12 bits, Binary = 8 bits

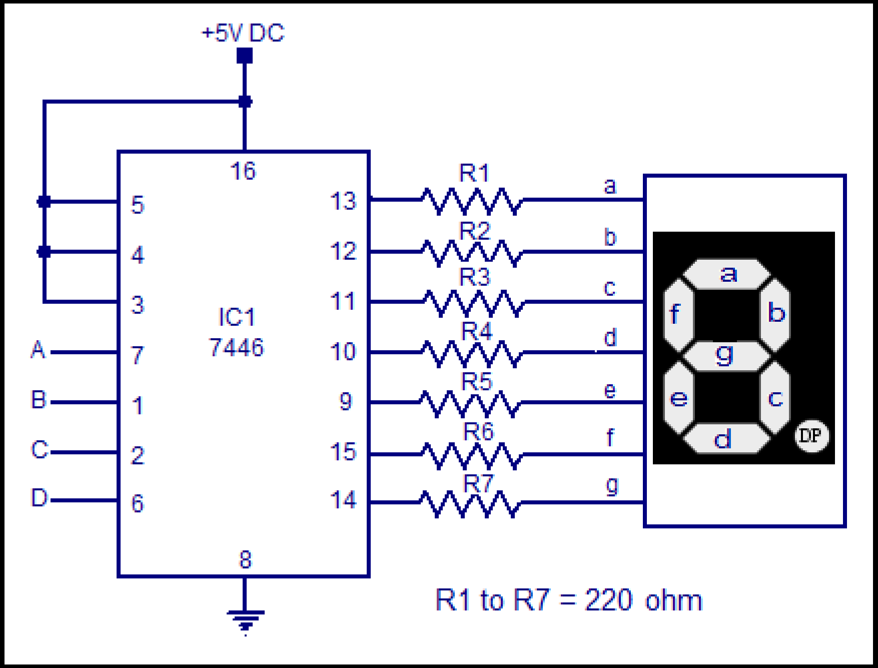
Some systems work directly on BCD (**IBM** Power6)

User enters decimal → BCD i/p→ compute in BCD → BCD o/p → Decimal output shown to user

Binary Codes - BCD

General digital systems

User enters decimal → BCD i/p→ Binary i/p → compute in binary → Binary o/p → BCD o/p → Decimal output shown to user



Binary Codes - BCD

BCD addition

4 + 5	4	0 1 0 0	
	5	0 1 0 1	
	9	1 0 0 1	Expected Result
4 + 8	4	0 1 0 0	
	8	1 0 0 0	
		1 1 0 0	Is this expected Result ?

Expected answer is BCD of 12 0001 0010

Binary Codes - BCD

BCD addition

4 + 8

4	0 1 0 0
8	1 0 0 0
Greater than 9	1 1 0 0
	0 1 1 0
0 0 0 1	0 0 1 0
1	2

Add correction of +6

= To skip 6 invalid states (10 - 15) BCDs

Binary Codes - BCD

BCD addition

9 + 9	9	1 0 0 1
	9	1 0 0 1
Carry out generated	1	0 0 1 0
		0 1 1 0
	0 0 0 1	1 0 0 0
	1	8

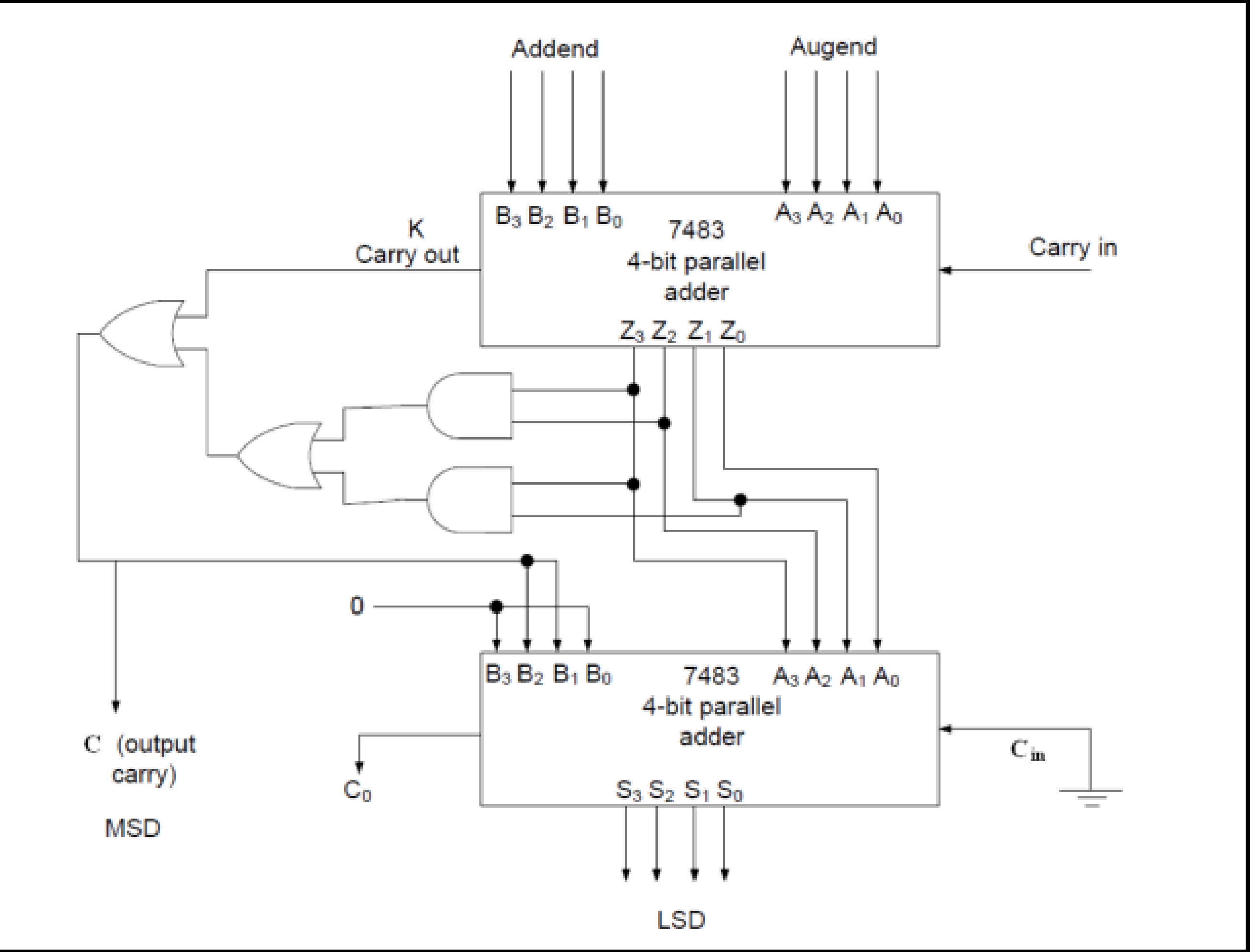
Expected result ?

Add correction of +6

After addition if carry out is generated or if sum is greater than 9 there is need for correction

Binary Codes - BCD

BCD addition



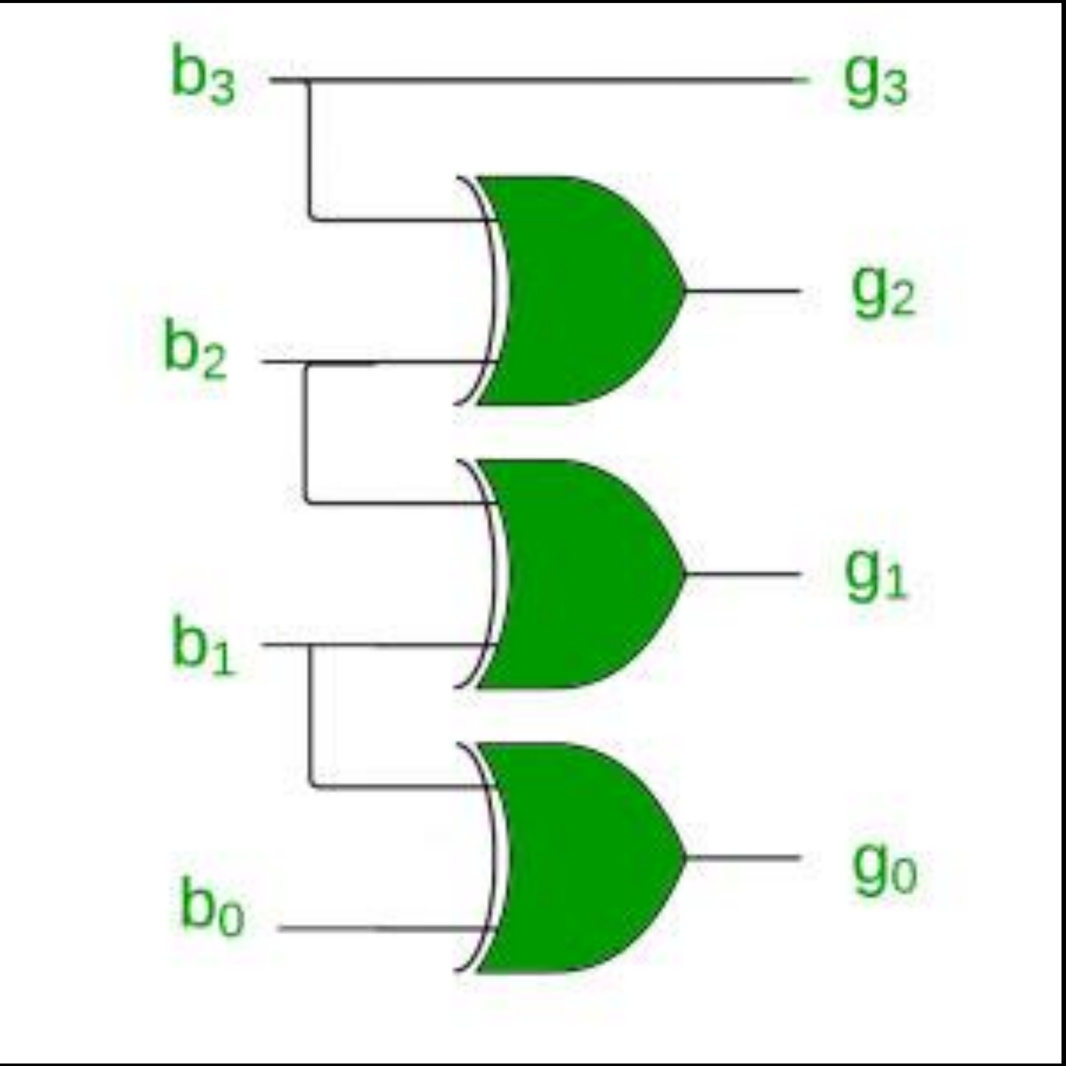
Binary Codes – Gray Code

THE GRAY CODE

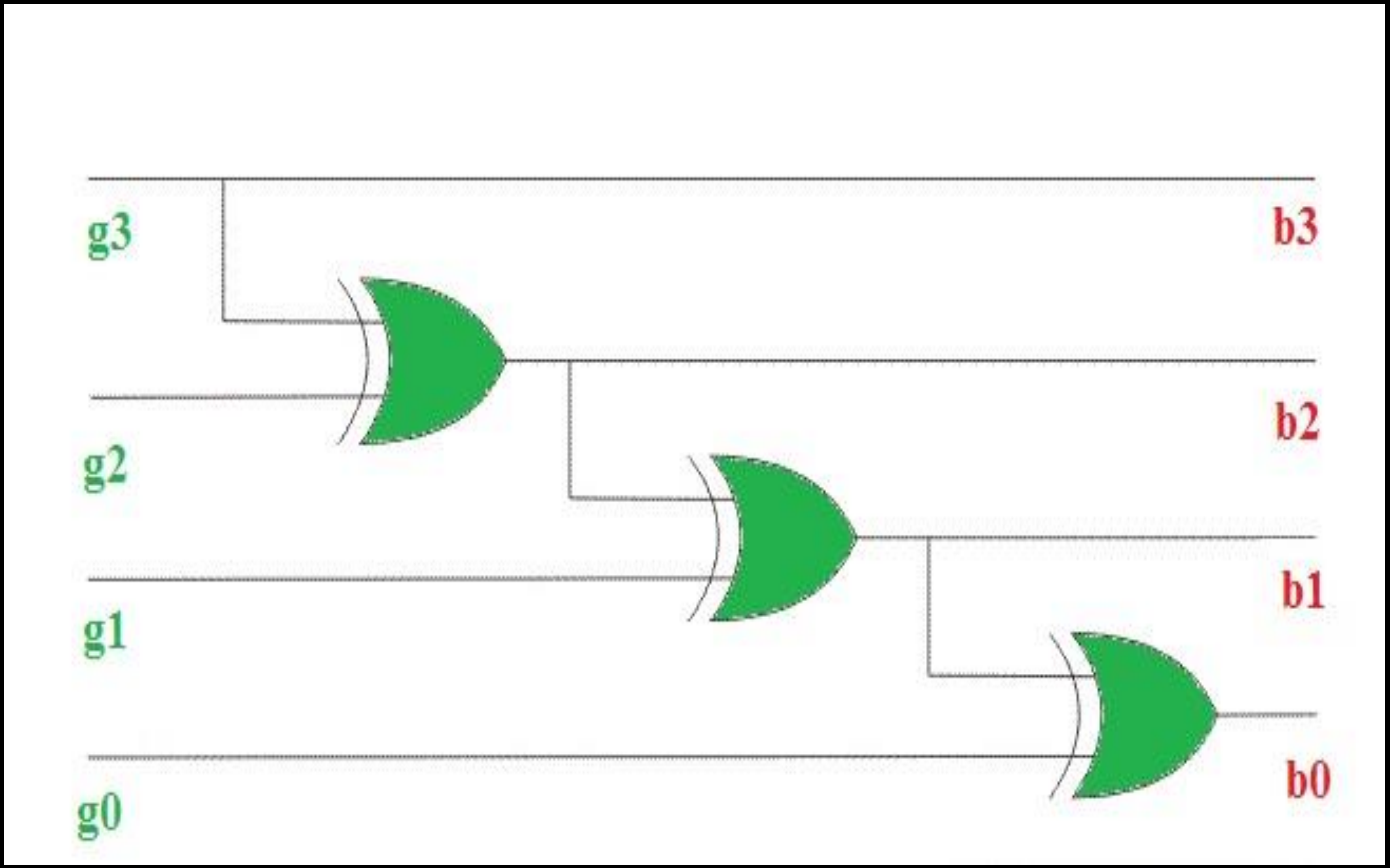
Decimal	Binary	Gray Code
0	0000	0000
1	0001	0001
2	0010	0011
3	0011	0010
4	0100	0110
5	0101	0111
6	0110	0101
7	0111	0100

Decimal	Binary	Gray Code
8	1000	1100
9	1001	1101
10	1010	1111
11	1011	1110
12	1100	1010
13	1101	1011
14	1110	1001
15	1111	1000

Binary Codes – Gray Code



Binary Codes – Gray Code



Thankyou