Electrical Science: 2021-22

Tutorial 7

Frequency Domain Analysis of RLC Circuit

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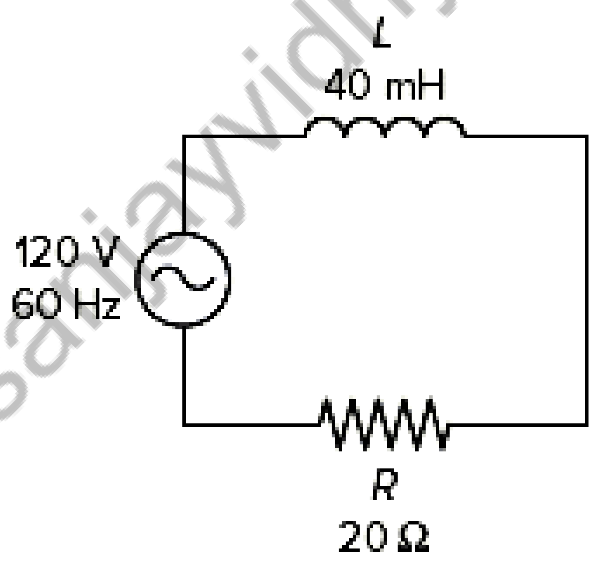
ELECTRICAL ELECTRONICS COMMUNICATION INSTRUMENTATION

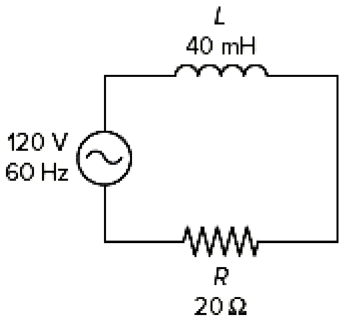
Example 1

For the series *RL* circuit shown in Figure determine:

1. Inductive reactance (X_L).
2. Impedance (Z).
3. Current (I).
4. Voltage drop across the resistor (E_R) and inductor (E_L).
5. The angle theta (θ) and power factor (PF) for the circuit.
6. True power (W), reactive power (VARs), apparent power (VA).



Example 1



	E	I	L	R/X_L/Z	W/VA/VARs	∠θ	PF
R			N/A	20 Ω		0°	N/A
L			40 mH			90°	
Total	120 V		N/A				

	E	I	L	R/X_L/Z	W/VA/VARs	∠θ	PF
R	95.6 V	4.78 A	N/A	20 Ω	457 W	0°	N/A
L	72.2 V	4.78 A	40 mH	15.1 Ω	345 VARs	90°	
Total	120 V	4.78 A	N/A	25.1 Ω	574 VA	37.1°	79.7%

$$X_L = 2\pi fL = 2 \times 3.14 \times 60 \times 0.04 = 15.1\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{20^2 + 15.1^2} = \sqrt{400 + 228} = 25.1\ \Omega$$

$$I_T = \frac{E_T}{Z} = \frac{120}{25.1} = 4.78\ \text{A}$$

$$I_T = I_R = I_L = 4.78\ \text{A}$$

$$\text{Cosine } \theta = \frac{R}{Z} = \frac{20}{25.1} = 0.797$$

$$\text{Angle } \theta = 37.1°$$

$$\text{Power factor} = \cos\theta = 0.797 \text{ or } 79.7\% \text{ lagging}$$

$$E_R = I \times R = 4.78 \times 20 = 95.6\ \text{V}$$

$$E_L = I \times X_L = 4.78 \times 15.1 = 72.2\ \text{V}$$

$$\text{W} = E_R \times I_R = 95.6 \times 4.78 = 457 \text{ watts}$$

$$\text{VARs} = E_L \times I_L = 72.2 \times 4.78 = 345 \text{ VARs}$$

$$\text{VA} = E_T \times I_T = 120 \times 4.78 = 574 \text{ VA}$$

Applying the theorem for AC circuits

- Thevenin’s theorem
- Norton’s theorem
- Nodal and Mesh analysis
- Superposition theorem
- Source transformation

Example 2

If I = 33∠-13º A, find the Thevenin's equivalent circuit to the left of terminals x-y in the network of figure.

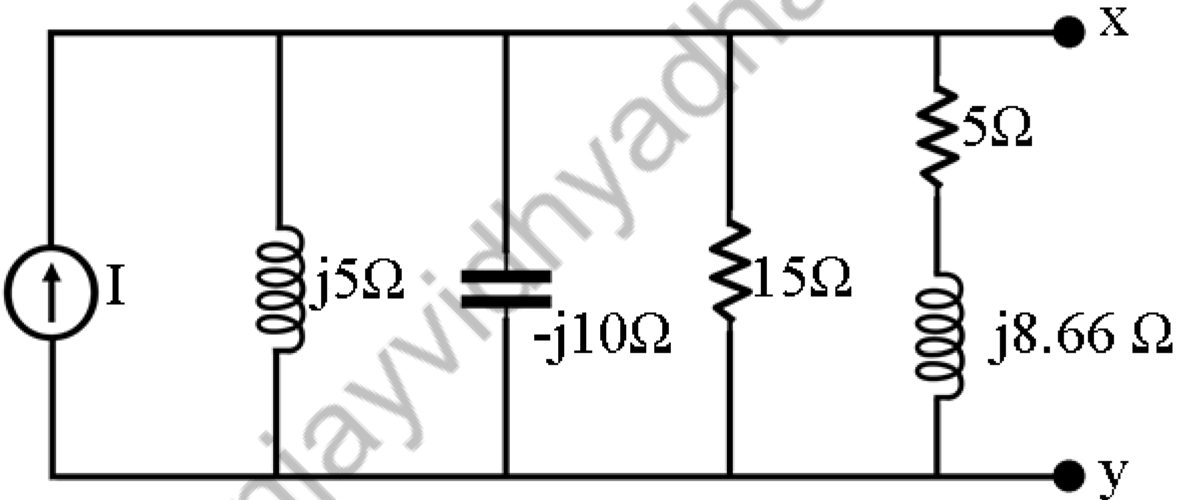


Figure: 1

Example 2

If I = 33∠-13º A, find the Thevenin's equivalent circuit to the left of terminals x-y in the network of figure.

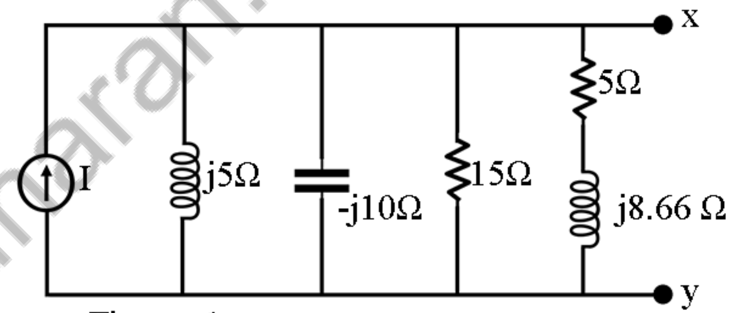


Figure: 1

$$Y_{eq} = Y_1 + Y_2 + Y_3 + Y_4$$

where Y_1, Y_2, Y_3 and Y_4 is the branch admittance

$$Y_1 = \frac{1}{j5} = -j0.2mho$$

$$Y_2 = \frac{1}{-j10} = j0.1mho$$

$$Y_3 = \frac{1}{15} = 0.067mho$$

$$Y_4 = \frac{1}{5 + j8.66} = \frac{1}{10\angle 60^\circ} = (0.05 - j0.0866)mho$$

$$\therefore Y_{eq} = (0.117 - j0.7866)mho$$

$$Z_{in} = \frac{1}{Y_{eq}} = \frac{1}{0.117 - j0.1866}$$

$$= \frac{1}{0.22\angle -58^\circ} = 4.54\angle 58^\circ ohm$$

$$V_{x-y}(= V_{o.c}) = \frac{I}{Y_{eq}}$$

$$= 150\angle 45^\circ V$$

Example 2

If I = 33∠-13° A, find the Thevenin's equivalent circuit to the left of terminals x-y in the network of figure.

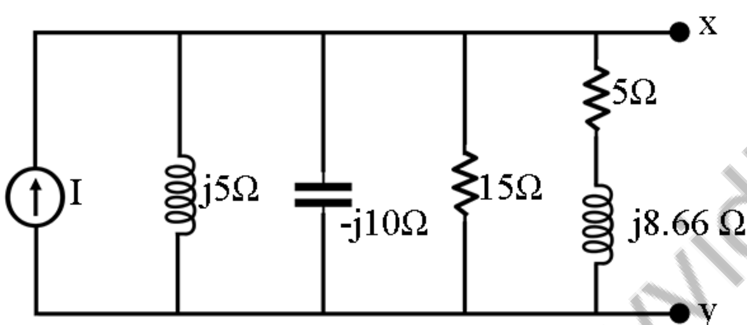
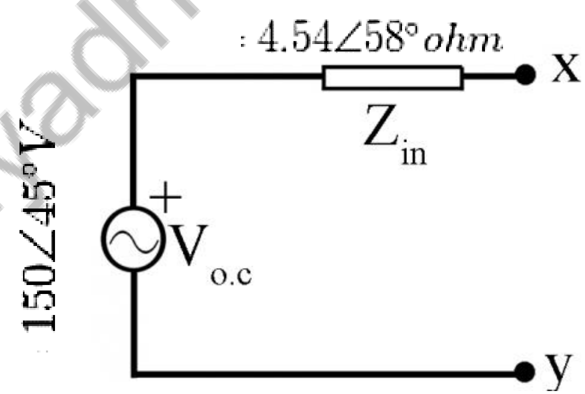
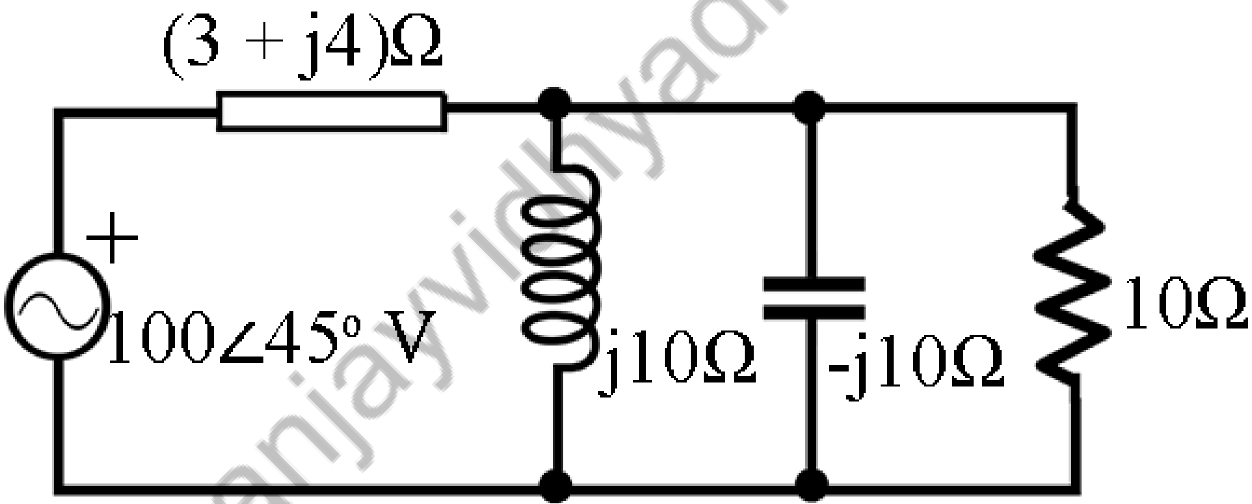


Figure: 1



Example 3

Find the current through 10Ω resistor using Thevenin's theorem



Example 3

Find the current through 10Ω resistor using Thevenin's theorem (figure 5).

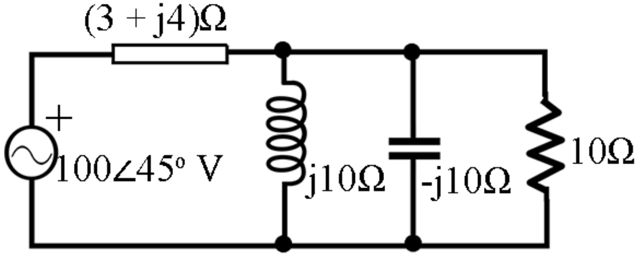


Figure: 5

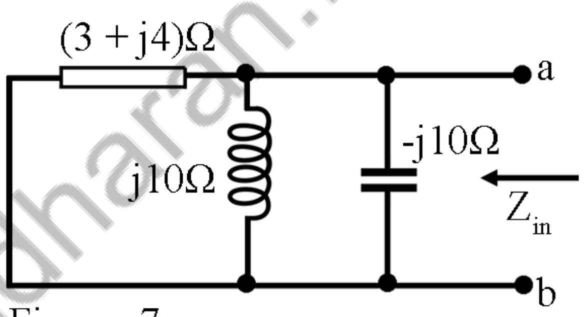


Figure: 7

$$Z_{int} = \frac{1}{Y_{int}}$$

$$= \frac{1}{\frac{1}{-j10} + \frac{1}{j10} + \frac{1}{3+j4}} = (3+j4)ohm$$

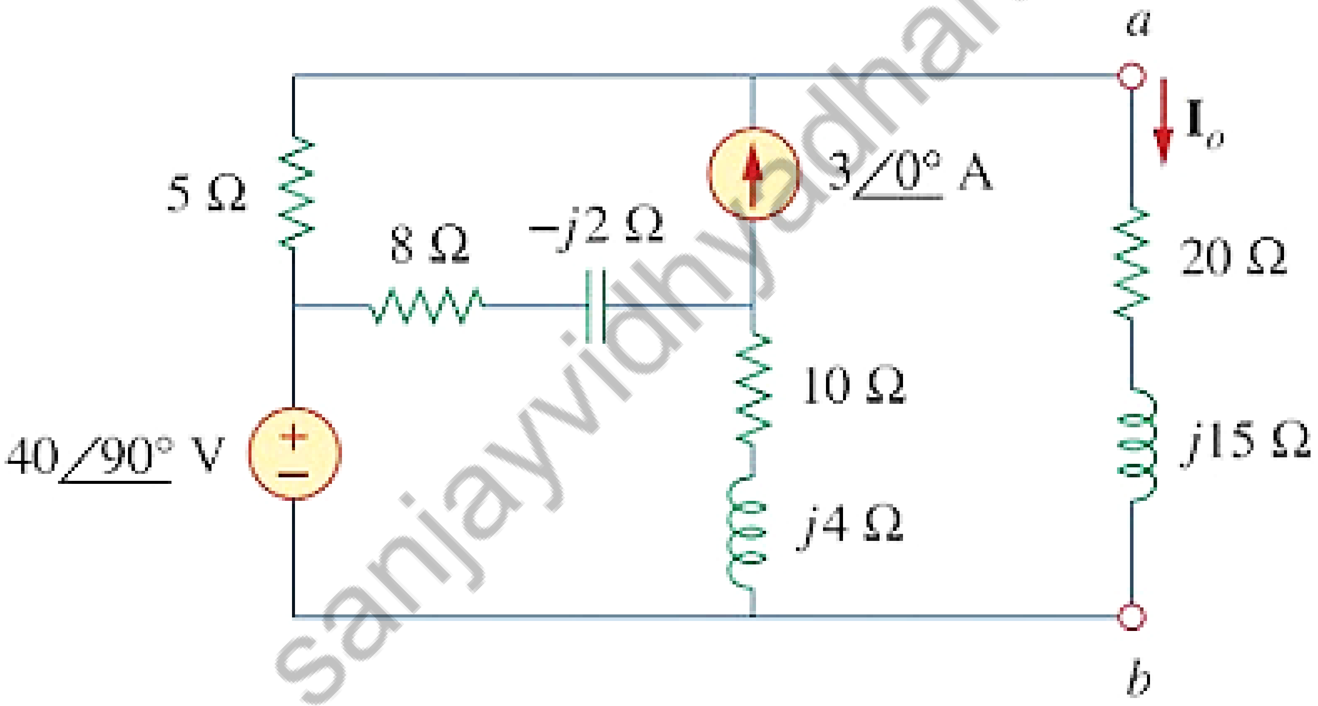
$$Z_{in} = (3+j4)\Omega$$

and $V_{o.c} = 100\angle 45^\circ V$

$$I_L = \frac{V_{o.c}}{Z_{in} + Z_L} = \frac{100\angle 45^\circ}{(3+j4)+10} = 7.35\angle 28.3^\circ A$$

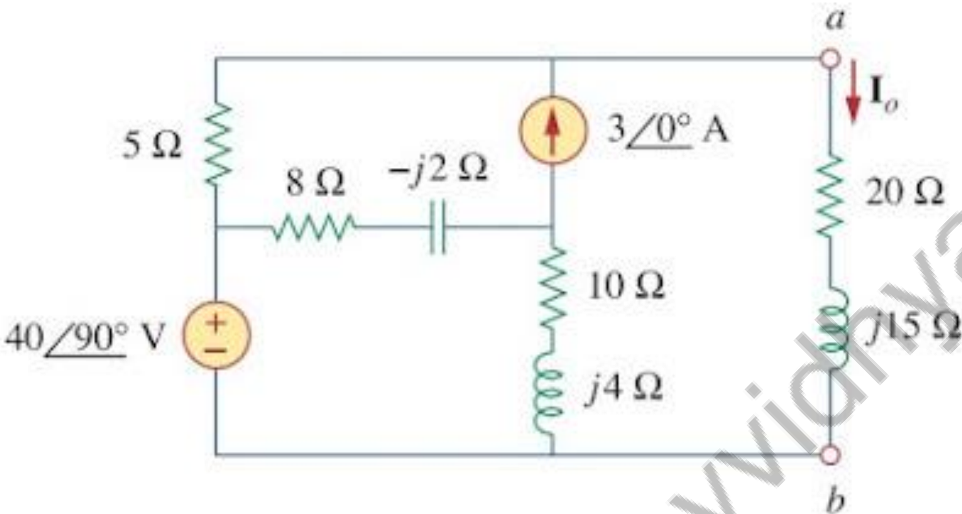
Example 4

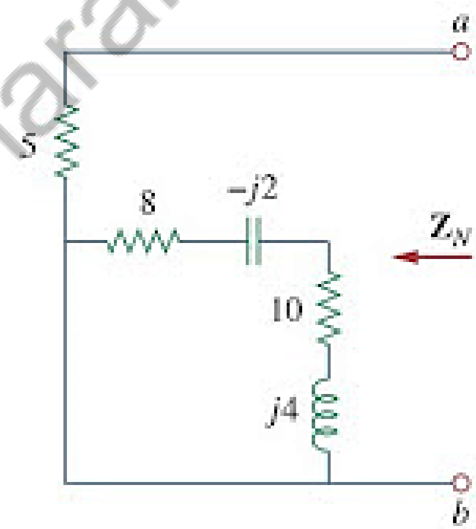
Find the current I_o using Nortons's theorem



Example 4

Find the current I_o using Nortons's theorem



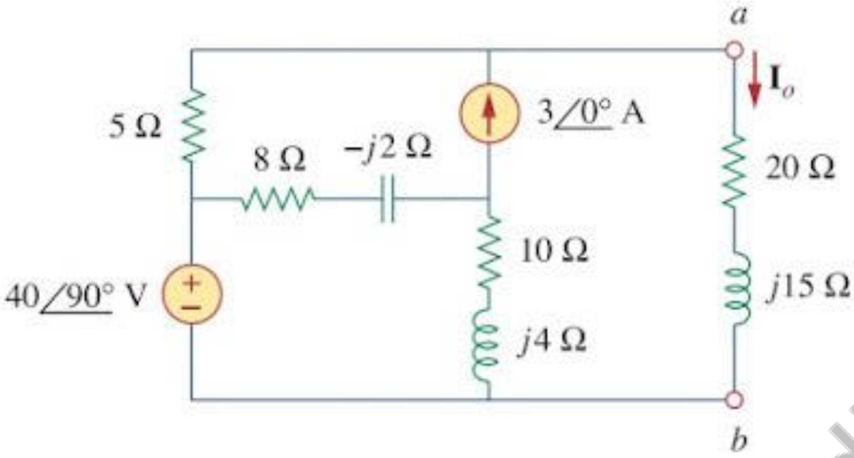


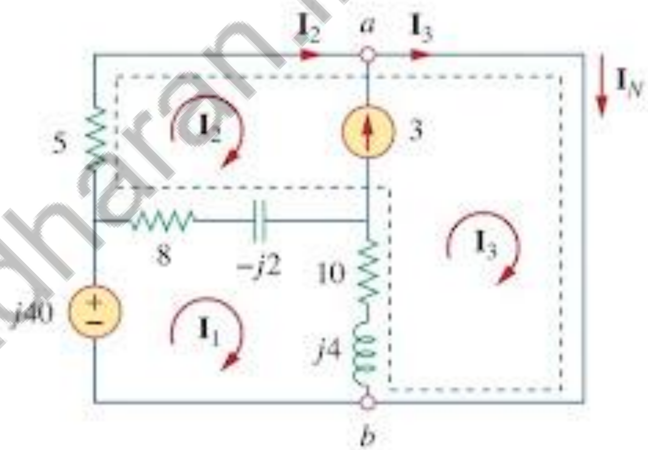
(a)

$$\mathbf{Z}_N = 5\ \Omega$$

Example 4

Find the current I_o using Nortons's theorem





For mesh 1,

$$-j40 + (18 + j2)\mathbf{I}_1 - (8 - j2)\mathbf{I}_2 - (10 + j4)\mathbf{I}_3 = 0$$

For the supermesh,

$$(13 - j2)\mathbf{I}_2 + (10 + j4)\mathbf{I}_3 - (18 + j2)\mathbf{I}_1 = 0$$

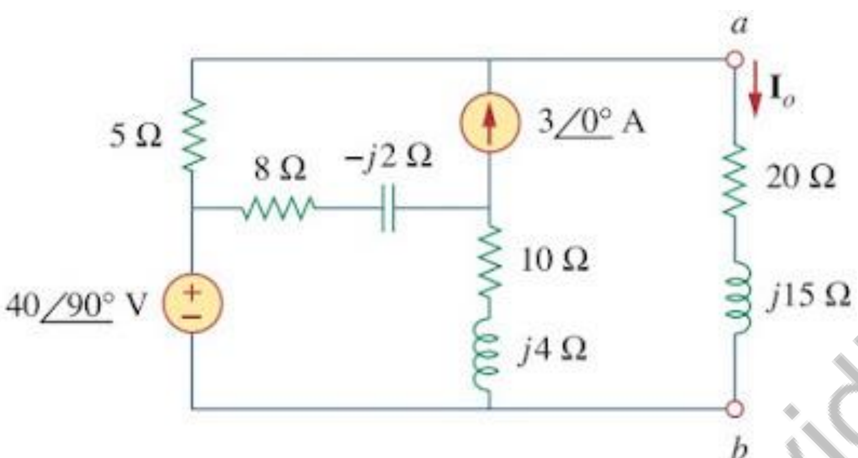
At node a, due to the current source between meshes 2 and 3,

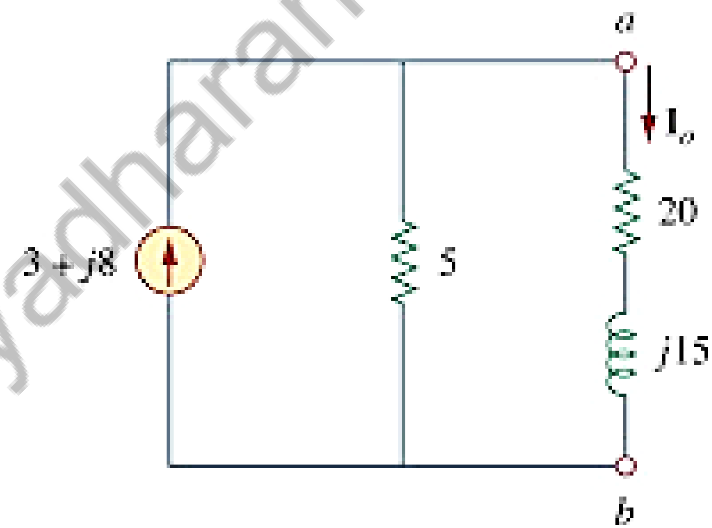
$$\mathbf{I}_3 = \mathbf{I}_2 + 3$$

$$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)\text{ A}$$

Example 4

Find the current I_o using Nortons's theorem





$$\mathbf{I}_N = \mathbf{I}_3 = (3 + j8)\ \text{A}$$

$$\mathbf{Z}_N = 5\ \Omega$$

$$\mathbf{I}_o = \frac{5}{5 + 20 + j15}\mathbf{I}_N = \frac{3 + j8}{5 + j3} = 1.465\angle 38.48^\circ\ \text{A}$$

Example 5

For the circuit of Figure 5.3.2, determine V_b using superposition. $I=2E-3\angle 90^\circ$ amps peak and $E=10\angle 0^\circ$ volts peak.

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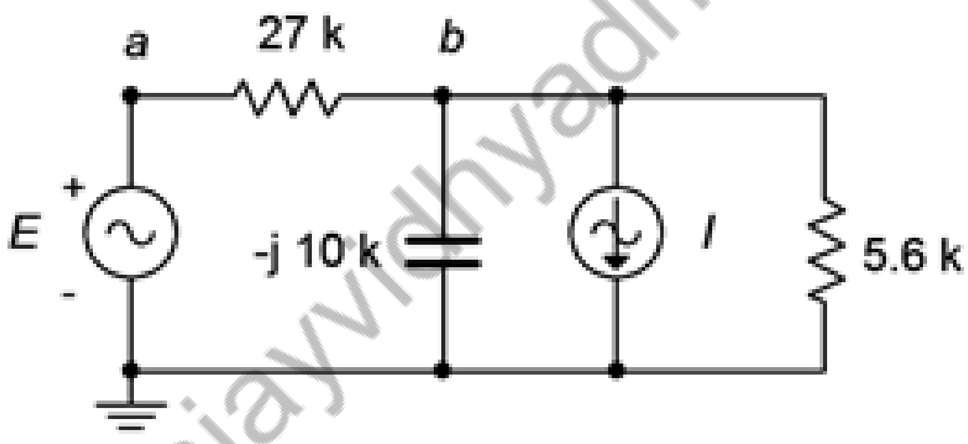
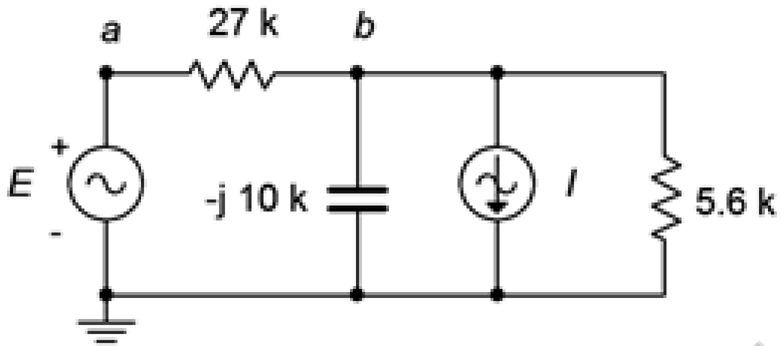
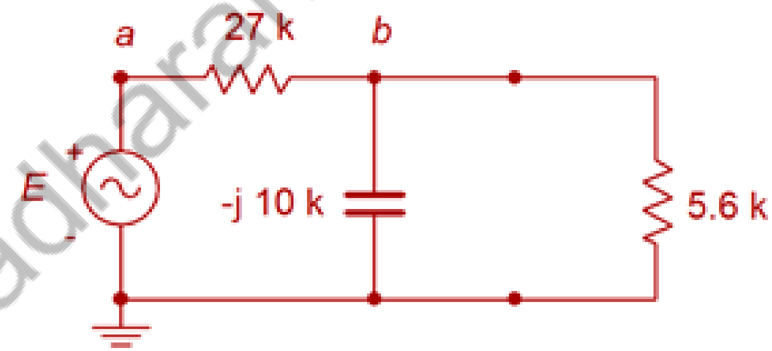


Example 5

For the circuit of Figure 5.3.2, determine V_b using superposition. I=2E−3∠90∘ amps peak and E=10∠0∘ volts peak.

- 



$$Z_{right2} = \frac{R \times jX_C}{R - jX_C}$$

$$Z_{right2} = \frac{5.6k\Omega \times (-j10k\Omega)}{5.6k\Omega - \text{j10K}}$$

$$Z_{right2} = 4886\angle -29.2^\circ\Omega$$

$$v_{b1} = E\frac{Z_{right2}}{Z_{right2} + R_1}$$

$$v_{b1} = 10\angle 0^\circ V\frac{4886\angle -29.2^\circ\Omega}{4886\angle -29.2^\circ\Omega + 27k\Omega}$$

$$v_{b1} = 1.558\angle -24.88^\circ V$$

Example 5

For the circuit of Figure 5.3.2, determine *vb* using superposition. *I*=2*E*−3∠90∘ amps peak and *E*=10∠0∘ volts peak.

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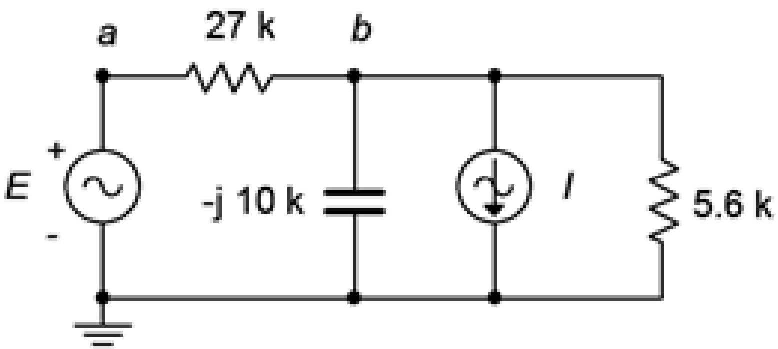
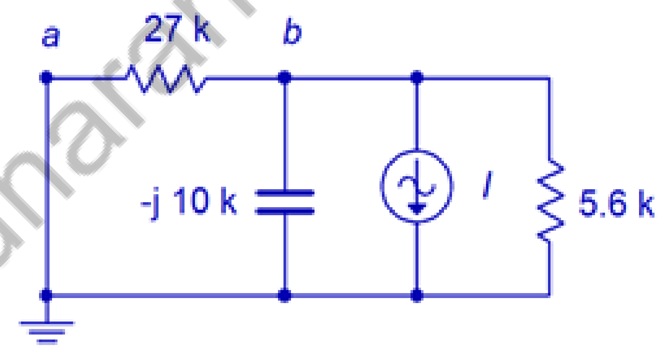




$$Z_{total} = \frac{1}{\frac{1}{X_C} + \frac{1}{R_1} + \frac{1}{R_2}}$$

$$Z_{total} = \frac{1}{\frac{1}{-j10k\Omega} + \frac{1}{27k\Omega} + \frac{1}{5.6k\Omega}}$$

$$Z_{total} = 4208\angle - 24.9°\Omega$$

$$v_{b2} = I \times Z_{total}$$

$$v_{b2} = -2E - 3\angle 90°\,A \times 4208\angle - 24.9°\Omega$$

$$v_{b2} = 8.416\angle - 114.9°V$$

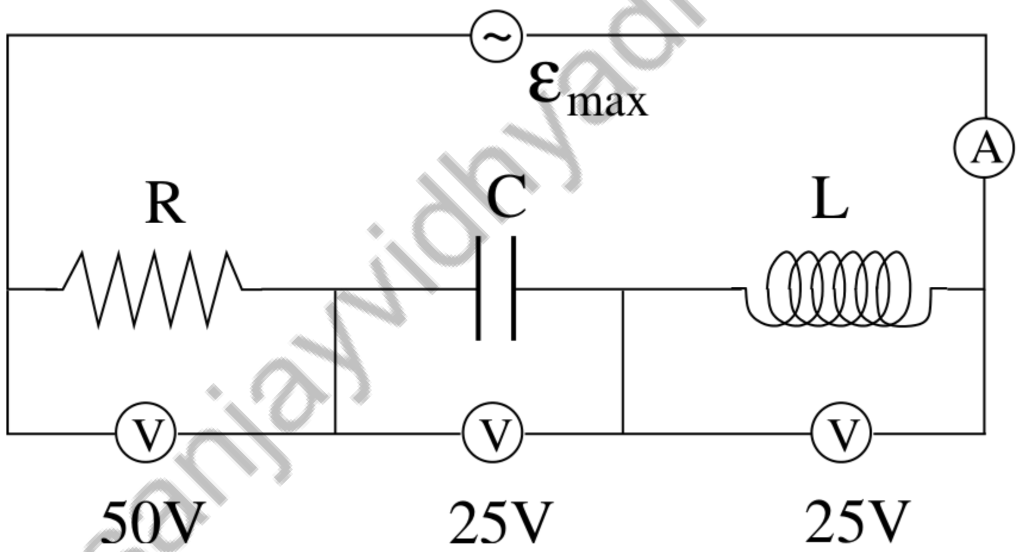
$$v_b = 1.558\angle - 24.88°V + 8.416\angle - 114.9°V$$

$$v_b = 8.558\angle - 104.4°V$$

Example 6

In this RLC circuit, we know the voltage amplitudes V_R ,V_C ,V_L across each device, the current amplitude I_{max} = 5A, and the angular frequency ω = 2rad/s.

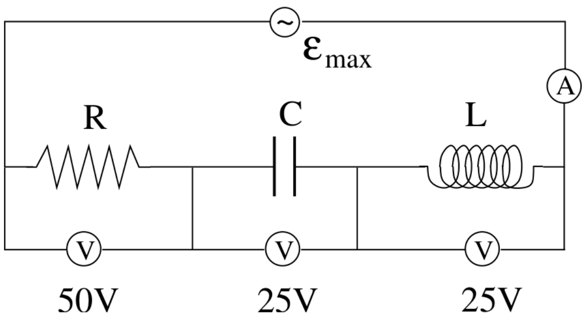
- Find the device properties R,C,L and the voltage amplitude E_{max} of the ac source.



Example 6

In this RLC circuit, we know the voltage amplitudes V_R ,V_C ,V_L across each device, the current amplitude I_{max} = 5A, and the angular frequency ω = 2rad/s.

- Find the device properties R,C,L and the voltage amplitude E_{max} of the ac source.



$$X_R = \frac{50\text{V}}{5\text{A}} = 10\Omega, \quad X_C = \frac{25\text{V}}{5\text{A}} = 5\Omega, \quad X_L = \frac{25\text{V}}{5\text{A}} = 5\Omega.$$

The device properties follow directly:

$$R = 10\Omega, \quad C = 0.1\text{F}, \quad L = 2.5\text{H}.$$

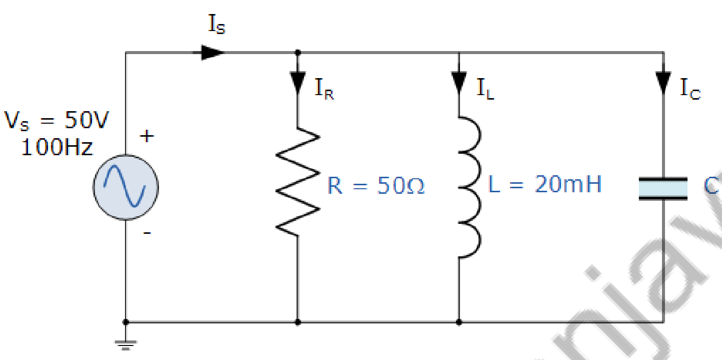
The general expression for the EMF is, we recall from the previous lecture,

$$\mathcal{E}_{max} = \sqrt{V_R^2 + (V_L - V_C)^2}.$$

$$\mathcal{E}_{max} = V_R = 50\text{V}.$$

Example 7

A 50Ω resistor, a 20mH coil and a 5uF capacitor are all connected in parallel across a 50V, 100Hz supply. Calculate the total current drawn from the supply, the current for each branch, the total impedance of the circuit and the phase angle. Also construct the current triangles representing the circuit.



1). Inductive Reactance, (X_L):

$$X_L = \omega L = 2\pi f L = 2\pi.100.0.02 = 12.6\Omega$$

2). Capacitive Reactance, (X_C):

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi.100.5\times10^{-6}} = 318.3\Omega$$

3). Impedance, (Z):

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2}} = \frac{1}{\sqrt{\left(\frac{1}{50}\right)^2 + \left(\frac{1}{318.3} - \frac{1}{12.6}\right)^2}}$$

$$Z = \frac{1}{\sqrt{0.0004 + 0.0058}} = \frac{1}{0.0788} = 12.7\Omega$$

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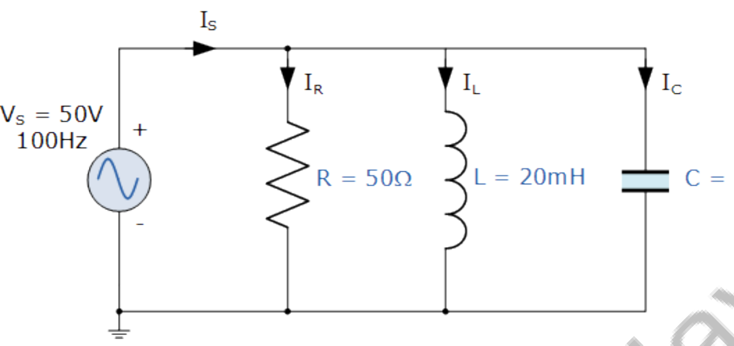


4). Current through resistance, R (I_R):

$$I_R = \frac{V}{R} = \frac{50}{50} = 1.0\,(A)$$

5). Current through inductor, L (I_L):

$$I_L = \frac{V}{X_L} = \frac{50}{12.6} = 3.9\,(A)$$

6). Current through capacitor, C (I_C):

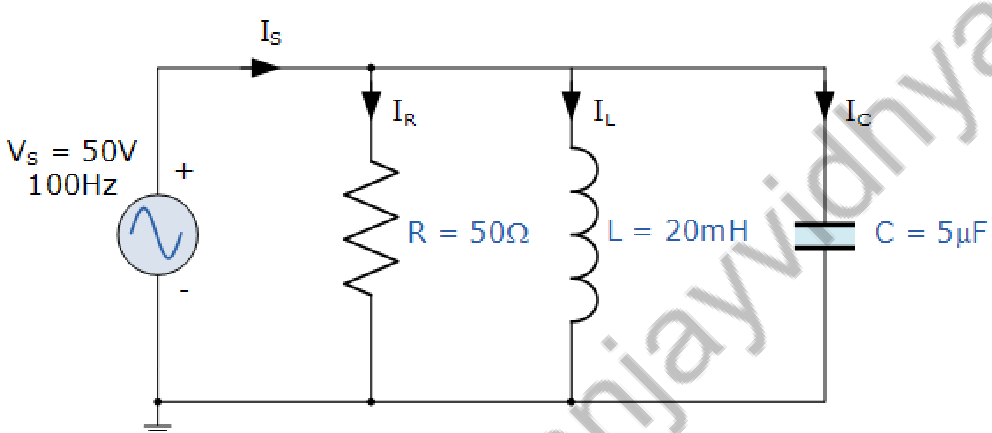
$$I_C = \frac{V}{X_C} = \frac{50}{318.3} = 0.16\,(A)$$

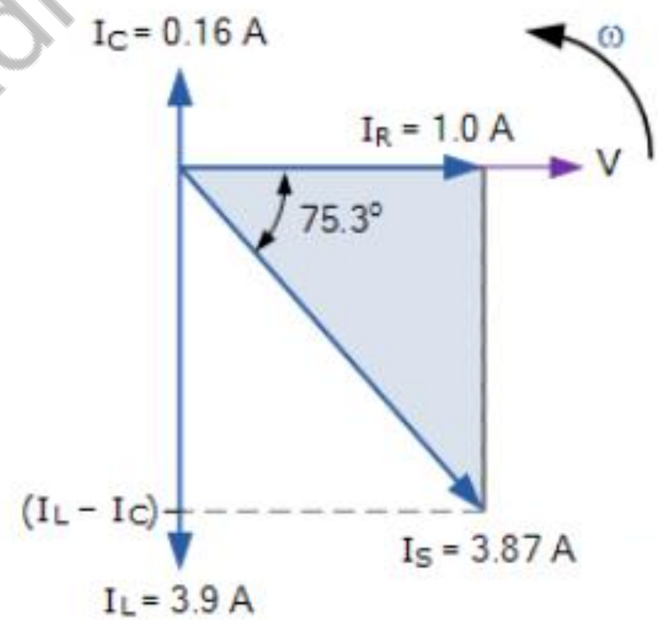
7). Total supply current, (I_S):

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2} = \sqrt{1^2 + (3.9 - 0.16)^2} = 3.87\,(A)$$

Example 7

A 50Ω resistor, a 20mH coil and a 5uF capacitor are all connected in parallel across a 50V, 100Hz supply. Calculate the total current drawn from the supply, the current for each branch, the total impedance of the circuit and the phase angle. Also construct the current triangles representing the circuit.

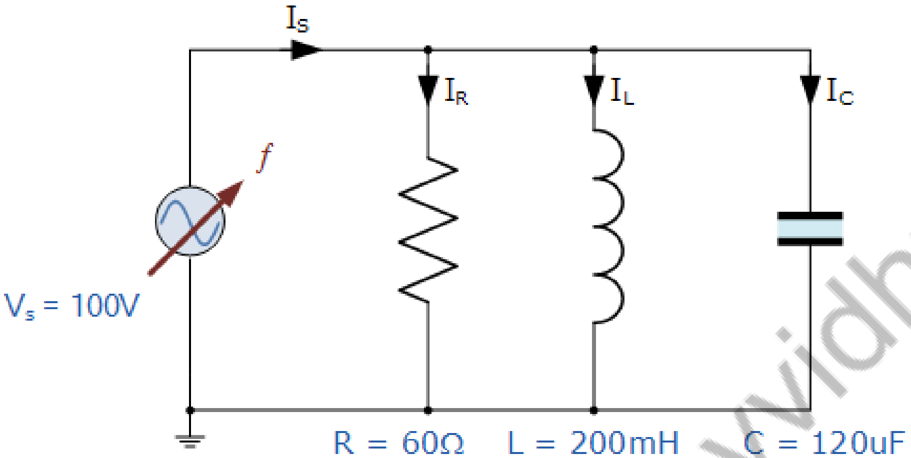


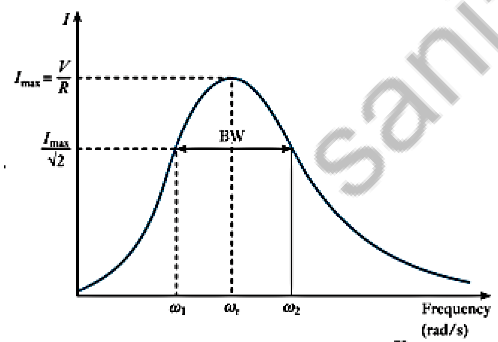


Current Triangle

Example 8

A parallel resonance network consisting of a resistor of 60Ω, a capacitor of 120uF and an inductor of 200mH is connected across a sinusoidal supply voltage which has a constant output of 100 volts at all frequencies. Calculate, the resonant frequency, the quality factor and the bandwidth of the circuit, the circuit current at resonance and current magnification.





1. Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.2.120.10^{-6}}} = 32.5\text{Hz}$$

2. Inductive Reactance at Resonance, X_L

$$X_L = 2\pi f L = 2\pi.32.5.0.2 = 40.8\Omega$$

3. Quality factor, Q

$$Q = \frac{R}{X_L} = \frac{R}{2\pi f L} = \frac{60}{40.8} = 1.47$$

4. Bandwidth, BW

$$BW = \frac{f_r}{Q} = \frac{32.5}{1.47} = 22\text{Hz}$$

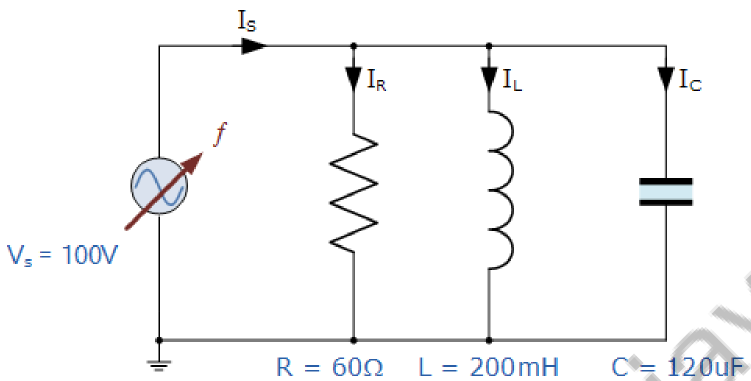
5. The upper and lower -3dB frequency points, f_H and f_L

$$f_L = f_r - \frac{1}{2}BW = 32.5 - \frac{1}{2}(22) = 21.5\text{Hz}$$

$$f_H = f_r + \frac{1}{2}BW = 32.5 + \frac{1}{2}(22) = 43.5\text{Hz}$$

Example 8

A parallel resonance network consisting of a resistor of 60Ω, a capacitor of 120uF and an inductor of 200mH is connected across a sinusoidal supply voltage which has a constant output of 100 volts at all frequencies. Calculate, the resonant frequency, the quality factor and the bandwidth of the circuit, the circuit current at resonance and current magnification.



6. Circuit Current at Resonance, I_T

At resonance the dynamic impedance of the circuit is equal to R

$$I_T = I_R = \frac{V}{R} = \frac{100}{60} = 1.67A$$

7. Current Magnification, I_{mag}

$$I_{MAG} = Q \times I_T = 1.47 \times 1.67 = 2.45A$$

Note that the current drawn from the supply at resonance (the resistive current) is only 1.67 amps, while the current flowing around the LC tank circuit is larger at 2.45 amps. We can check this value by calculating the current flowing through the inductor (or capacitor) at resonance.

$$I_L = \frac{V}{X_L} = \frac{V}{2\pi f L} = \frac{100}{2\pi .32.5.0.2} = 2.45A$$

Example 9

You have a parallel RLC circuit with a 16 Ω resistor, 8 Ω inductor, 20 Ω capacitor, and a 120-V power supply what are the following values?

a. Current through the resistor (I_R). $I_R = V_s / R = 120 / 16 = 7.5$ A

b. Current through the inductor (I_L). $I_L = V_s / X_L = 120 / 8 = 15$ A

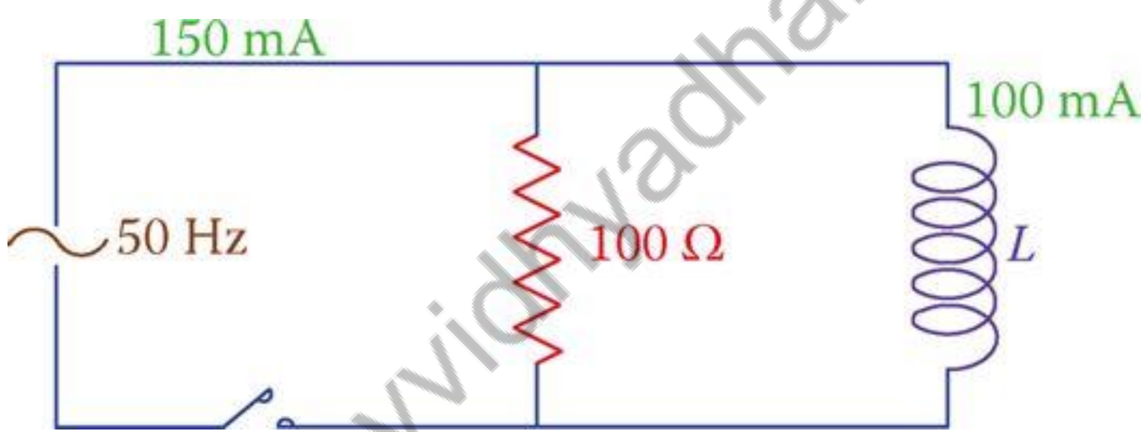
c. Current through the capacitor (I_C). $I_C = V_s / X_C = 120 / 20 = 6$ A

d. Net reactive current (I_X). $I_X = I_L - I_C = 9$ A

e. Total line current (I_T). $I_T = \sqrt{I^2_R + (I_L - I_C)^2} = \sqrt{7.5^2 + 9^2} = 11.71$A

Example 10

In the circuit shown in Figure, the total current is 150 mA and the current through the inductor is 100 mA. Determine what the applied voltage is. Also, knowing that the frequency is 50 Hz, find the value of *L*.



$$I_R = \sqrt{I^2 - I_L^2} = \sqrt{150^2 - 100^2} = 0.1118A$$

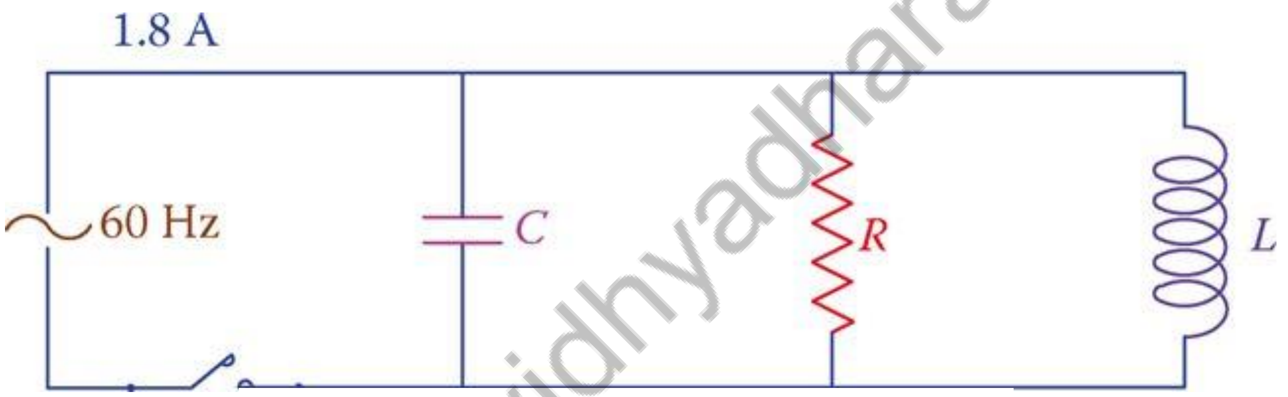
$V = 100 *$ **11.8** $= 11.18V$

$$X_L = 11.18 \div 0.100 = 111.8\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{111.8}{2\pi * 50} = 35.6mH$$

Example 11

In the circuit shown in Figure, R = 55 Ω, L = 0.08 H, and C = 1 μF, find the impedance of the circuit and the applied voltage.



$$X_L = 2 * 3.14 * 60 * 0.08 = 30.16\Omega$$

$$X_C = \frac{1}{2 * 3.14 * 60 * 0.000001} = 26.5\Omega$$

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{55}\right)^2 + \left(\frac{1}{55} - \frac{1}{26.5}\right)^2} = \frac{1}{53.33}$$

$$Z = 53.33\Omega$$

Applied voltage = $V = ZI$ = (53.33) (1.8) = **96 V.**

Thank you