Electrical Science: 2021-22

Tutorial 6

Basics of AC Circuit

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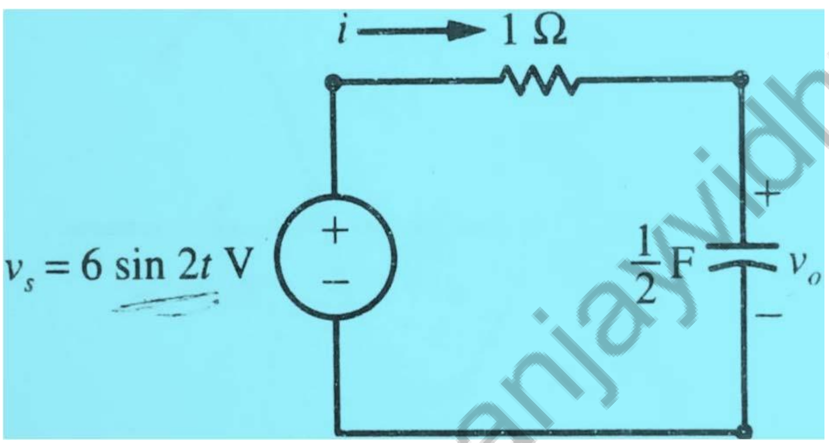
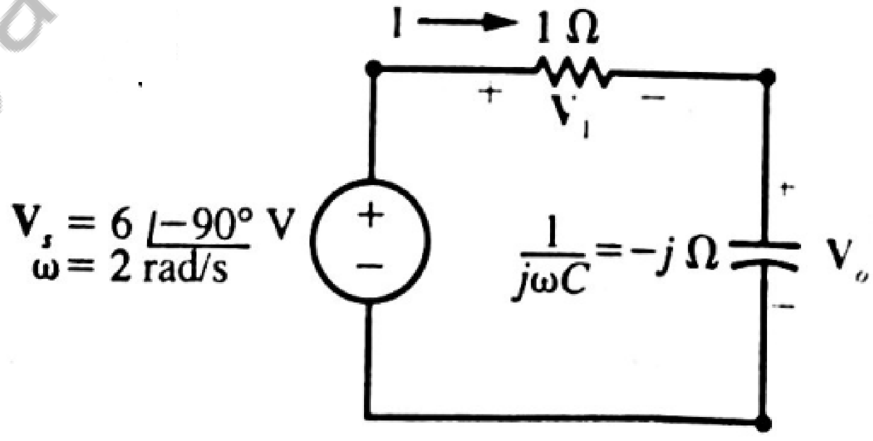
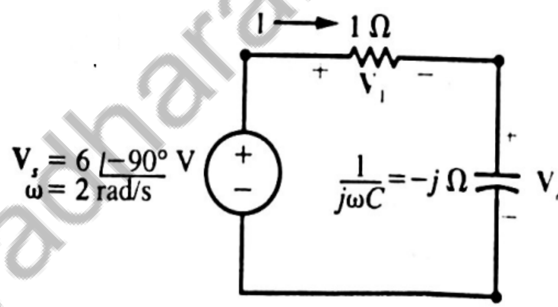
Example 1

- Express the sinusoidal time function by phasor representation
- Label resistors, inductors and capacitors (or combination of them) by their impedances or admittances. Find V_0





Example 1



$$\mathbf{V}_s = 1\mathbf{I} - j\mathbf{I}$$

$$6\angle -90° = (1 - j)\mathbf{I} = \sqrt{2}\angle -45°\,\mathbf{I} \quad \Rightarrow$$

$$\mathbf{I} = \frac{6\angle -90°}{\sqrt{2}\angle -45°} = 3\sqrt{2}\angle -45° \text{ A}$$

$$\mathbf{V}_o = -j\mathbf{I} = (1\angle -90°)(3\sqrt{2}\angle -45°) = 3\sqrt{2}\angle -135° \text{ V}$$

Example 2

A sinusoidal alternating current of 6 amps is flowing through a resistance of 40Ω. Calculate the average voltage and the peak voltage of the supply.

The R.M.S. Voltage value is calculated as: $V_{RMS} = I \times R = 6 \times 40 = 240V$

The Average Voltage value is calculated as: $\text{Form Factor} = \frac{V_{RMS}}{V_{average}}$

$$\therefore V_{average} = \frac{V_{RMS}}{\text{Form Factor}} = \frac{240}{1.11} = 216.2 \text{ volts}$$

The Peak Voltage value is calculated as: $\text{Peak Voltage} = \text{R.M.S.} \times 1.414$

$$\therefore 240 \times 1.414 = 339.4 \text{ volts}$$

Example 3

Converting Polar Form into Rectangular Form, (P→R) $6\angle 30^{o} = x + jy$

$$x = A.\cos\theta \quad y = A.\sin\theta$$

$$\begin{aligned} 6\angle 30^{0} &= (6\cos\theta) + j(6\sin\theta) \\ &= (6\cos 30^{o}) + j(6\sin 30^{o}) \\ &= (6 \times 0.866) + j(6 \times 0.5) \\ &= 5.2 + j3 \end{aligned}$$

Example 4

Converting Rectangular Form into Polar Form, (R→P) $\left(5.2 + j3\right)$

$$\left(5.2 + j3\right) = A\angle\theta$$

$$\text{where:} \quad A = \sqrt{5.2^2 + 3^2} = 6$$

$$\text{and} \quad \theta = \tan^{-1}\frac{3}{5.2} = 30^o$$

$$\text{Hence,} \quad \left(5.2 + j3\right) = 6\angle 30^0$$

Example 5

Multiplying together 6 ∠30º and 8 ∠– 45º

$$Z_1 \times Z_2 = A_1 \times A_2 \angle \theta_1 + \theta_2$$

$$Z_1 \times Z_2 = 6 \times 8 \angle 30^0 + (-45^0) = 48 \angle -15^0$$

Division in Polar Form

Likewise, to divide together two vectors in polar form, we must divide the two modulus and then subtract their angles as shown.

$$\frac{Z_1}{Z_2} = \left(\frac{A_1}{A_2}\right) \angle \theta_1 - \theta_2$$

$$\frac{Z_1}{Z_2} = \left(\frac{6}{8}\right) \angle 30^0 - (-45^0) = 0.75 \angle 75^0$$

Example 6

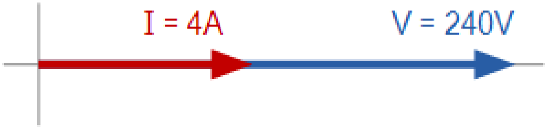
An electrical heating element which has an AC resistance of 60 Ohms is connected across a 240V AC single phase supply. Calculate the current drawn from the supply and the power consumed by the heating element. Also draw the corresponding phasor diagram showing the phase relationship between the current and voltage.

1. The supply current: $I = \frac{V}{R} = \frac{240}{60} = 4.0\,A$

2. The Active power consumed by the AC resistance is calculated as:

$$P = I^2R = 4^2.60 = 960\,W$$

3. As there is no phase difference in a resistive component, (θ = 0), the corresponding phasor diagram is given as:



Example 7

A sinusoidal voltage supply defined as: V(t) = 100 x cos(ωt + 30°) is connected to a pure resistance of 50 Ohms. Determine its impedance and the peak value of the current flowing through the circuit. Draw the corresponding phasor diagram.

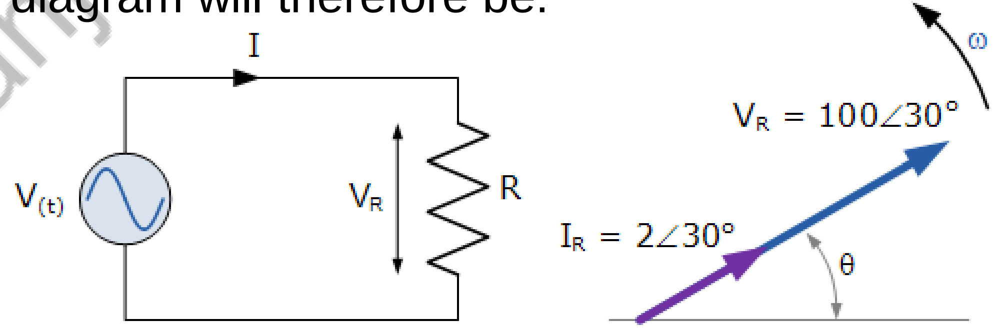
The sinusoidal voltage across the resistance will be the same as for the supply in a purely resistive circuit. Converting this voltage from the time-domain expression into the phasor-domain expression gives us:

$$V_{R(t)} = 100\cos(\omega t + 30^\circ) \quad \Rightarrow \quad V_R = 100\angle 30^\circ \text{ volts}$$

Applying Ohms Law gives us:

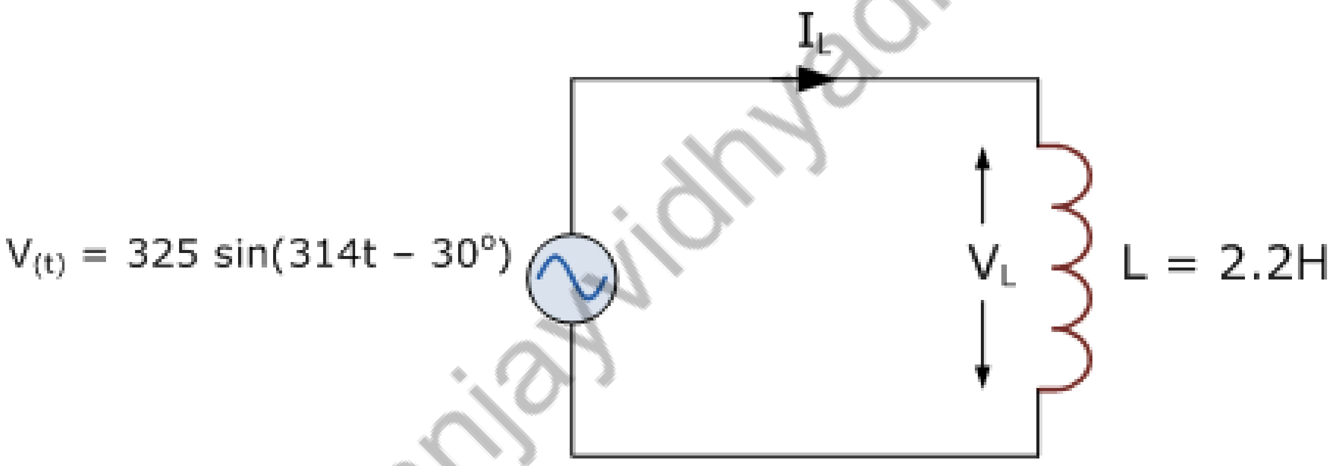
$$I_R = \frac{V_R}{R} = \frac{100\angle 30^\circ}{50\Omega} = 2\angle 30^\circ \text{ Amps}$$

The corresponding phasor diagram will therefore be:



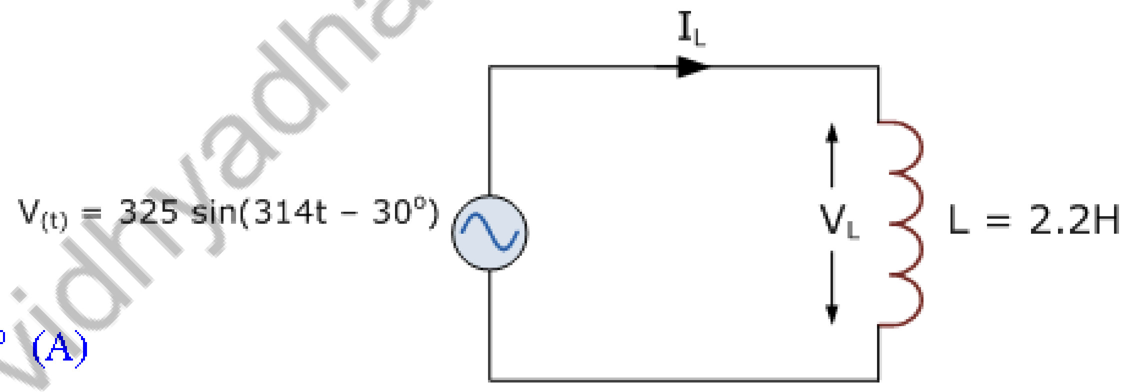
Example 8

In the following circuit, the supply voltage is defined as: $V_{(t)} = 325 \sin(314t - 30^o)$ and L = 2.2H. Determine the value of the rms current flowing through the coil and draw the resulting phasor diagram.



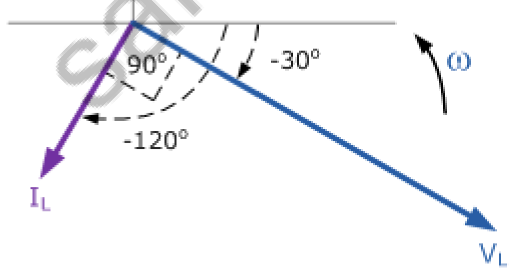
Example 8

The rms voltage across the coil will be the same as from the supply voltage. If the power supplies peak voltage is 325V, then the equivalent rms value will be 230V. Converting this time domain value into its polar form gives us: $V_L = 230 \angle -30^o$ (volts). The inductive reactance of the coil is: $X_L = \omega L = 314 \times 2.2 = 690\Omega$. Then the RMS current flowing through the coil can be found using Ohms law as:



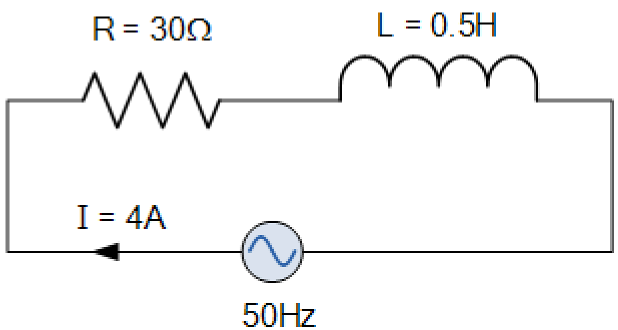
$$I_L = \frac{V_L}{jX_L} = \frac{230 \angle -30^o}{690 \angle 90^o} = 0.33 \angle -120^o \ (A)$$

With the current lagging the voltage by 90º the phasor diagram will be.



Example 9

A coil has a resistance of 30Ω and an inductance of 0.5H. If the current flowing through the coil is 4amps. What will be the rms value of the supply voltage if its frequency is 50Hz.



$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157\Omega$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = \sqrt{30^2 + 157^2}$$

$$Z = 159.8\Omega$$

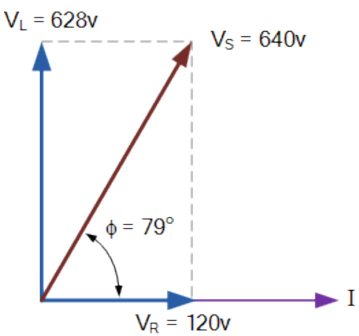
$$\tan^{-1}\phi = \frac{X_L}{R} = \frac{157}{30} = 79.2^o$$

Then the voltage drops across each component is calculated as:

$$V_S = I.Z = 4 \times 159.8 = 640v$$

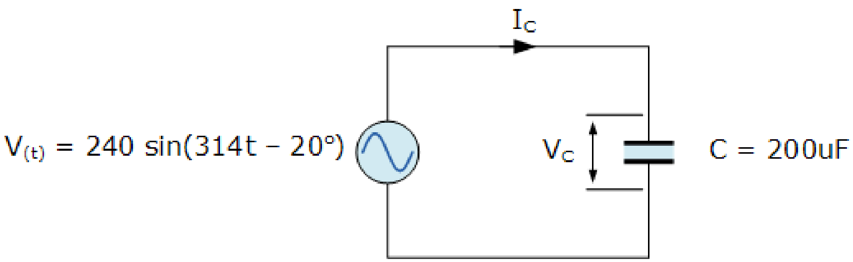
$$V_R = I.R = 4 \times 30 = 120v$$

$$V_L = I.X_L = 4 \times 157 = 628v$$



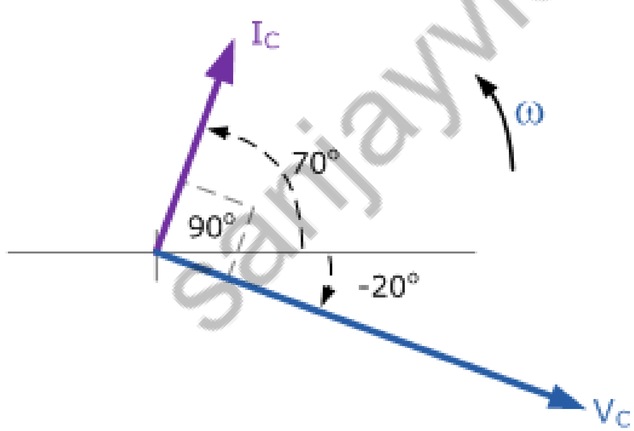
Example 10

A single-phase sinusoidal AC supply voltage defined as: $V_{(t)} = 240 \sin(314t - 20^o)$ is connected to a pure AC capacitance of 200uF. Determine the value of the current flowing into the capacitor and draw the resulting phasor diagram.



$$X_C = \frac{1}{j\omega C} = \frac{1}{314 \times 200\mu F} = 16\angle\text{-}90^\circ$$

$$I_C = \frac{V_C}{jX_C} = \frac{240\angle\text{-}20^\circ}{16\angle\text{-}90^\circ} = 15\angle 70^\circ \ (A)$$



Thank you