Electrical Science: 2021-22
Tutorial 5
Second Order Circuits

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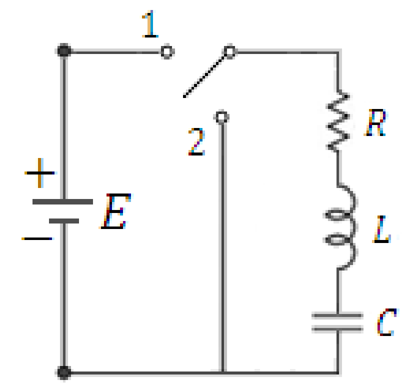
Problem 1

1.The Switch is kept at position 1 for a log time and toggled to position 2 at t=0. R= 40 Ω, L = 4 H, and C = 1/4 F. Calculate the characteristic roots of the circuit. Is the circuit overdamped, underdamped, or critically damped?



R= 40 Ω, L = 4 H, C = 1/4 F => $\alpha = \frac{R}{2L} = 5;\ \omega_0 = \frac{1}{\sqrt{LC}} = 1$

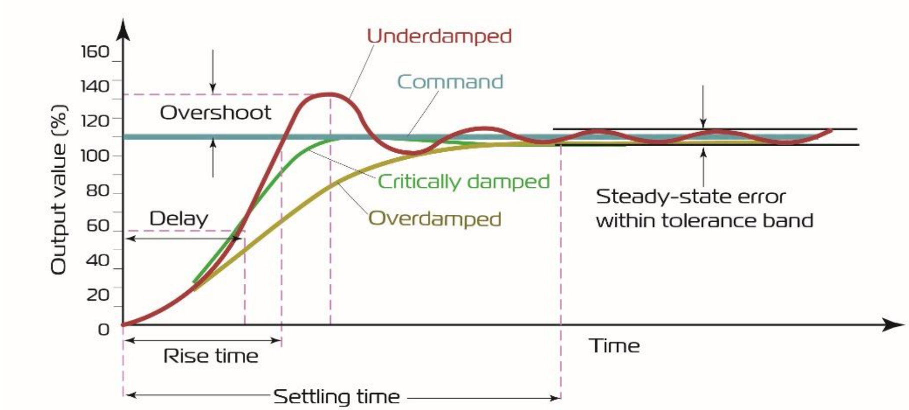
The roots are, $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -5 + \sqrt{5^2 - 1}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -5 - \sqrt{5^2 - 1}$

$y(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

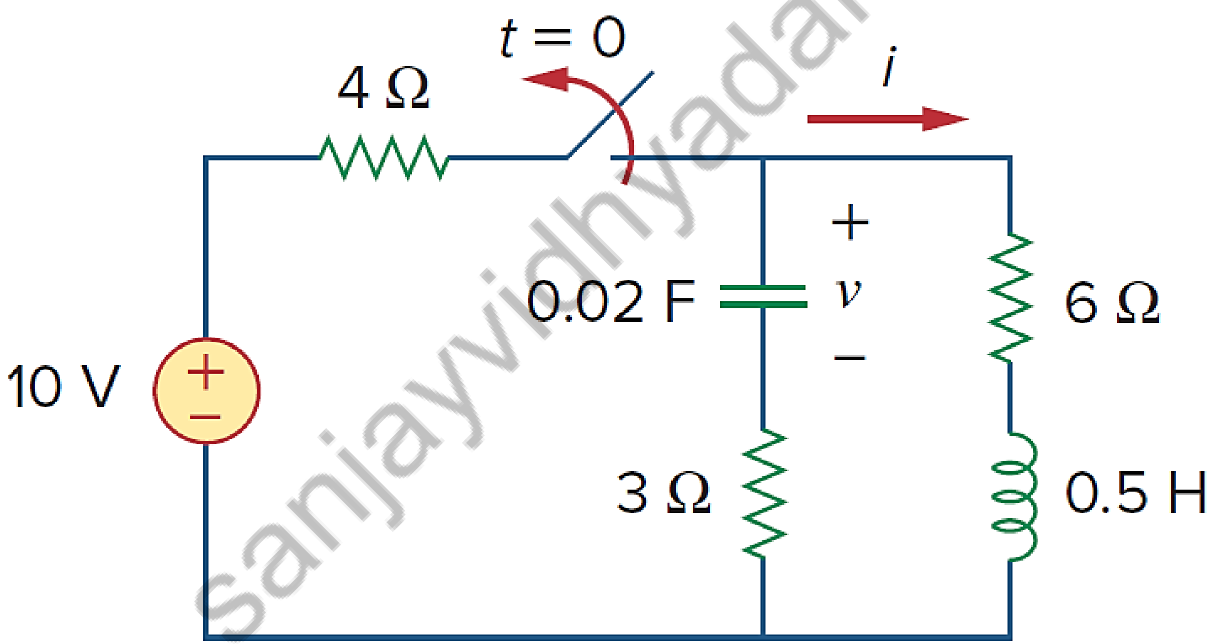
(A_1 *and* A_2 arearbitraryconstants and are determined from the initial conditions)

Since $\boldsymbol{\alpha} > \boldsymbol{\omega_0}$, we conclude that the response is ***overdamped***.



Problem 2

Find i(t) for t > 0. Assume that the circuit has reached steady state before the switch is opened.



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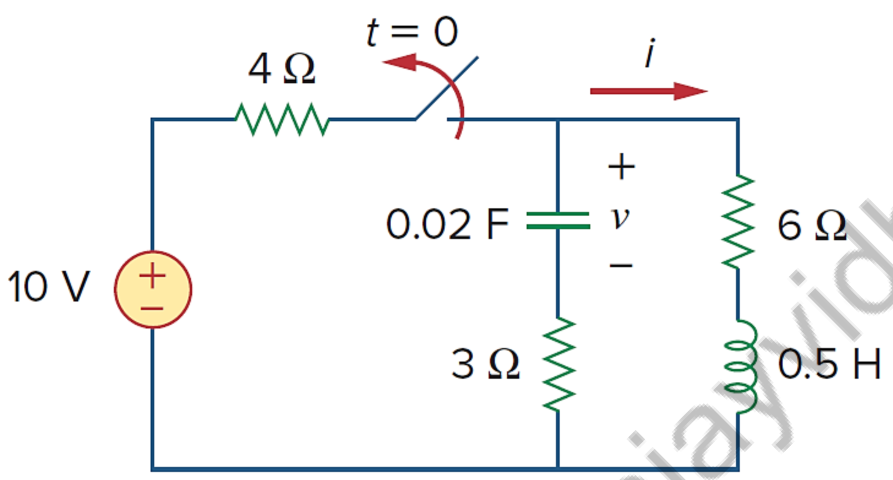

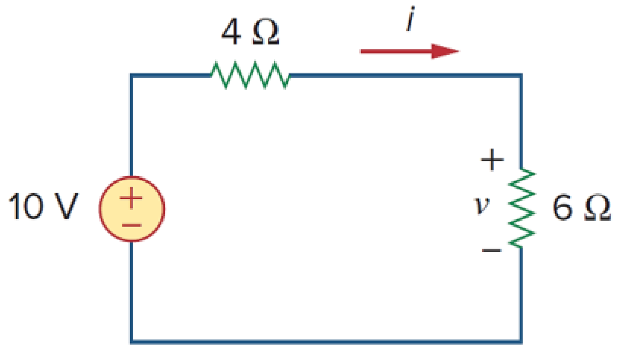

(a) for $t < 0$

$$i(0) = \frac{10}{4+6} = 1\text{A};\ \ v(0) = 6i(0) = 6\text{V}$$

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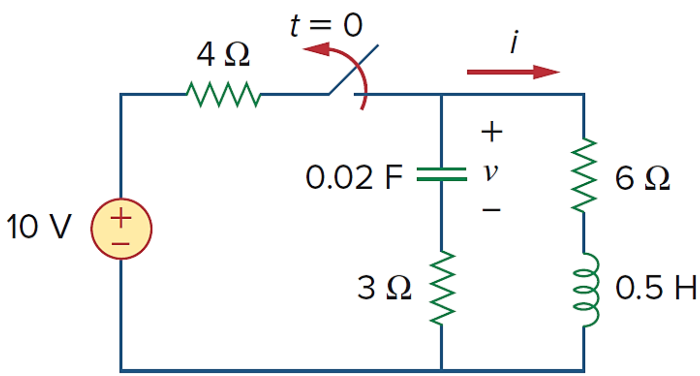
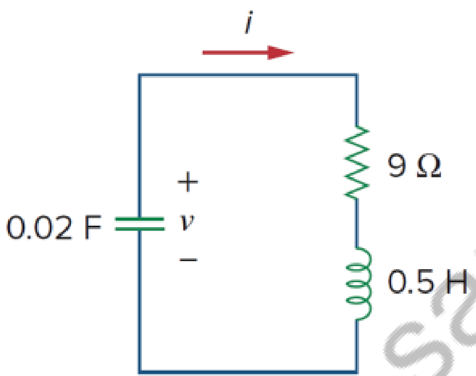
Find i(t) for t > 0. Assume that the circuit has reached steady state before the switch is opened.





(b) for $t > 0$.

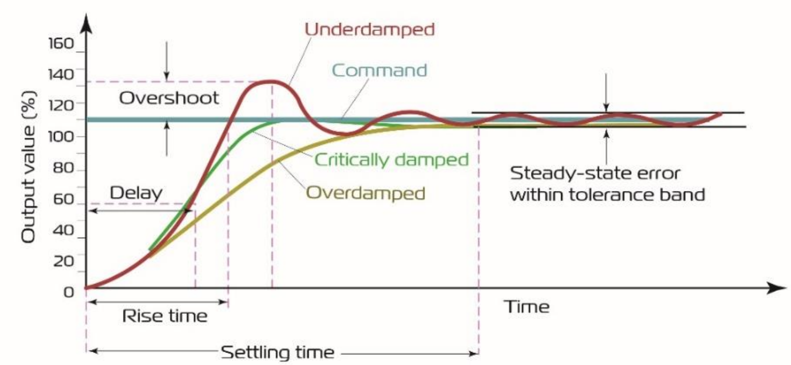
$$\alpha = \frac{R}{2L} = 9;\ \omega_0 = \frac{1}{\sqrt{LC}} = 10$$

$$s_1 = -9 + \sqrt{9^2 - 100}$$

$$s_2 = -9 - \sqrt{9^2 - 100}$$

Under-damped response $s_{1,2} = -9 \pm j4.359$

Hence, $i(t) = e^{-9t}[A_1(\cos 4.359t) + A_2(\sin 4.359t)]$



Problem 2

Find i(t) for t > 0. Assume that the circuit has reached steady state before the switch is opened.

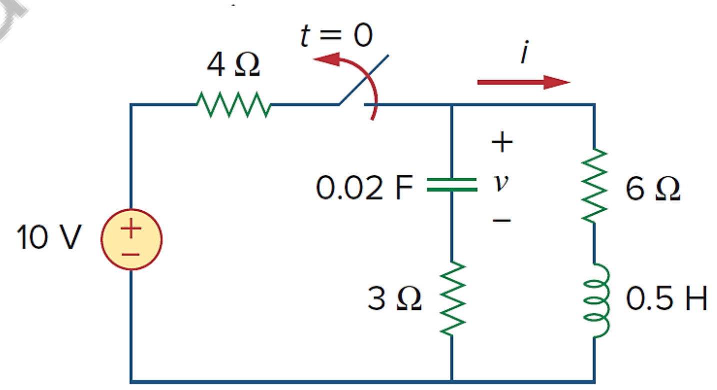
$$\text{Hence, } \boxed{i(t) = e^{-9t}[A_1(\cos 4.359t) + A_2(\sin 4.359t)]}$$

A_1 and A_2 are found using the initial conditions.
At $t = 0$, $i(0) = \boxed{1 = A_1}$

$$\left.\frac{di}{dt}\right|_{t=0} = -\frac{1}{L}[Ri(0) - v(0)] = -6A/s$$

Taking the derivative of $i(t)$

$$\frac{di}{dt} = -9e^{-9t}[A_1(\cos 4.359t) + A_2(\sin 4.359t)]$$
$$+ e^{-9t}(4.359)[-A_1(\sin 4.359t) + A_2(\cos 4.359t)]$$



Problem 2

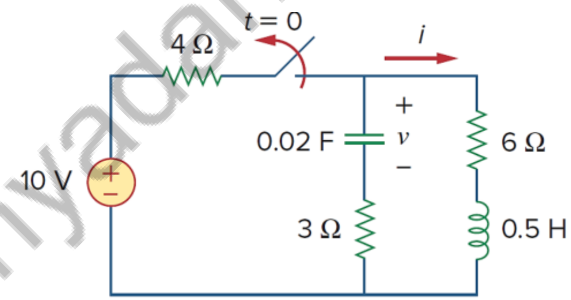
Find i(t) for t > 0. Assume that the circuit has reached steady state before the switch is opened.

$$-6 = -9(A_1 + 0) + 4.359(-0 + A_2)$$

substituting $A_1 = 1$,

$$-6 = -9 + 4.359A_2$$

$$\Rightarrow \boxed{A_2 = 0.6882}$$



Substituting the values of A_1 and A_2 yields the complete solution as,

$$\boxed{i(t) = e^{-9t}[(\cos 4.359t) + 0.6882(\sin 4.359t)]\,\text{A}} \quad \text{for } t > 0$$

Problem 3

The switch has been closed for a sufficiently long time and then it is opened at (see fig.11.6(a)). Find the expression for (a) $v(t)$, (b) $i_c(t)$, $t>0$ for $L=0.5$ H

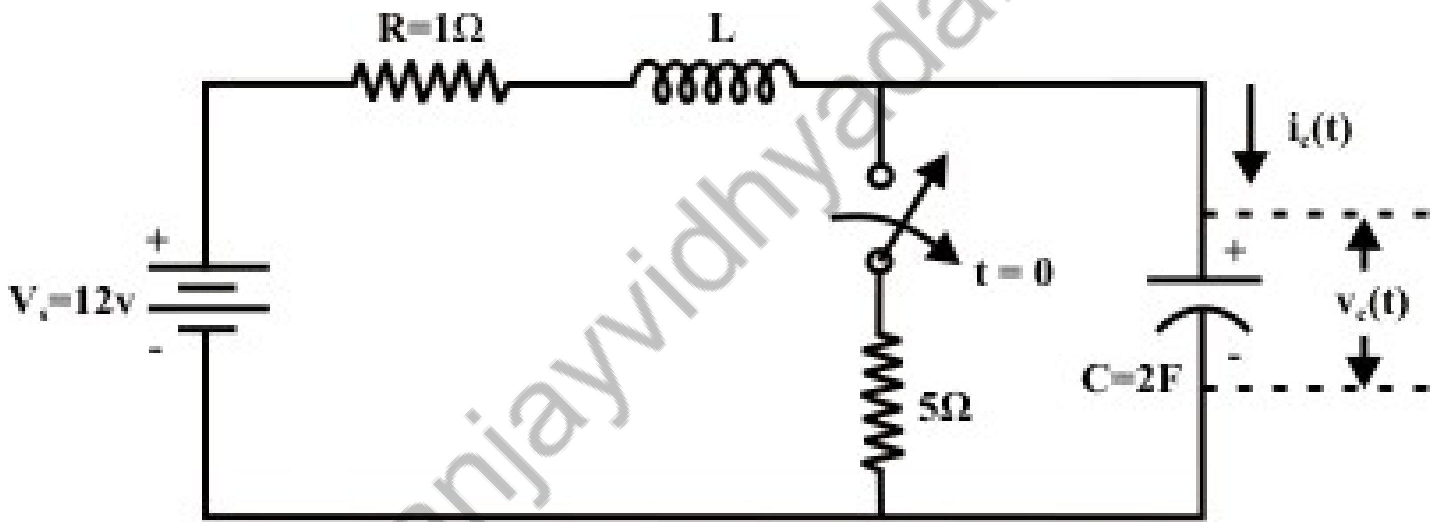


Fig. 11.6(a)

Problem 3

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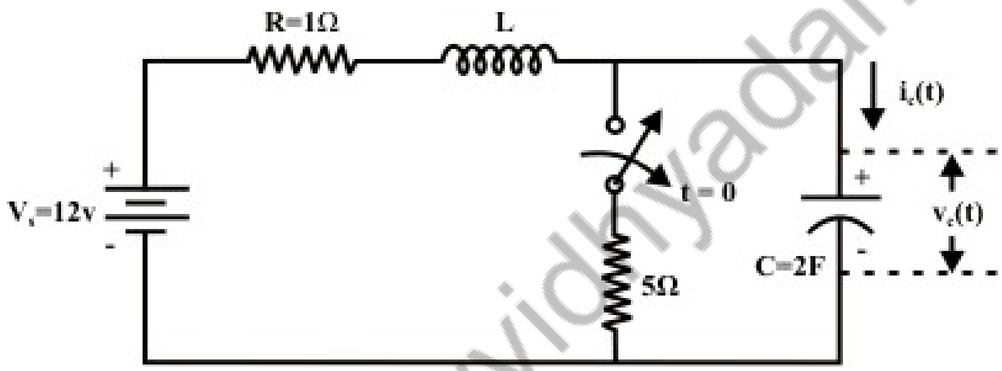

Fig. 11.6(a)

$$i_L(0^+) = i_L(0^-) = \frac{12}{1+5} = 2A$$

$$v_c(0^+) = v_c(0^-) = 10\,volt.$$

Problem 3

The switch has been closed for a sufficiently long time and then it is opened at (see fig.11.6(a)). Find the expression for (a) v(t) , (b) $i_c(t)$, t>0 for L=0.5 H

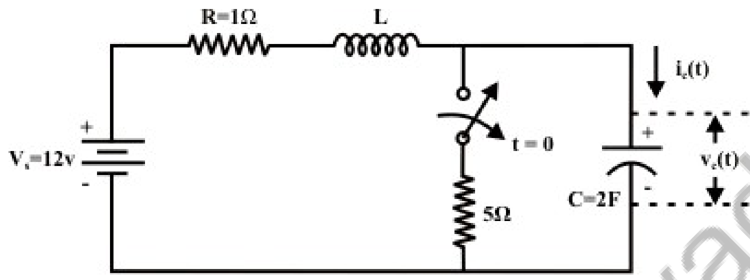

Fig. 11.6(a)

$$i_L(0^+) = i_L(0^-) = \frac{12}{1+5} = 2A$$

$$v_c(0^+) = v_c(0^-) = 10\,volt.$$

Case-1: $L = 0.5H$, $R = 1\Omega$ *and* $C = 2F$

$$V_s = R\,i(t) + L\frac{di(t)}{dt} + v_c(t) \Rightarrow V_s = RC\frac{dv_c(t)}{dt} + LC\frac{d^2 v_c(t)}{dt^2} + v_c(t)$$

The solution of the above differential equation is given by

$v_c(t) = v_{cn}(t) + v_{cf}(t)$

Problem 3

The switch has been closed for a sufficiently long time and then it is opened at (see fig.11.6(a)). Find the expression for (a) v(t) , (b) $i_c(t)$, t>0 for L=0.5 H

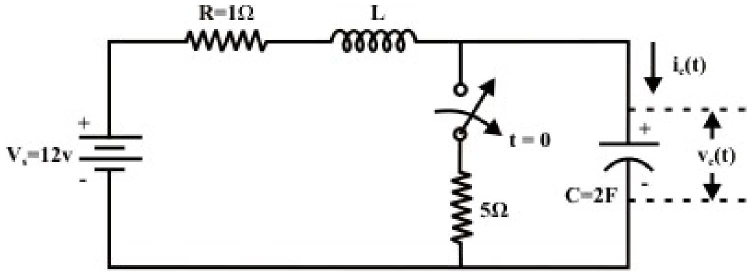

Fig. 11.6(a)

Case-1: $L=0.5H \;, R=1\Omega \; and \; C=2F$

$$\frac{d^2v_c(t)}{dt^2}+\frac{R}{L}\frac{dv_c(t)}{dt}+\frac{1}{LC}v_c(t)=0$$

$$\alpha=\frac{R}{2L};\; \omega_n=\frac{1}{\sqrt{LC}};$$

$$v_{cn}(t)=\left(A_1 t+A_2\right)e^{\alpha t} \quad (\text{where } \alpha=\alpha_1=\alpha_2=-1)$$

$$v_c(t)=\left(A_1 t+A_2\right)e^{\alpha t}+A$$

$$i_L(0^+)=i_L(0^-)=\frac{12}{1+5}=2A$$

$$v_c(0^+)=v_c(0^-)=10\,volt.$$

At $t=0^+$;

$$v_c(t)\big|_{t=0^+}=A_2\,e^{-1\times 0}+A=A_2+A \quad \Rightarrow \; A_2+A=10$$

$$\frac{dv_c(t)}{dt}=\alpha\left(A_1t+A_2\right)e^{\alpha t}+A_1\,e^{\alpha t}=-\left(A_1t+A_2\right)e^{-t}+A_1\,e^{-t}$$

$$\frac{dv_c(t)}{dt}\bigg|_{t=0^+}=A_1-A_2 \Rightarrow \; A_1-A_2=1$$

$$(\text{note, } C\frac{dv_c(0^+)}{dt}=i_c(0^+)=i_L(0^+)=2 \Rightarrow \frac{dv_c(0^+)}{dt}=1\,volt/\text{sec.})$$

$$v_c(\infty)=A \Rightarrow A=12$$

$$A_1=-1;\; A_2=-2\,.$$

$$v_c(t)=-(t+2)e^{-t}+12=\;12-(t+2)e^{-t};$$

$$i(t)=C\frac{dv_c(t)}{dt}=2\times\left[(t+2)e^{-t}-e^{-t}\right]=2\times(t+1)e^{-t}$$

Problem 4

The switch ' ' in the circuit of Fig. 11.7(a) was closed in position '1' sufficiently long time and then kept in position '2'. Find (a) $v_c(t)$, (b) $i_c(t)$, for $t \geq 0$ if C = 1/9 F

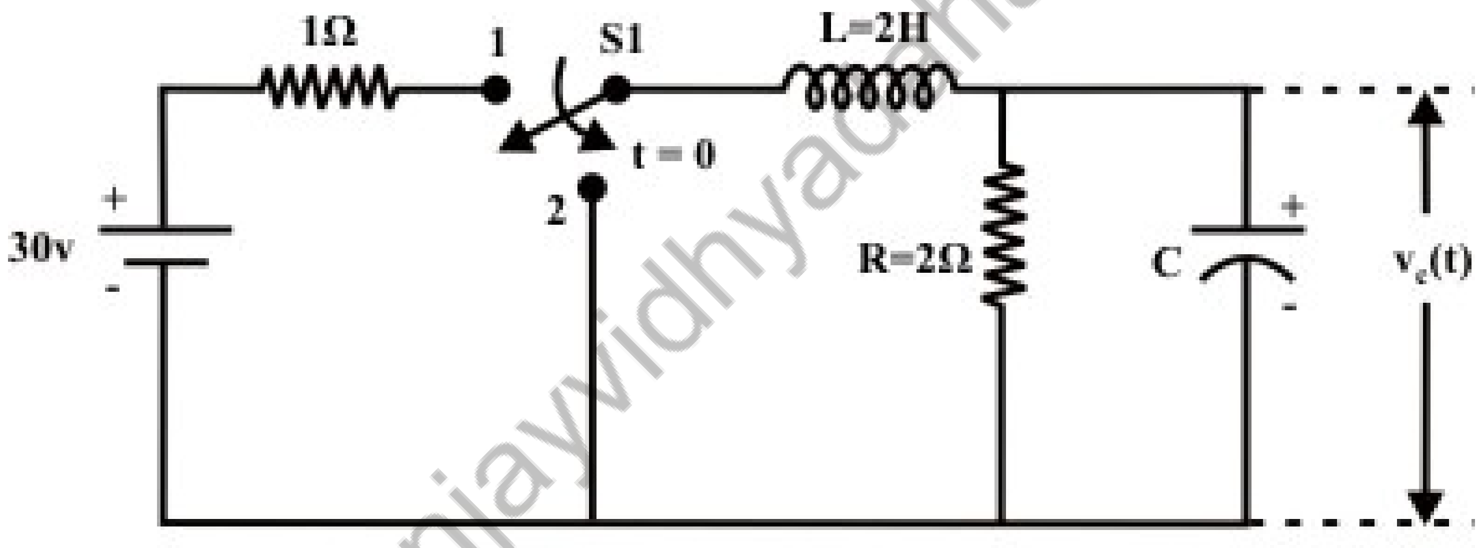

Fig. 11.7(a)

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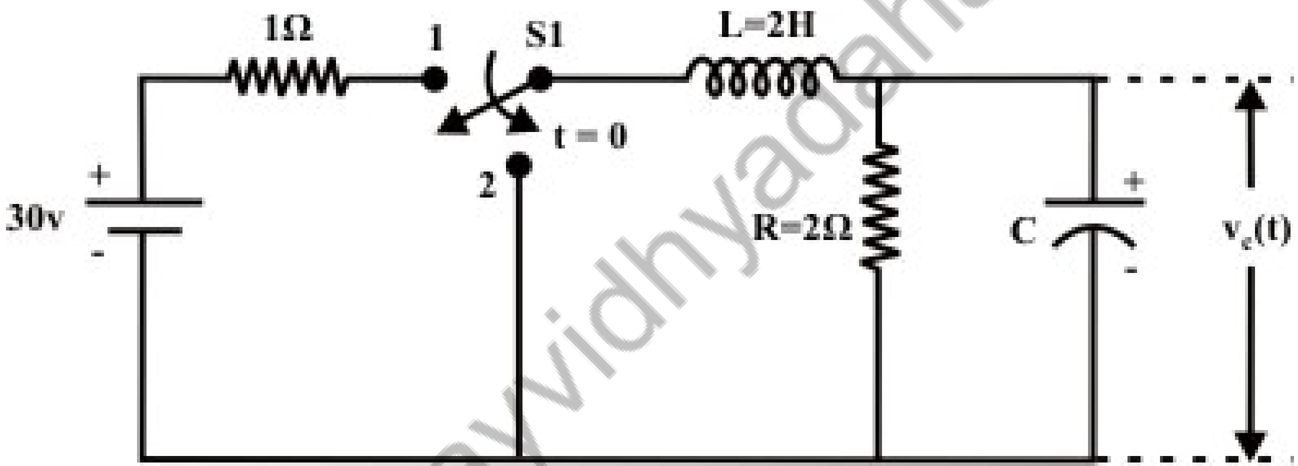


Fig. 11.7(a)

When the switch was in position ' 1 ', the steady state current in inductor is given by

$$i_L(0^-) = \frac{30}{1+2} = 10\text{A}, \quad v_c(0^-) = i_L(0^-)R = 10 \times 2 = 20 \text{ volt.}$$

Problem 4

The switch ‘ ’ is kept in position ‘2’ and corresponding circuit diagram is shown in Fig.11.7 (b)

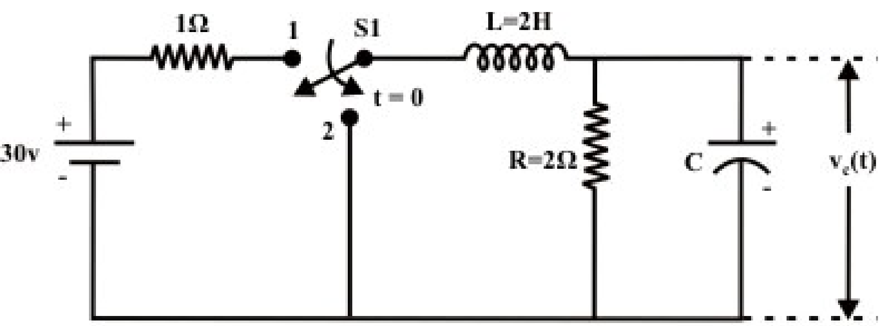


Fig. 11.7(a)

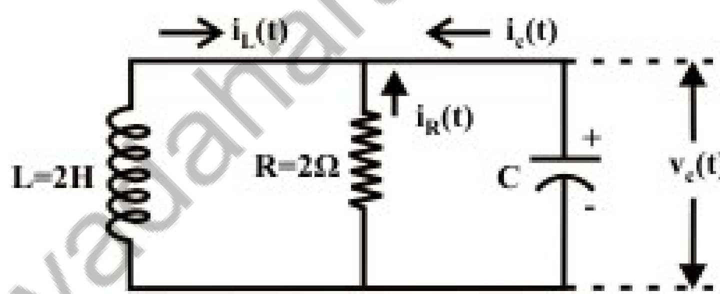


Fig. 11.7(b)

$$\frac{v_C(t)}{R} + i_C(t) + i_L(t) = 0$$

$$\frac{v_C(t)}{R} + C\frac{dv_C(t)}{dt} + i_L(t) = 0$$

$$\frac{L}{R}\frac{di_L(t)}{dt} + C.L\frac{d^2 i_L(t)}{dt^2} + i_L(t) = 0 \quad [\text{note: } v_c(t) = L\frac{di_L(t)}{dt}]$$

$$\frac{d^2 i_L(t)}{dt^2} + \frac{1}{RC}\frac{di_L(t)}{dt} + \frac{1}{LC} i_L(t) = 0$$

Problem 4

The switch ‘ ’ is kept in position ‘2’ and corresponding circuit diagram is shown in Fig.11.7 (b)

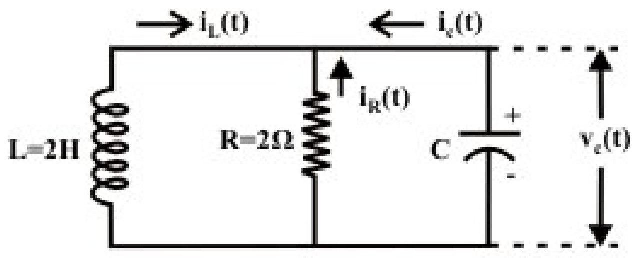


Fig. 11.7(b)

$$\frac{d^2 i_L(t)}{dt^2} + \frac{1}{RC}\frac{di_L(t)}{dt} + \frac{1}{LC} i_L(t) = 0$$

$$\alpha_1 = \frac{-\frac{1}{RC} + \sqrt{\left(\frac{1}{RC}\right)^2 - 4/LC}}{2} = \frac{-\frac{9}{2} + \sqrt{\left(\frac{9}{2}\right)^2 - \frac{4\times 9}{2}}}{2} = -1.5$$

$$\alpha_2 = \frac{-\frac{1}{RC} - \sqrt{\left(\frac{1}{RC}\right)^2 - 4/LC}}{2} = \frac{-\frac{9}{2} - \sqrt{\left(\frac{9}{2}\right)^2 - \frac{4\times 9}{2}}}{2} = -3.0$$

$$s_{1,2} = \frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$,	$\alpha = \frac{1}{2RC}$,	$\omega_0 = \frac{1}{\sqrt{LC}}$

The transient or neutral solution of the homogeneous equation is given by

$$i_L(t) = A_1 e^{-1.5t} + A_2 e^{-3.0t}$$

To determine A_1 and A_2, the following initial conditions are used.

At $t = 0^+$;

$$i_L(0^+) = i_L(0^-) = A_1 + A_2$$

$$10 = A_1 + A_2$$

$$v_c(0^+) = v_c(0^-) = v_L(0^+) = L\frac{di_L(t)}{dt}\Big|_{t=0^+}$$

$$20 = 2\times\left[A_1 \times -1.5\,e^{-1.5t} - 3.0 \times A_2 e^{-3.0t}\right]$$

$$= 2\left[-1.5A_1 - 3A_2\right] = -3A_1 - 6A_2$$

Solving equations (11.31) and (11,32) we get , $A_2 = -16.66$, $A_1 = 26.666$.

The natural response of the circuit is

$$i_L = \frac{80}{3}e^{-1.5t} - \frac{50}{3}e^{-3.0t} = 26.66e^{-1.5t} - 16.66e^{-3.0t}$$

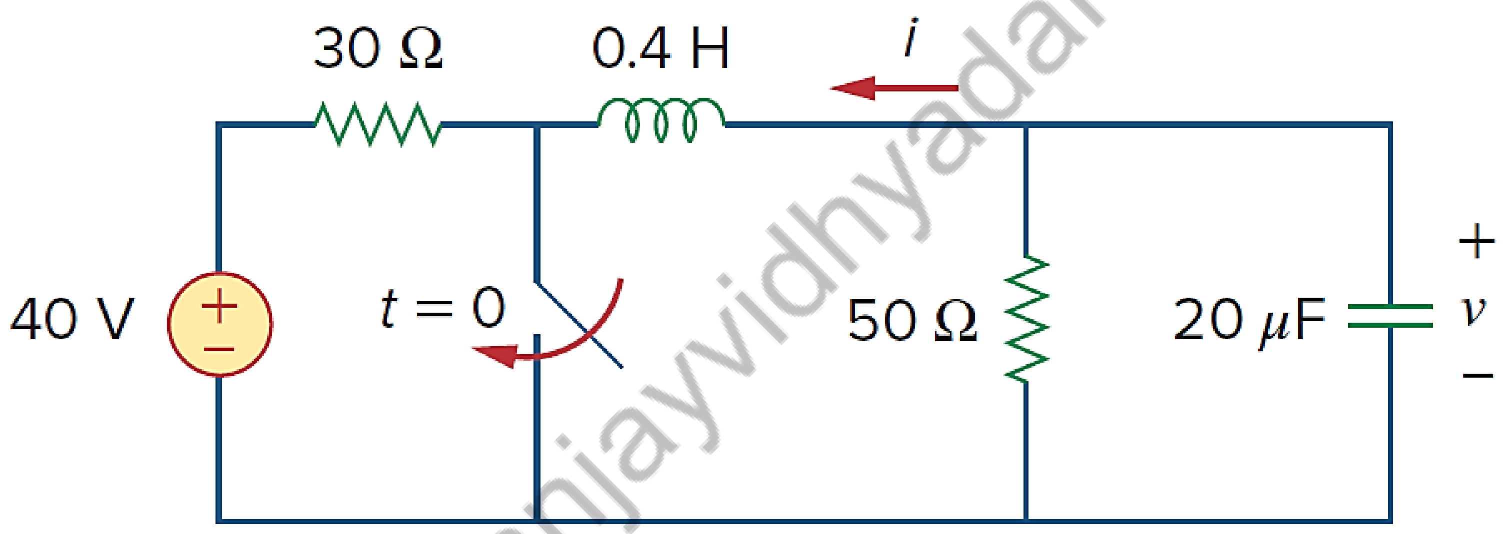
$$L\frac{di_L}{dt} = 2\left[26.66\times -1.5e^{-1.5t} - 16.66\times -3.0e^{-3.0t}\right]$$

$$v_L(t) = v_c(t) = \left[100e^{-3.0t} - 80e^{-1.5t}\right]$$

$$i_c(t) = c\frac{dv_c(t)}{dt} = \frac{1}{9}\left(-300.0e^{-3.0t} + 120e^{-1.5t}\right) = \left(13.33e^{-1.5t} - 33.33e^{-3.0t}\right)$$

Problem 5

Find v(t) for t > 0 in the RLC circuit. Assume that the switch has been open for a long time before closing.

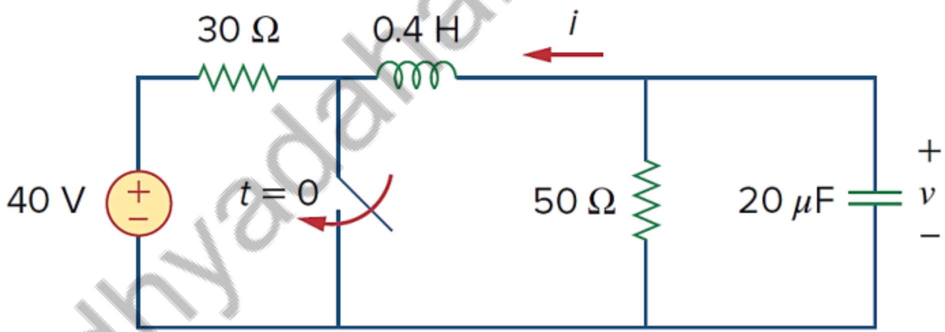


Problem 5

Find v(t) for t > 0 in the RLC circuit. Assume that the switch has been open for a long time before closing.

Solution:

- For $t < 0$, the switch is open.
- Inductor acts like a short circuit, capacitor behaves like an open circuit.
- The initial voltage across the capacitor is the same as the voltage across the 50-Ω resistor.



$$v(0) = \frac{50}{30+50}(40) = 25;$$

$$i(0) = \frac{40}{30+50} = -0.5A$$

$$\frac{dv(0)}{dt} = -\frac{v(0)+Ri(0)}{RC} = 0$$

Problem 5

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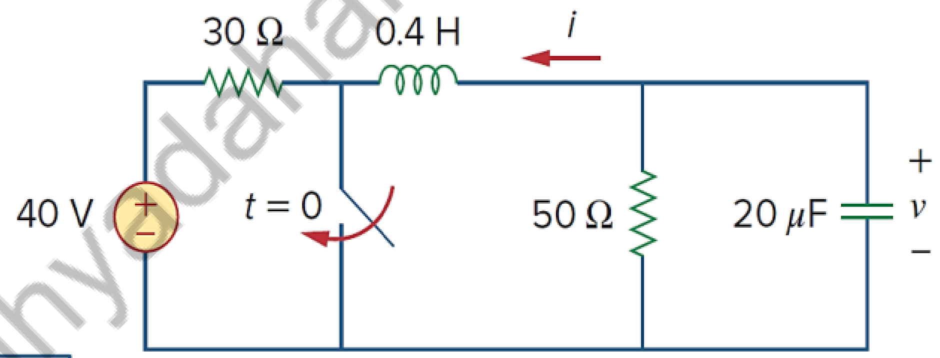


For $t > 0$, the switch is closed. The voltage source along with the 30-Ω resistor is separated from the rest of the circuit.

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times 50\times 20\times 10^{-6}} = 500;$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 354$$

Since $\boldsymbol{\alpha} > \boldsymbol{\omega_0}$, we have the **overdamped** response.

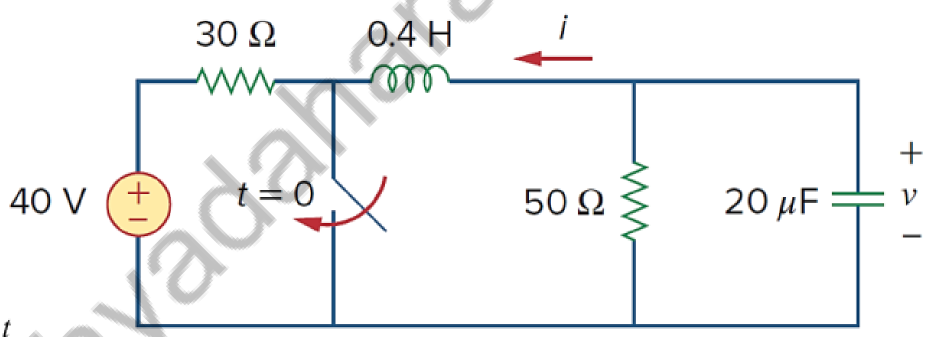
$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -500 \pm 354$$

$$s_1 = -854; s_2 = -146$$

$$v(t) = A_1 e^{-854t} + A_2 e^{-146t}$$

Problem 5

Find v(t) for t > 0 in the RLC circuit. Assume that the switch has been open for a long time before closing.



Applying intial conditions,

$$v(0)=25=A_1 + A_2$$

on differentiating,

$$\frac{dv}{dt} = -854A_1e^{-854t} + -146A_2e^{-146t}$$

$$\frac{dv(0)}{dt} = 0 = -854A_1 - 146A_2$$

$$A_1 = -5.156;\ A_2 = 30.16$$

Thus, the complete solution is given as,

$$v(t) = -5.156e^{-854t} + 30.16e^{-146t} V$$

Thank you