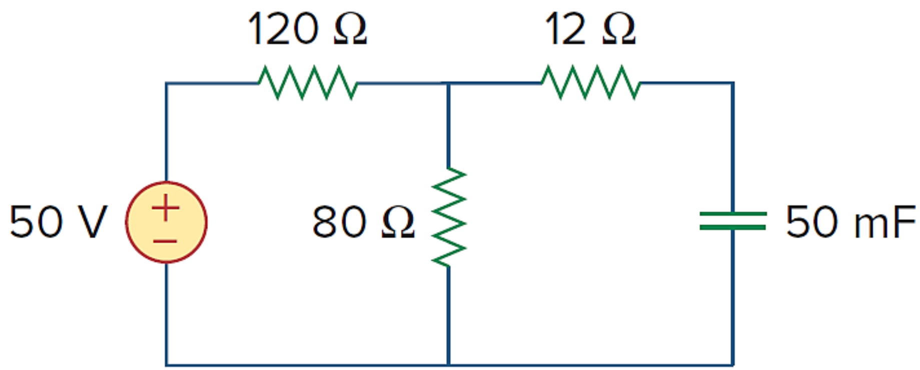
Electrical Science: 2021-22
Tutorial 4
First Order Circuits

By Dr. Sanjay Vidhyadharan

Problem 1

Find the time constant for the RC circuit.



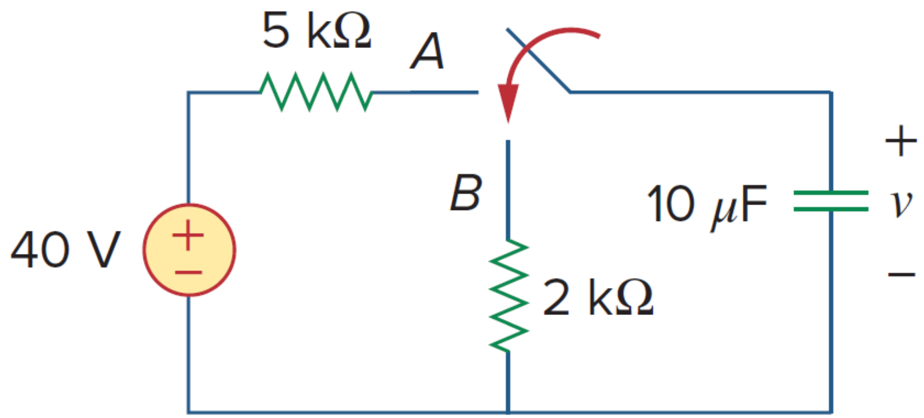
R_{th}=12 + (120 || 80) = 60

T= 60 Ohm x 50 mF = 3 ms

Source: Notes ELL 100 IITD

Problem 2

The switch in Fig. has been in position A for a long time. Assume the switch moves instantaneously from A to B at t = 0. Find v for t > 0.



$V(0) = 40$ V

$T = 2$ KOhm x 10 μF = 20 ms

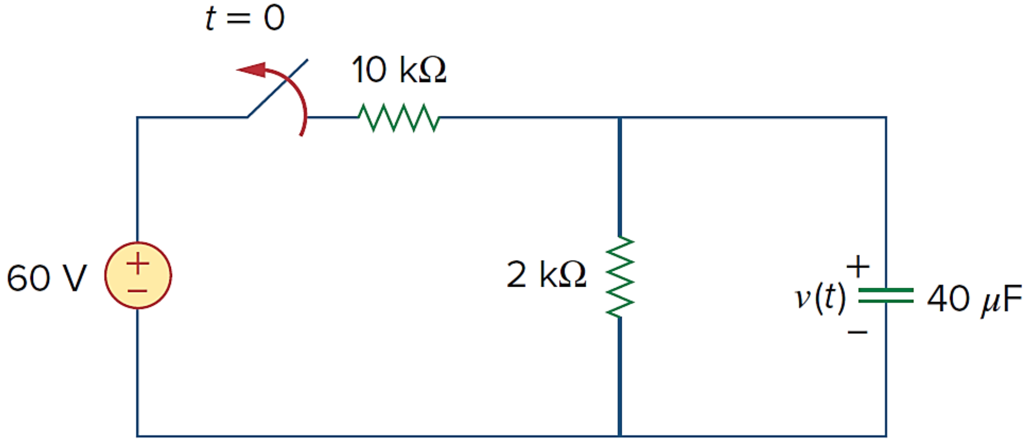
$1/T = 1/20$ ms $= 50\ s^{-1}$

Ans: $v(t) = 40e^{-50t}$ V

Source: Notes ELL 100 IITD

Problem 3

The switch in Fig. has been closed for a long time, and it opens at t = 0. Find v(t) for t ≥ 0.



$$V(0) = \frac{60 * 2}{2+10} = 10\ V$$

T= 2 KOhm x 40 μF = 80 ms

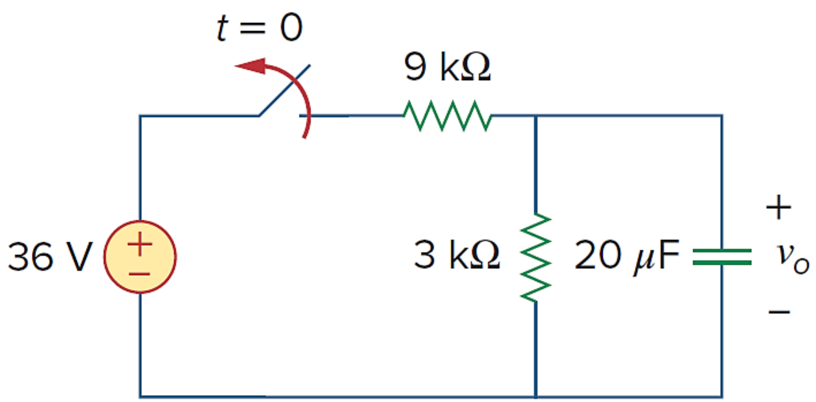
1/T= 1/ 80 ms = 12.5 s^{-1}

Ans: $v(t) = 10e^{-12.5t}$ V

Source: Notes ELL 100 IITD

Problem 4

For the circuit in Fig., find v_o (t) for t > 0. Determine the time necessary for the capacitor voltage to decay to one-third of its value at t = 0.



$$V(0) = \frac{36 * 3}{9+3} = 9\ V$$

T= 3 KOhm x 20 μF = 60 ms

1/T= 1/ 60 ms = 16.67 s^{-1}

Ans: $v_0(t) = 9e^{-16.67t}$ V

t = 65.92 ms

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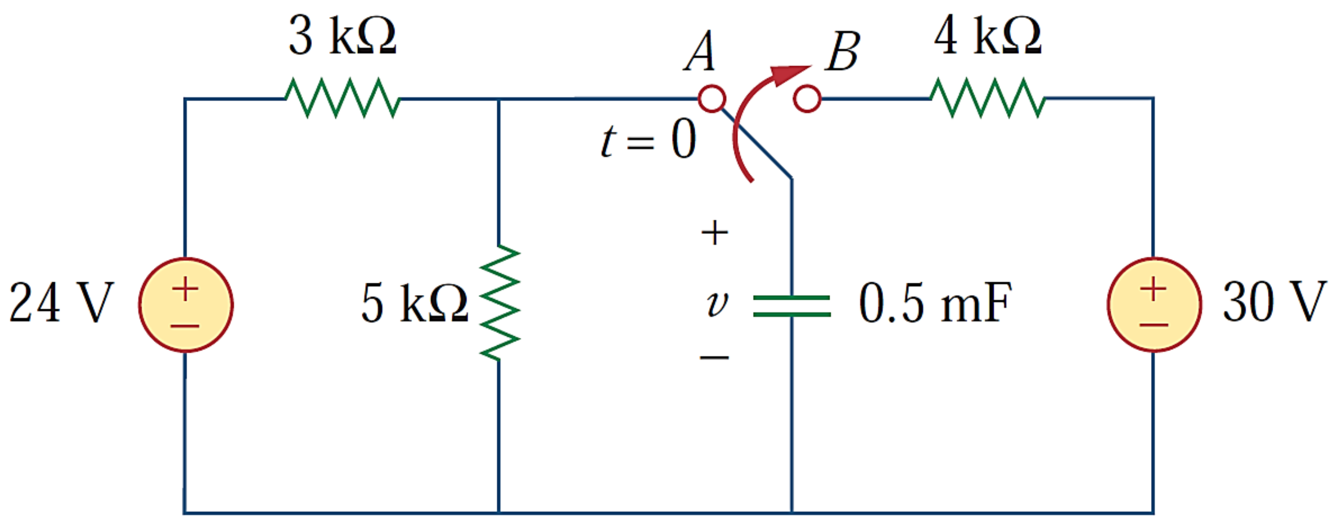
Source: Notes ELL 100 IITD

Problem 5

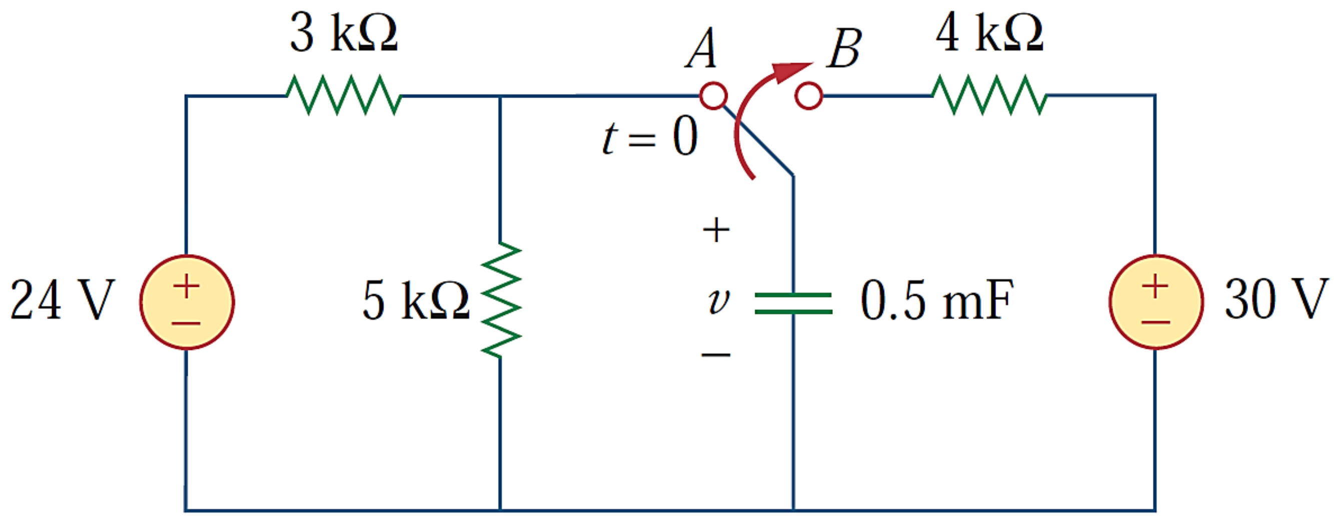
The switch is kept a position A for a long time and moved to B at time t=0.
Determine the voltage across the capacitor at t = 1s and 4s.



Source: Notes ELL 100 IITD

Problem 5

The switch is kept a position A for a long time and moved to B at time t=0. Determine the voltage across the capacitor at t = 1s and 4s.

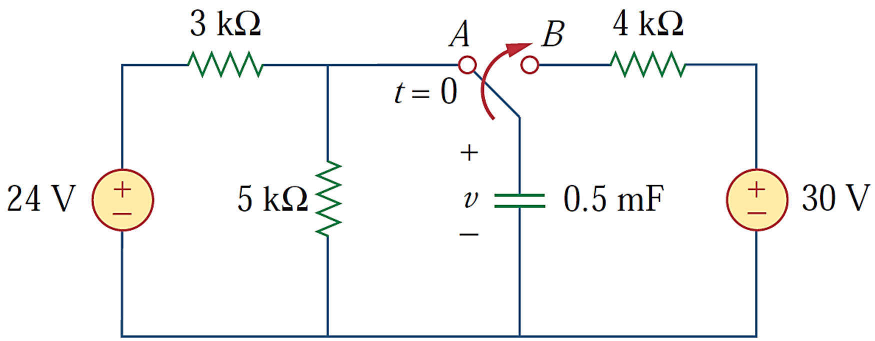


For $t < 0$, the switch is at position A. The capacitor acts like an open circuit to dc, so v is the same as the voltage across the 5kΩ resistor. Hence, the voltage across the capacitor before $t=0$ is obtained by voltage division as,

$$v(0^-) = \frac{5}{5+3}(24) = 15V$$

Source: Notes ELL 100 IITD

Problem 5



$$\tau = RC = 4 \times 10^3 \times 0.5 \times 10^{-3} = 2\text{ s}$$

$v(0) = 15$ V
$v(\infty) = 30$ V

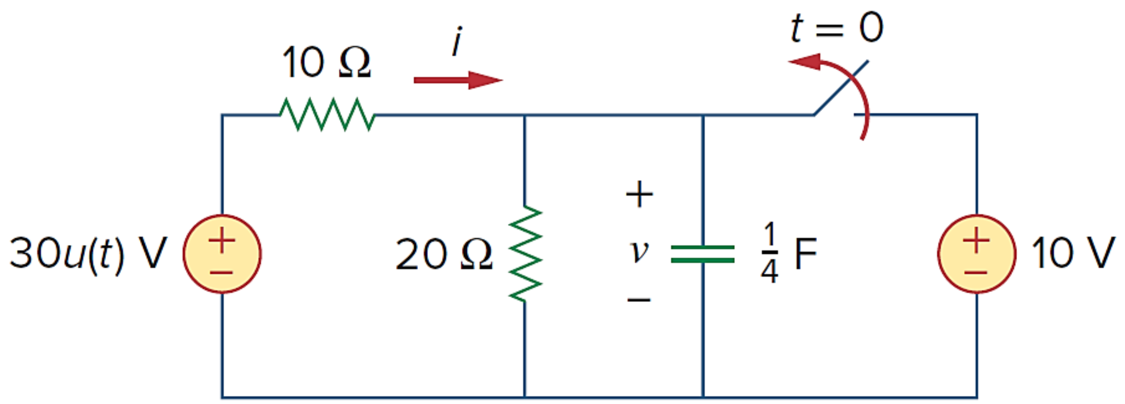
$$\text{Thus, } v(t) = v(\infty) + [v(0) - v(\infty)]e^{-t/\tau}$$
$$= 30 + (15 - 30)e^{-t/2}$$
$$= (30 - 15e^{-0.5t})\text{ V}$$

At $t = 1$, $v(1) = 30 - 15\,e^{-0.5} = 20.9$ V
At $t = 4$, $v(4) = 30 - 15e^{-2} = 27.97$ V

Source: Notes ELL 100 IITD

Problem 6

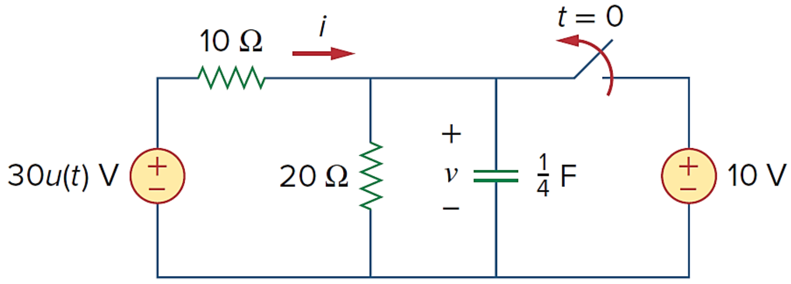
Find i and v if the switch is opened at t = 0 after keeping it closed for a long time.



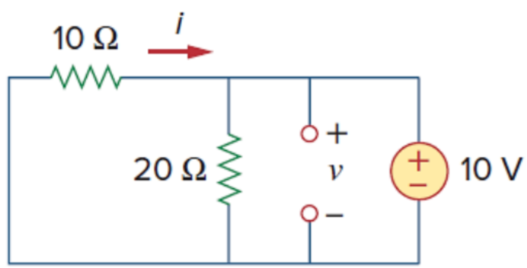
Source: Notes ELL 100 IITD

Problem 6

Find i and v if the switch is opened at t = 0 after keeping it closed for a long time.



For $\boldsymbol{t < 0}$:



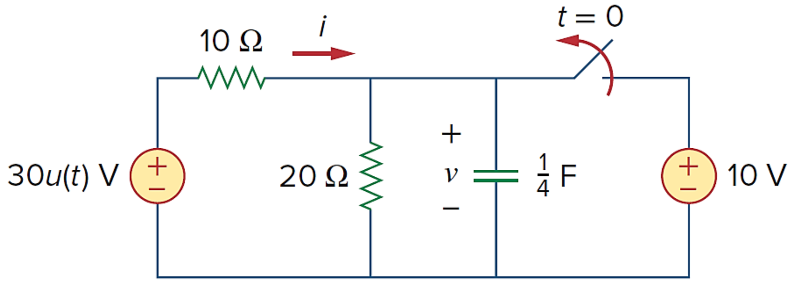
By definition of the unit step function, $30u(t) = \begin{cases} 0 & t < 0 \\ 30 & t > 0 \end{cases}$

For $t < 0$, $v = 10V, i = -\frac{v}{10} = -1A$

The capacitor voltage cannot change instantaneously, $v(0) = v(0^-) = 10V$

Problem 6

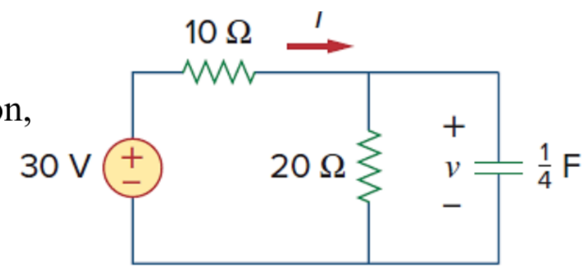
Find i and v if the switch is opened at t = 0 after keeping it closed for a long time.



For $t > 0$, the 10V voltage source is disconnected and the 30V voltage source is now operative

$v(\infty)$ is obtained using voltage division,

$$v(\infty) = \frac{20}{20+10}(30) = 20V$$



The Thevenin resistance at the capacitor terminals is $R_{Th} = 10\|20 = 20/3\ \Omega$

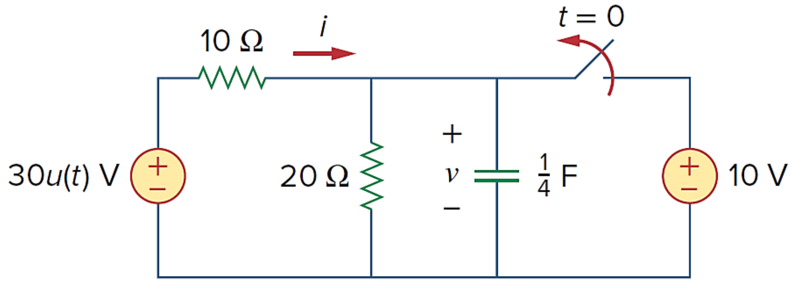
Time constant is $\tau = R_{Th}C = \frac{20}{3}.\frac{1}{4} = \frac{5}{3}s$

Thus, $$v(t) = v(\infty) + [v(0) - v(\infty)\ ]e^{-t/\tau}$$
$$= 20 + (10 - 20)e^{-(3/5)t} = (20 - 10e^{-0.6t})\ \text{V}$$

Source: Notes ELL 100 IITD

Problem 6

Find i and v if the switch is opened at t = 0 after keeping it closed for a long time.

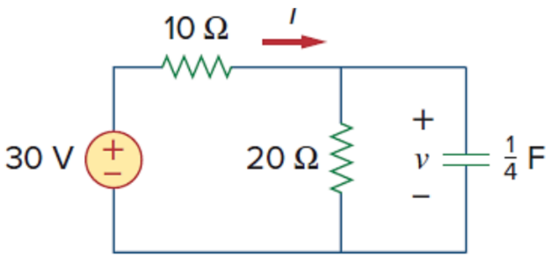


i is the sum of the currents through the 20-Ω resistor and the capacitor,

$$i = \frac{v}{20} + C\frac{dv}{dt}$$

$$i = 1 - 0.5e^{-0.6t} + 0.25(-0.6)(-10)e^{-0.6t} = (1 + e^{-0.6t})\text{ A}$$

Or $v + 10i = 30$ (KVL in outer loop)



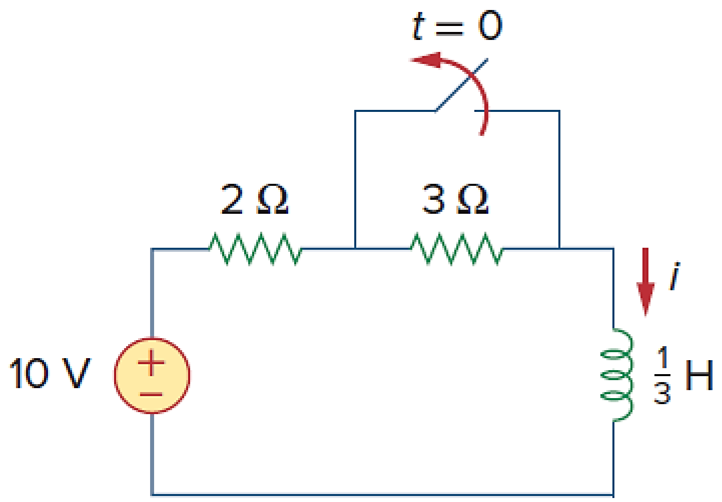
$$v = \begin{cases} 10V & t < 0 \\ (20 - 10e^{-0.6t})V & t > 0 \end{cases}$$

$$i = \begin{cases} -1A & t < 0 \\ (1 + e^{-0.6t})\text{ A} & t > 0 \end{cases}$$

Source: Notes ELL 100 IITD

Problem 7

Find i(t) in the circuit below for t > 0. Assume that the switch has been closed for a long time before opening.



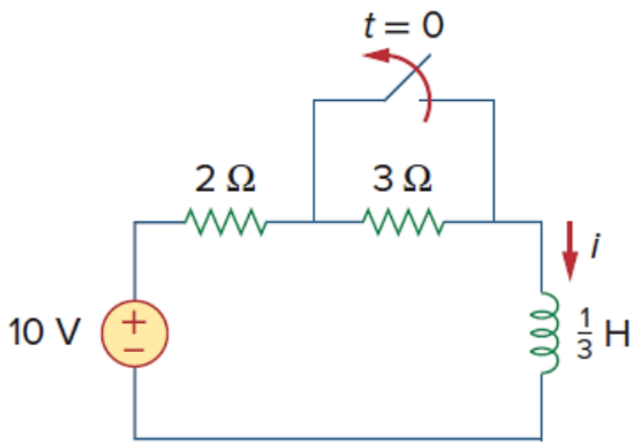
Source: Notes ELL 100 IITD

Problem 7

Find i(t) in the circuit below for t > 0. Assume that the switch has been closed for a long time before opening.

Solution:

For $\boldsymbol{t < 0}$, the 3-Ω resistor is short-circuited, and the inductor acts like a short circuit. The current through the inductor at $t = 0^-$ is $i(0^-) = 10/2 = 5\text{A}$



Since inductor current cannot change instantaneously $i(0) = i(0^+) = i(0^-) = 5\text{A}$

For $\boldsymbol{t > 0}$, the 3-Ω resistor comes in series with the 2-Ω resistor

The steady-state inductor current is $i(\infty) = \dfrac{10}{2+3} = 2\text{A}$

Source: Notes ELL 100 IITD

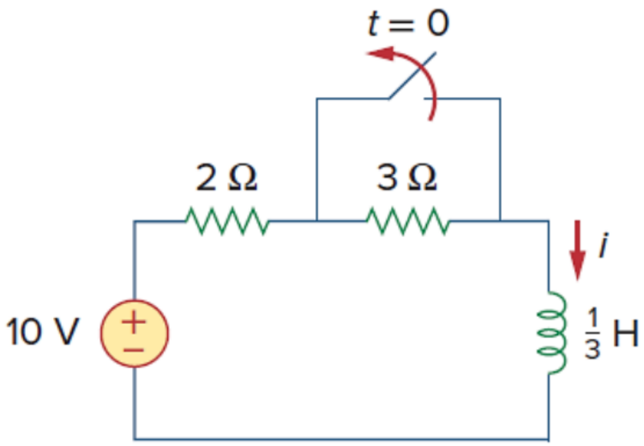
Problem 7

Find i(t) in the circuit below for t > 0. Assume that the switch has been closed for a long time before opening.

The Thevenin resistance across the inductor terminals is

$$R_{TH} = 2 + 3 = 5\ \Omega$$

Time constant $\tau = \frac{L}{R_{TH}} = \frac{\frac{1}{3}}{5} = \frac{1}{15} s$



$$\begin{aligned} i(\mathrm{t}) &= i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} \\ &= 2 + (5-2)e^{-15t} \\ &= 2 + 3e^{-15t}\ \mathrm{A}, \quad t > 0 \end{aligned}$$

Source: Notes ELL 100 IITD

Thank you