Electrical Science: 2021-22

Lecture 19

Three Phase AC Circuits - Part 3

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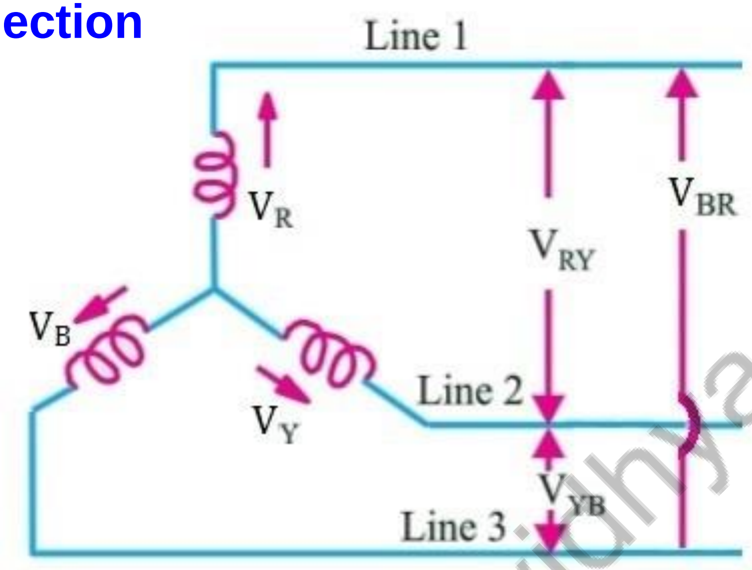
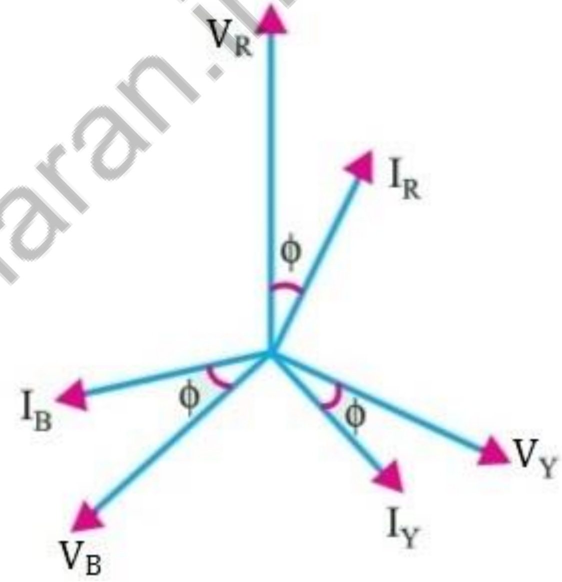
Power in Three Phase Circuits

Star Connection





$P = 3 \times V_{PH} \times I_{PH} \times \text{Cos}\Phi$

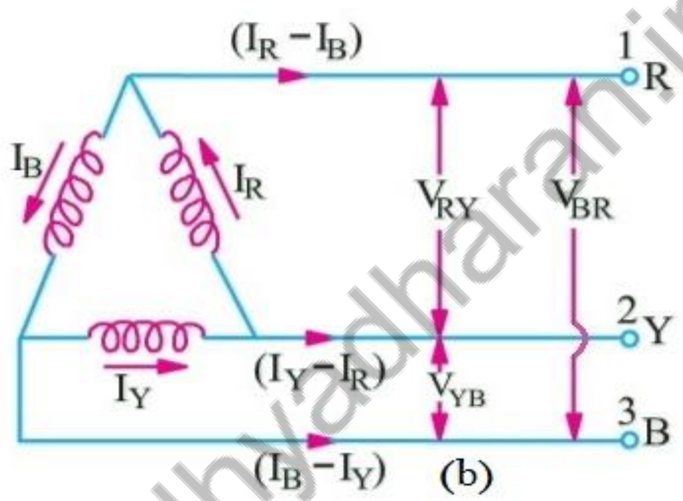
$I_L = I_{PH}$

$V_{PH} = V_L / \sqrt{3}$ ….. (From $V_L = \sqrt{3}\, V_{PH}$)

$P = \sqrt{3} \times V_L \times I_L \times \text{Cos}\Phi$

Power in Three Phase Circuits

Delta Connection



Power / Phase = $V_{PH} \times I_{PH}$ x CosΦ

And the total power of three phases;

Total Power = P = 3 x $V_{PH} \times I_{PH}$ x CosΦ

$I_{PH} = I_L / \sqrt{3}$ ….. (From $I_L = \sqrt{3}\, I_{PH}$)

$V_{PH} = V_L$

P = 3 x $V_{PH} \times I_{PH}$ x CosΦ …. or

P = $\sqrt{3}$ x V_L x I_L x CosΦ

Power in Three Phase Circuits

A three phase 400V, 50 Hz, balanced supply feeds a balanced load consisting of (a) three equal single phase loads of (40 + j 30) Ω connected in star, and (b) a three phase heating load (purely resistive) of 1.8 kW.

Determine the supply current, supply power factor, active and reactive power supplied.

Solution

(i) Using one-phase diagram (figure 8)

$$Z_{L1} = 40 + j\,30\ \Omega$$

$$E_p = 400/\sqrt{3} = 230.9\angle 0$$

$$P_p = 1.8/3 = 0.6\text{ kW} = 600\text{ W}$$

$$\therefore I_{p1} = \frac{230.9\angle 0}{40 + j30} = \frac{230.9\angle 0}{50\angle 36.87^0} = 4.619\angle -36.87^0$$

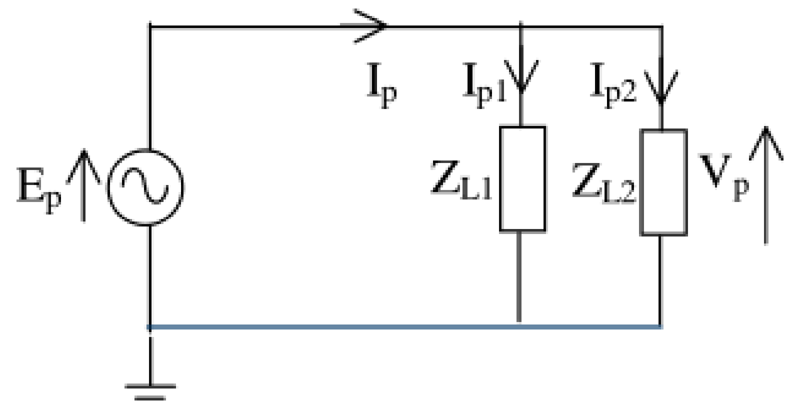


Figure 8 – Single phase diagram

Power in Three Phase Circuits

A three phase 400V, 50 Hz, balanced supply feeds a balanced load consisting of (a) three equal single phase loads of (40 + j 30) Ω connected in star, and (b) a three phase heating load (purely resistive) of 1.8 kW.

Determine the supply current, supply power factor, active and reactive power supplied.

In order to calculate I_{p2}, we need not calculate Z_{L2}, but can use $P = V\, I \cos\phi$.

$$\therefore I_{p2} = \frac{600}{230.9 \times 1} = 2.598\angle 0$$ [Note: angle is zero because it is purely resistive]

Thus $I_p = I_{p1} + I_{p2} = 4.619\angle{-36.87^0} + 2.598 = 6.293 - j\,2.771 = 6.876\angle{-23.77^0}$ A

$\therefore$ supply current $= 6.876\angle{-23.77^0}$ A

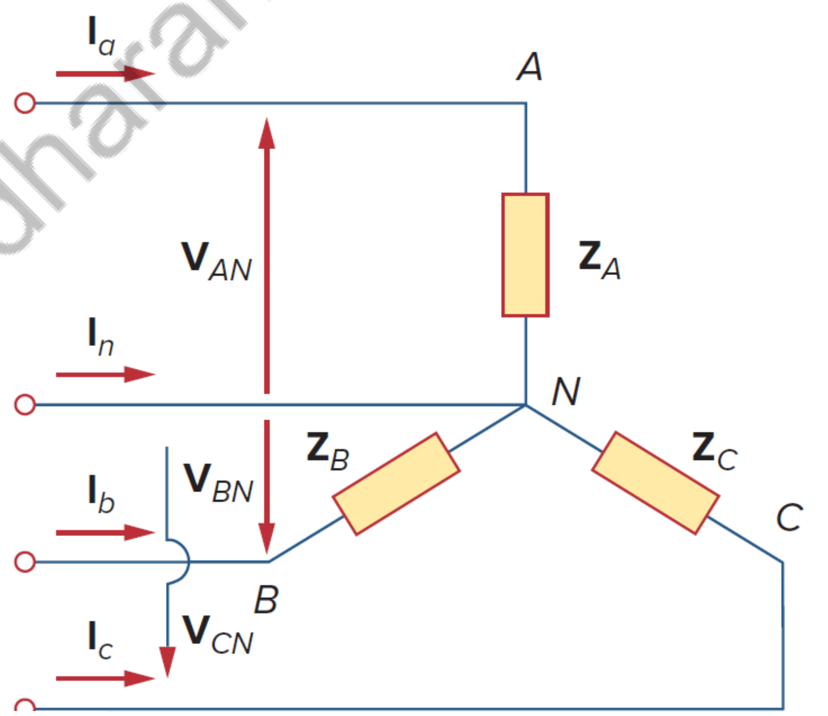
supply power factor $= \cos(0 - (-23.77)) = 0.915$ lag

active power supplied $= \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 6.876 \times 0.915 = 4360$ W

reactive power supplied $= \sqrt{3} V_L I_L \sin\phi = \sqrt{3} \times 400 \times 6.876 \times \sin(-23.77) = 1920$ var

Unbalanced Loads

The unbalanced Y-load of Figure below has balanced phase voltage of 100 V and the acb sequence. Calculate the line currents and the neutral current.
Z_A = 15 Ω, Z_B = 10 + j5 Ω, Z_C = 6 − j8 Ω.



The line currents are,

$$I_a = \frac{100\angle 0^0}{15} = 6.67\angle 0^0 \text{ A}$$

$$I_b = \frac{100\angle 120^0}{10+j5} = 8.94\angle 93.44^0 \text{ A};$$

$$I_c = \frac{100\angle -120^0}{6-j8} = 10\angle -66.87^0 \text{ A}$$

The current in the neutral line is:

$$I_n = -(I_a + I_b + I_c) = -(6.67 - 0.54 + j8.92 + 3.93 - j9.2)$$

$$= -10.06 + j0.28 = 10.06\angle 178.4^0 \text{ A}$$

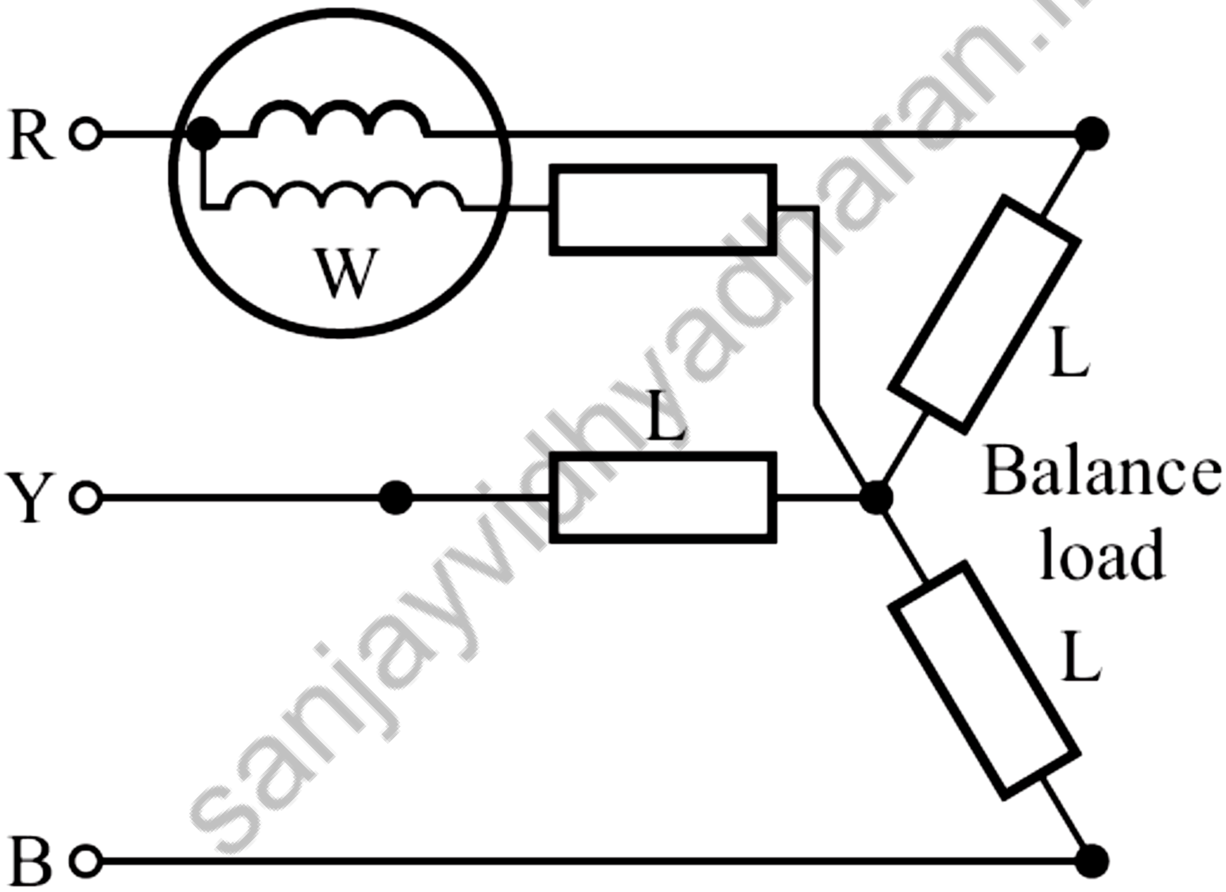
Measurement of Power

(i) **Three-Wattmeter Method :** This is simplest and straight forward method.

(ii) **Two-Wattmeter Method :** This can be used for any balanced or unbalanced load, star- or delta-connected.

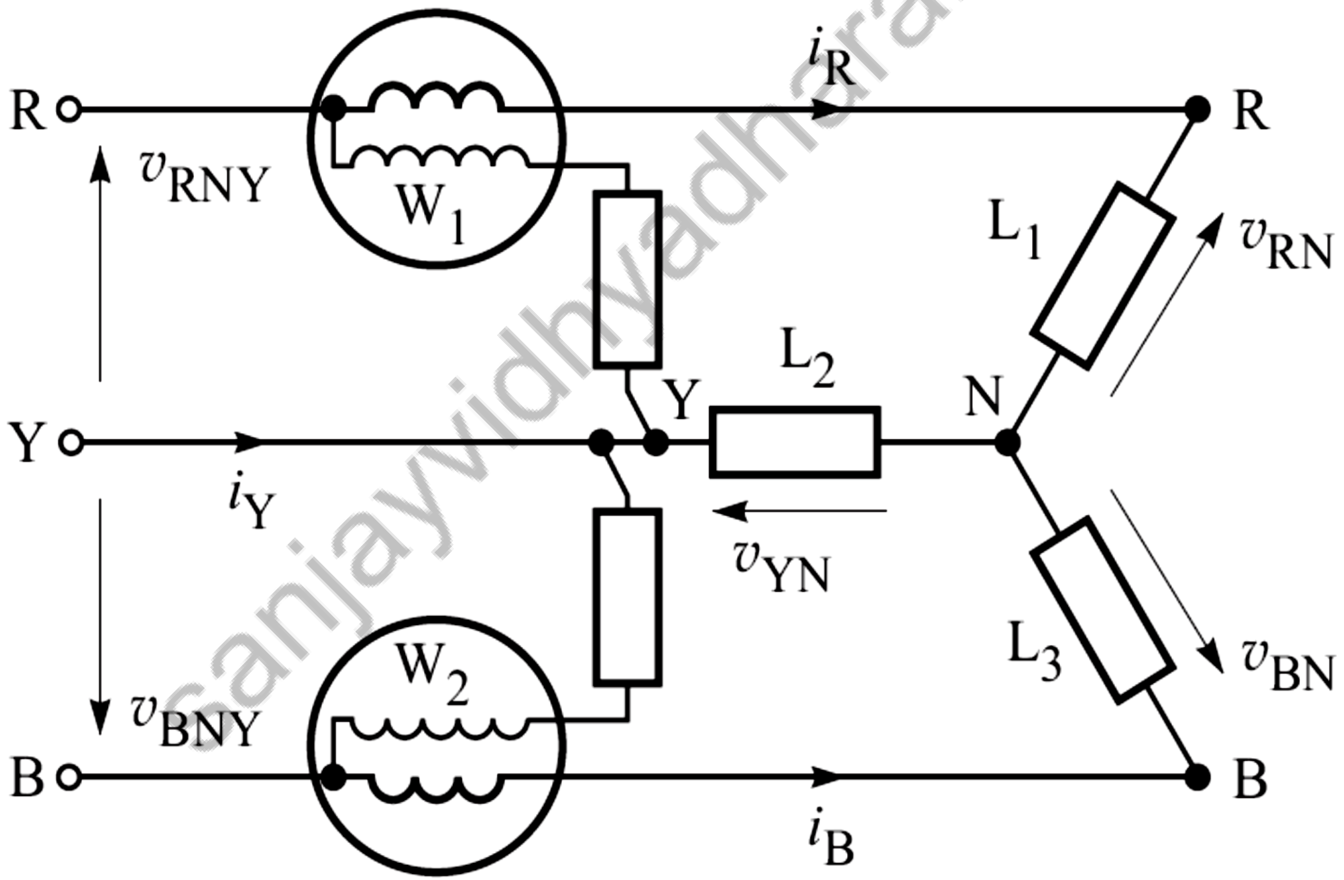
(iii) **One-Wattmeter Method :** This can be used only for a star-connected balanced load.

Measurement of Power



Measurement of Power

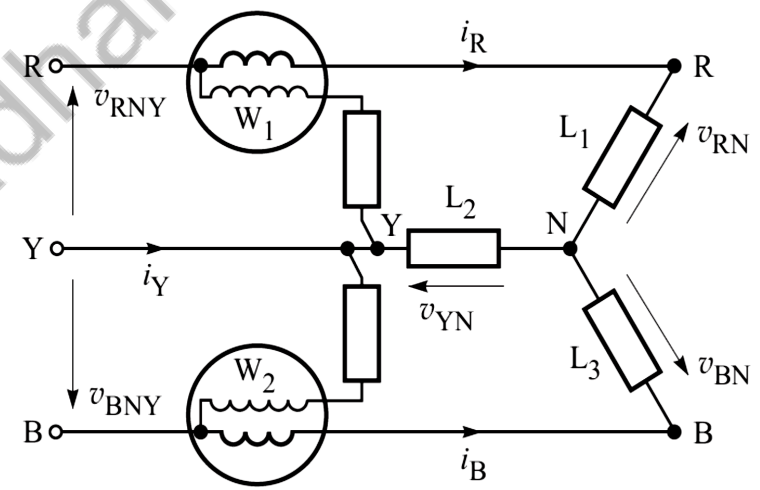
Two-Wattmeter Method



Measurement of Power

Two-Wattmeter Method

The sum of the wattmeter readings gives the average value of the total power absorbed by the three phases



Total instantaneous power

$$= i_R v_{RN} + i_Y v_{YN} + i_B v_{BN}.$$

The instantaneous power measured by W_1,

$$p_1 = i_R (v_{RN} - v_{YN})$$

The instantaneous power measured by W_2,

$$p_2 = i_B (v_{BN} - v_{YN})$$

$$\therefore \quad p_1 + p_2 = i_R (v_{RN} - v_{YN}) + i_B (v_{BN} - v_{YN})$$
$$= i_R v_{RN} + i_B v_{BN} - (i_R + i_B) v_{YN}$$

By KCL, $i_R + i_Y + i_B = 0 \Rightarrow (i_R + i_B) = -i_Y$

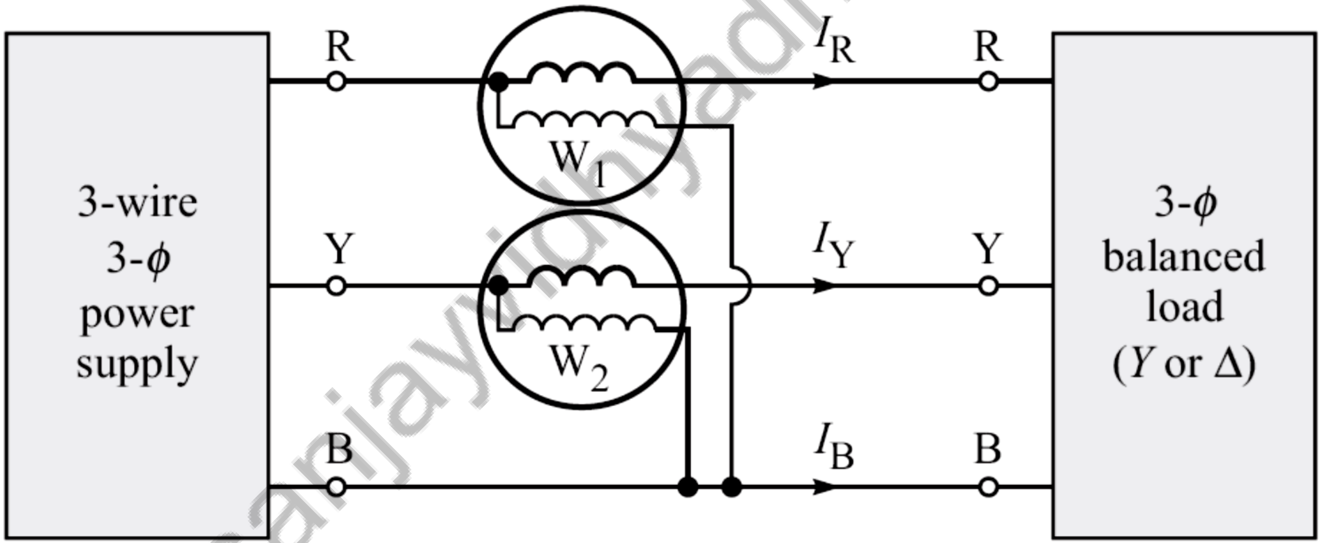
$$\therefore \quad p_1 + p_2 = i_R v_{RN} + i_B v_{BN} + i_Y v_{YN}$$
$$= \text{total instantaneous power}$$

Since we did not assume a balanced load or a sinusoidal waveform, it follows that the sum of the two wattmeter readings gives the total power under all conditions.

Measurement of Power

Power Factor Measurement by Two-Wattmeter Method

Concept of 'power factor' is meaningful only if the load is balanced.

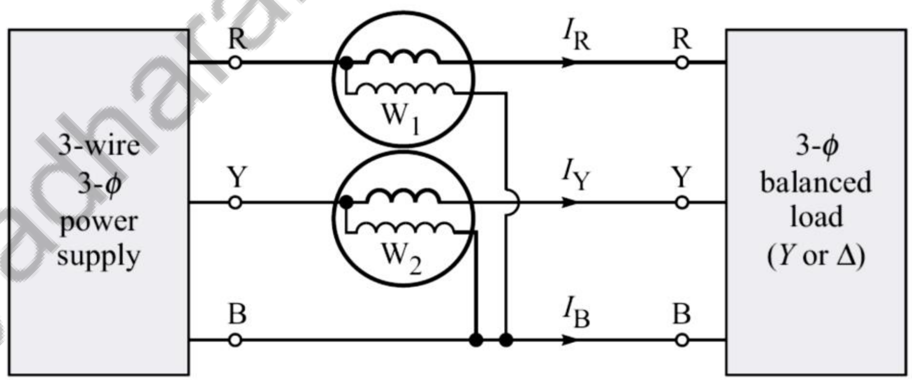


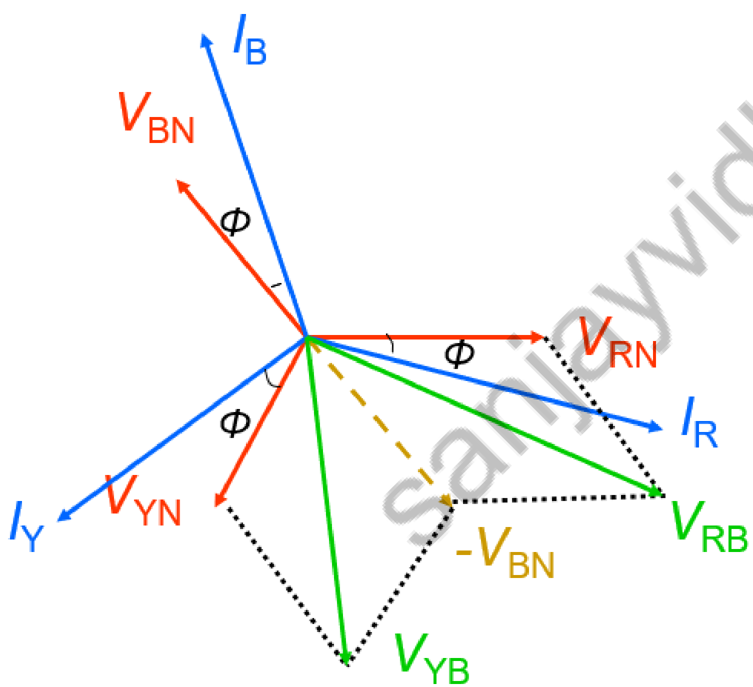
Measurement of Power

Power Factor Measurement by Two-Wattmeter Method

$$I_R = I_Y = I_B = I_L \text{ (say)}$$

and $$V_{RN} = V_{YN} = V_{BN} = V_{ph} \text{ (say)}$$





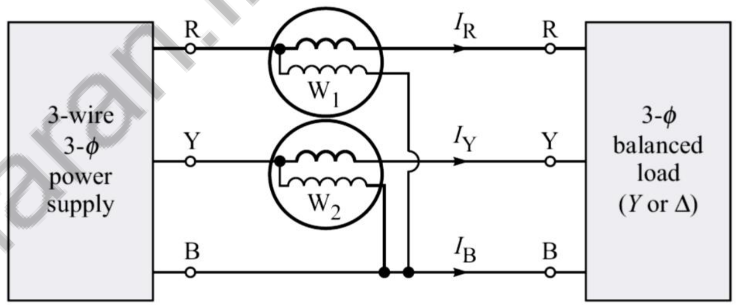
$$P_1 = V_{RB} I_R \cos(\varphi) = \sqrt{3}\, V_{RN} I_R \cos(-30° - \varphi)$$

$$= \sqrt{3}\, V_{RN} I_R \cos(30° + \varphi)$$

$$P_2 = V_{YB} I_Y \cos(\varphi) = = \sqrt{3}\, V_{YN} I_Y \cos(30° - \varphi)$$

Measurement of Power

Power Factor Measurement by Two-Wattmeter Method



$$P_1 = \sqrt{3}\, V_{RN} I_R \cos(30° + \varphi)$$

$$P_2 = \sqrt{3}\, V_{YN} I_Y \cos(30° - \varphi)$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{\cos(30° - \phi)}{\cos(30° + \phi)}$$

By applying componendo and dividendo

$$\frac{P_1 - P_2}{P_1 + P_2} = \frac{\cos(30° - \phi) - \cos(30° + \phi)}{\cos(30° - \phi) + \cos(30° + \phi)}$$

$$= \frac{2\sin 30° \sin\phi}{2\cos 30° \cos\phi} = \tan 30° \tan\phi$$

$$\therefore \boxed{\tan\varphi = \sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right]}$$

Measurement of Power

Important Points :

- If $\Phi = 0°$, $P_1 = P_2$.
- If $\Phi < 60°$, both P_1 and P_2 are positive; and

$$P = P_1 + P_2$$

- If $\Phi = 60°$, P_2 is zero.
- If $\Phi > 60°$, (i.e., if $pf < 0.5$), P_2 is negative.

  In such case, the connection of either the current coil or the potential coil has to be reversed to make positive deflection.

  The value P_2 should then be taken as negative while calculating the power factor or the total power.

Measurement of Power

Example:

- Two-wattmeter method was used to determine the input power to a three-phase motor. The readings were 5.2 kW and -1.7 kW, and the line voltage was 415 V. Calculate

  (*a*) the total power,

  (*b*) the power factor, and

  (*c*) the line current.

Measurement of Power

Solution :

(*a*) The total power,

$$P = P_1 + P_2 = 5.2\ \text{kW} - 1.7\ \text{kW} = \mathbf{3.5\ kW}$$

(*b*) $$\tan\phi = \sqrt{3}\left[\frac{P_1 - P_2}{P_1 + P_2}\right] = \sqrt{3}\left[\frac{5.2-(-1.7)}{5.2+(-1.7)}\right] = 3.41$$

$$\therefore \quad \phi = \tan^{-1} 3.41 = 73°39'$$

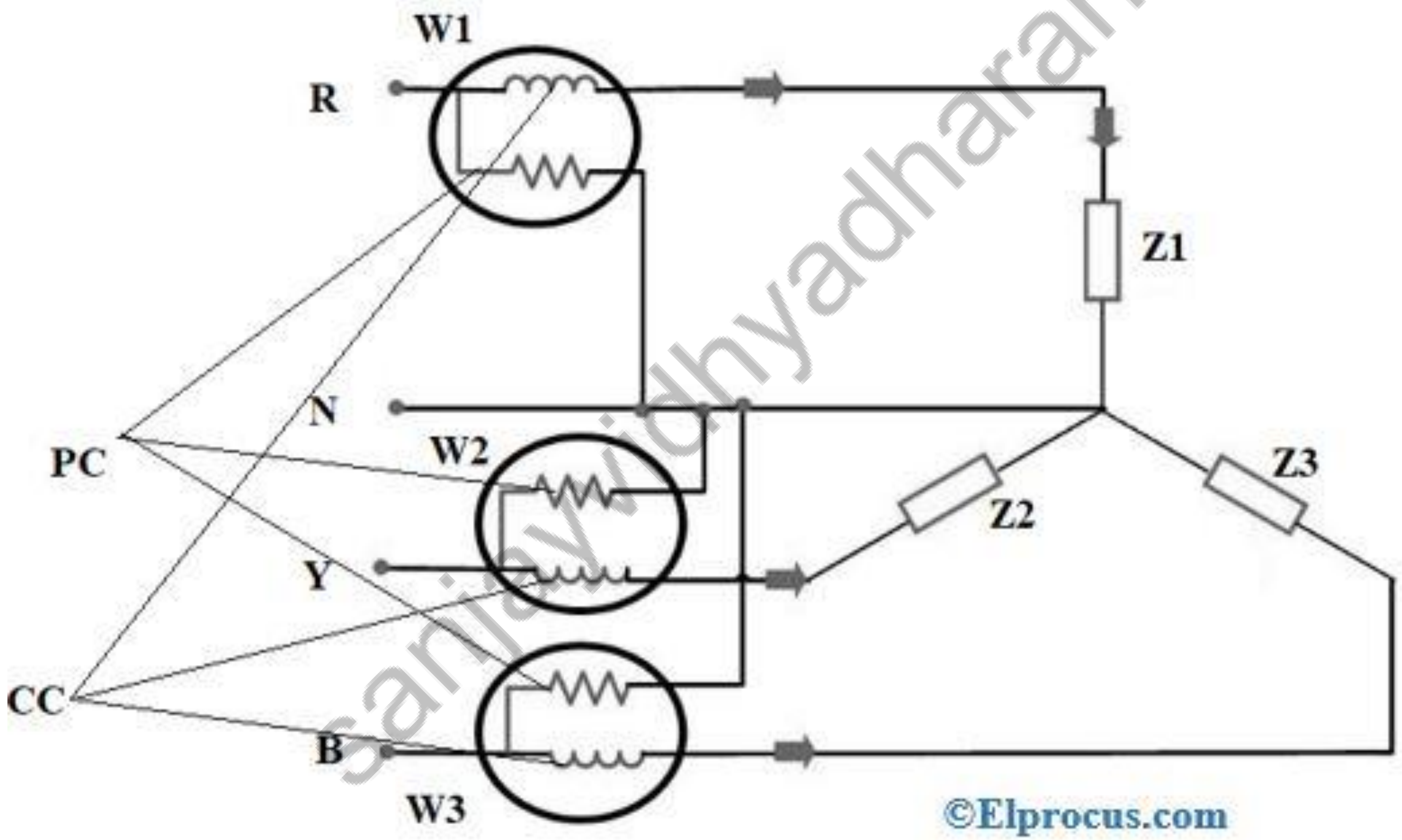
$$\therefore \quad pf = \cos\phi = \cos 73°39' = \mathbf{0.281}$$

(*c*) $$3500 = \sqrt{3} \times 415 \times I_L \times 0.281$$

$$\Rightarrow \quad I_L = \mathbf{17.3\ A}$$

Measurement of Power

Three-Wattmeter Method



Measurement of Power

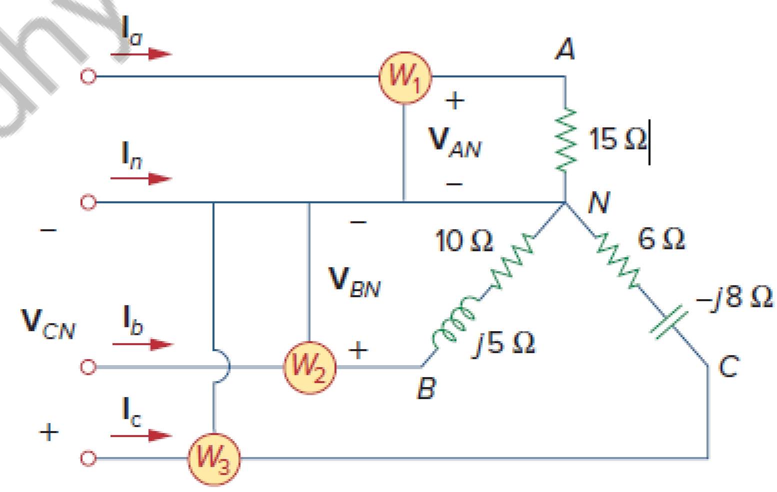
Three-Wattmeter Method

Three wattmeters W1, W2, and W3 are connected, respectively, to phases A, B, and C of an unbalanced Y-connected load as in figure. The balanced source is Y-connected with phase voltage 100 V in negative (acb) sequence.

Find

(a) the wattmeter readings

(b) the total power absorbed by the load.



Measurement of Power

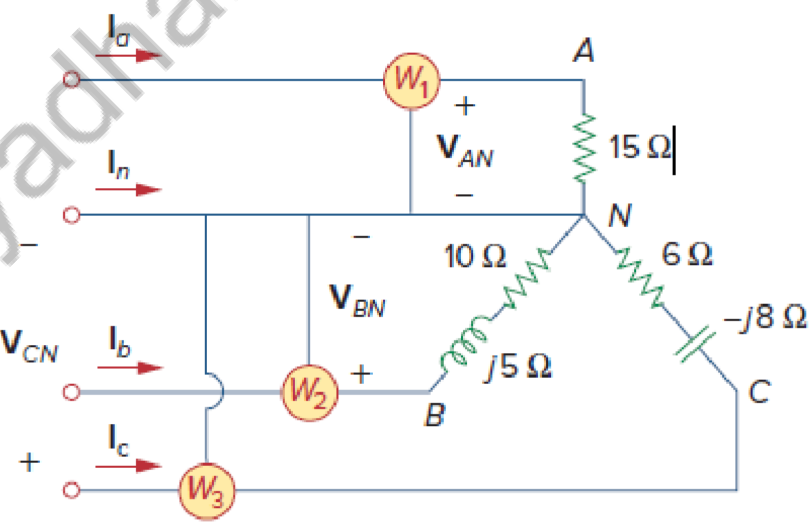
Three-Wattmeter Method

Solution: The line currents are,

$$I_a = \frac{100\angle 0°}{15} = 6.67\angle 0° A$$

$$I_b = \frac{100\angle 120°}{10 + j5} = 8.94\angle 93.44° A$$

$$I_c = \frac{100\angle -120°}{6 - j8} = 10\angle -66.87° A$$



Measurement of Power

Three-Wattmeter Method

(a) The wattmeter readings are,

$$P_1 = \text{Re}(V_{AN}I_a^*) = V_{AN}I_a \cos(\theta_{V_{AN}} - \theta_{I_a}) = 100 * 6.67 * \cos(0^0 - 0^0) = 667W$$

$$P_2 = \text{Re}(V_{BN}I_b^*) = V_{BN}I_b \cos(\theta_{V_{BN}} - \theta_{I_b}) = 100 * 8.94 * \cos(120^0 - 93.44^0) = 800W$$

$$P_3 = \text{Re}(V_{CN}I_c^*) = V_{CN}I_c \cos(\theta_{V_{CN}} - \theta_{I_c}) = 100 * 10 * \cos(-120^0 + 66.87^0) = 600W$$

(b) The total power absorbed is $P_T = P_1 + P_2 + P_3 = 667 + 800 + 600 = 2067W$

The power absorbed can also be calculated as the power dissipated across the resistors,

$$P_T = |I_a|^2(15) + |I_b|^2(10) + |I_c|^2(6) = 6.67^2(15) + 8.94^2(10) + 10^2(6) = 667 + 800 + 600 = 2067W$$

Thank you