Electrical Science: 2021-22

Lecture 18

Three Phase AC Circuits - Part 2

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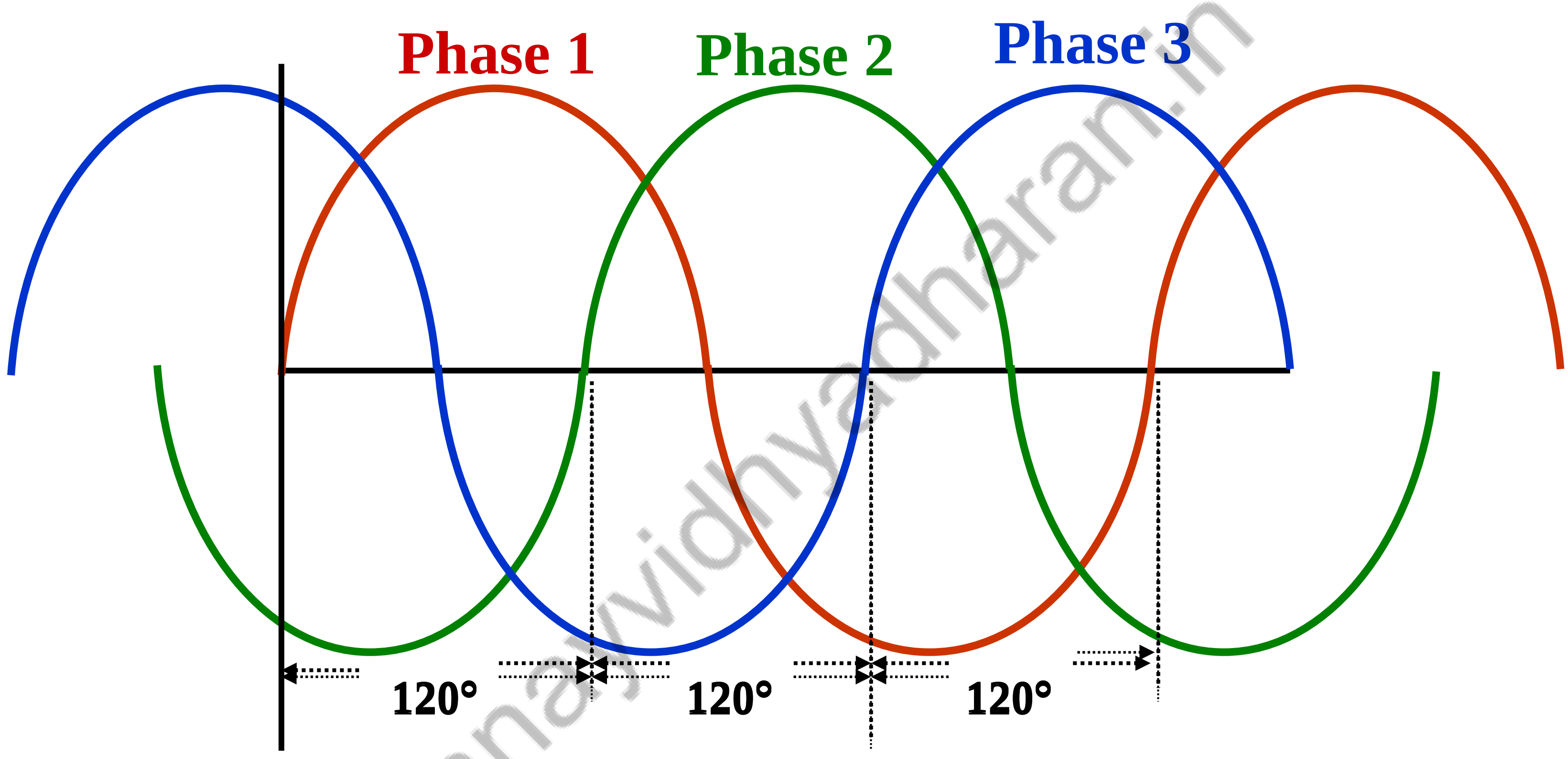
BALANCED THREE-PHASE VOLTAGES

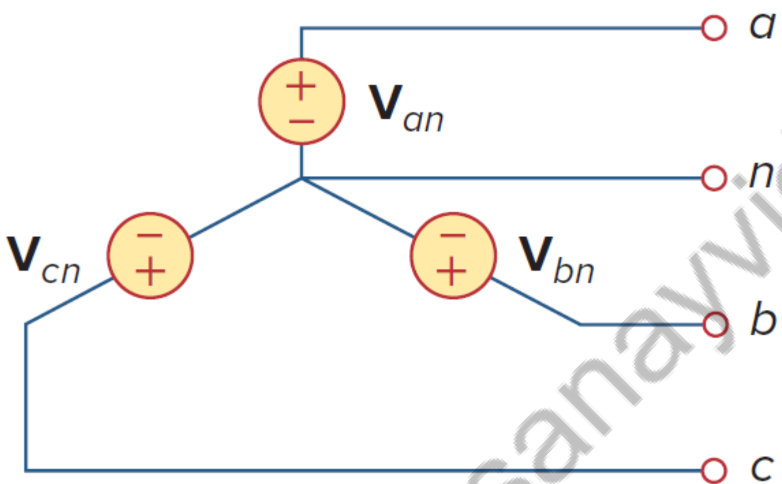


Phase 2 lags phase 1 by 120°. Phase 2 leads phase 3 by 120°.

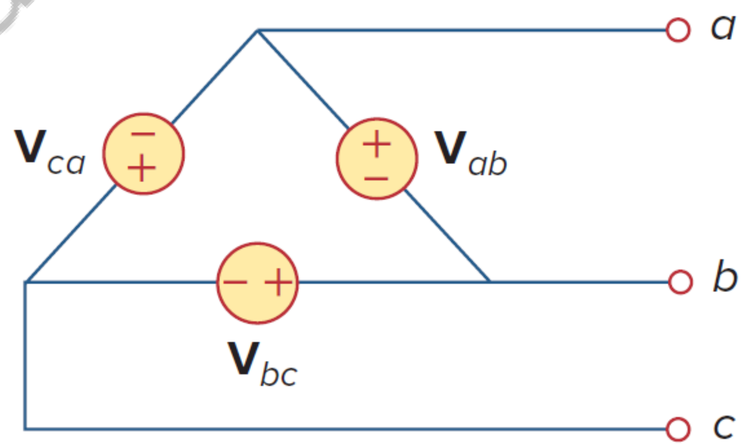
THREE-PHASE VOLTAGES

A typical three-phase system consists of three voltage sources connected to loads by three or four wires (or transmission lines).

The voltage sources can be either wye connected or delta-connected.



Y-connected source



Δ-connected source

WYE CONNECTED THREE-PHASE SYSTEM

Balanced Phase Voltage

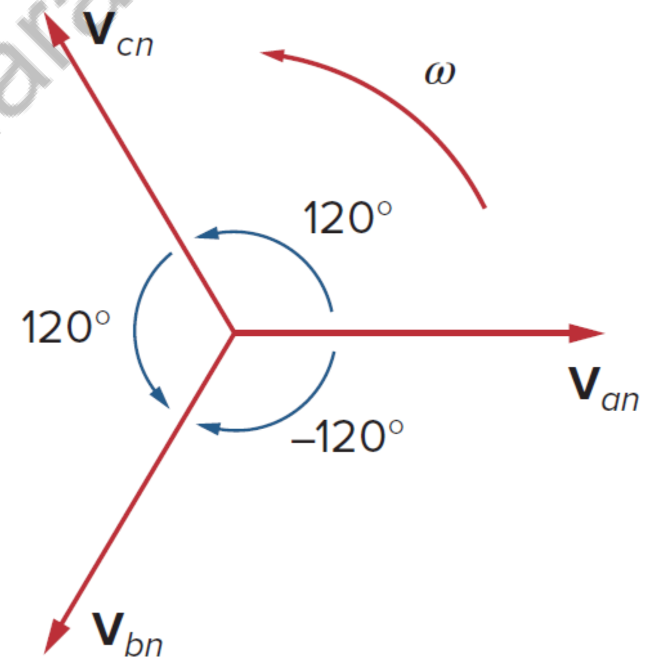
abc Sequence or Positive Sequence

Rotor has to rotate counter-clockwise.

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

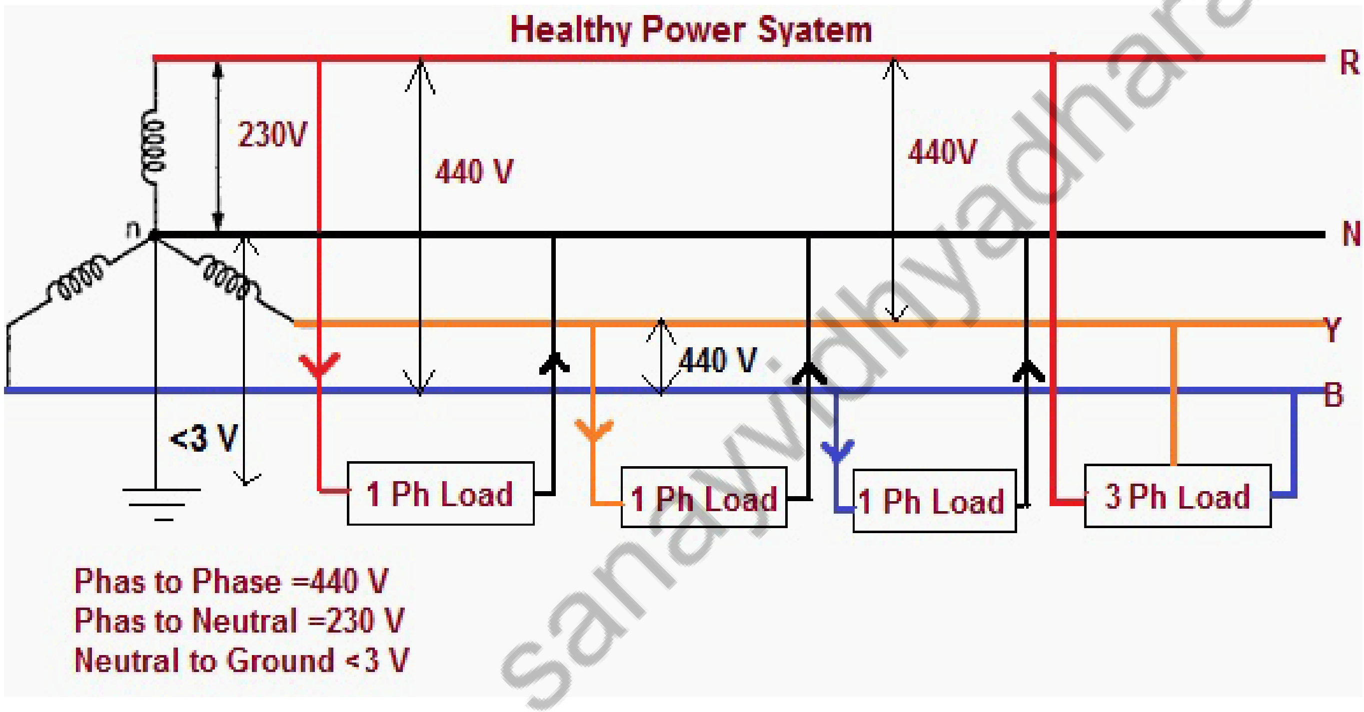
$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$



where V_p is the effective or **rms value** of the phase voltages.

WYE CONNECTED LINE VOLTAGE

Line voltage is measured between any two of the three lines: line to line voltage.



$$V_L = \sqrt{3} V_p$$

WYE-WYE CONNECTED THREE-PHASE SYSTEM

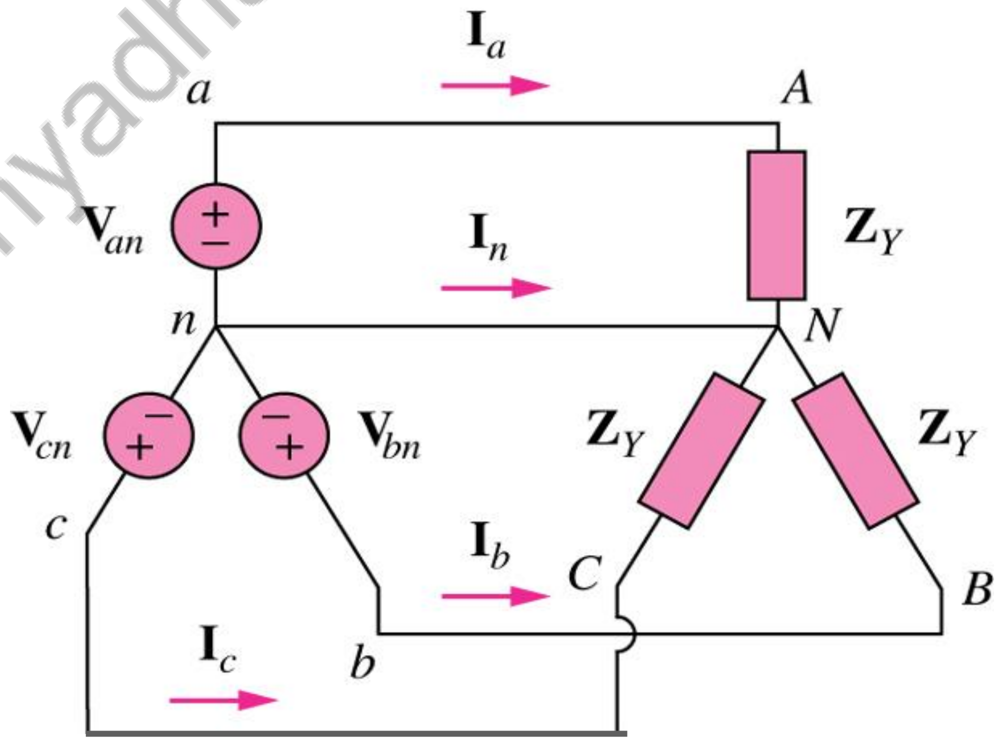
Balanced Load

The phase impedances are equal in magnitude and in phase.

$$I_a = \frac{V_{an}}{Z_Y}$$

$$I_b = \frac{V_{bn}}{Z_Y} = \frac{V_{an}\angle -120^\circ}{Z_Y} = I_a\angle -120^\circ$$

$$I_c = \frac{V_{cn}}{Z_Y} = \frac{V_{an}\angle -240^\circ}{Z_Y} = I_a\angle -240^\circ$$



Balanced Wye-Delta Connection

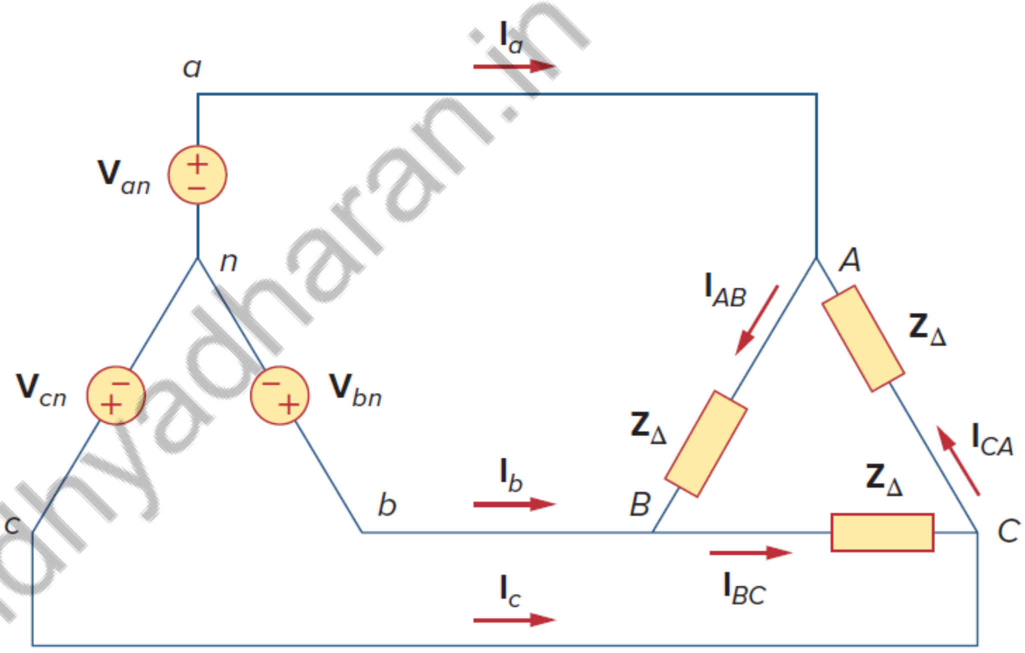
$$V_{an} = V_p\angle 0^\circ,\ V_{bn} = V_p\angle -120^\circ,$$
$$V_{cn} = V_p\angle +120^\circ$$

The line voltages are:

$$V_{ab} = \sqrt{3}V_p\angle 30^\circ = V_{AB}$$

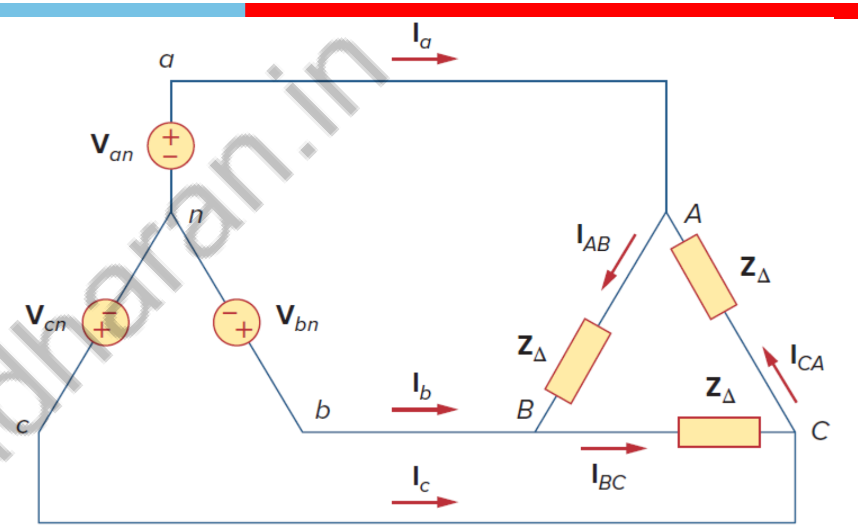
$$V_{bc} = \sqrt{3}V_p\angle -90^\circ = V_{BC}$$

$$V_{ca} = \sqrt{3}V_p\angle 150^\circ = V_{CA}$$



Balanced Wye-Delta Connection

$$I_{AB} = \frac{V_{AB}}{Z_\Delta},\ I_{BC} = \frac{V_{BC}}{Z_\Delta},\ I_{CA} = \frac{V_{CA}}{Z_\Delta}$$



These currents have the same magnitude but are out of phase with each other by 120°.

The line currents are obtained from the phase currents by applying KCL at Nodes A, B, and C. Thus,

$$I_a = I_{AB} - I_{CA},\ I_b = I_{BC} - I_{AB},\ I_c = I_{CA} - I_{BC}$$

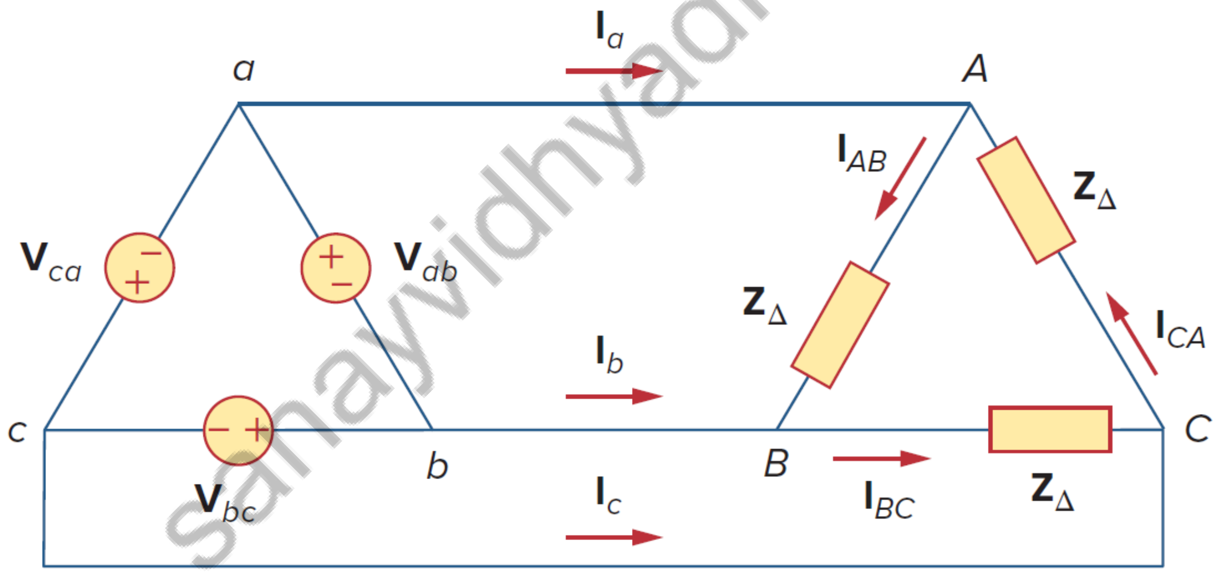
$$\begin{aligned} I_a = I_{AB} - I_{CA} &= I_{AB}\left(1 - 1\angle -240^\circ\right) \\ &= I_{AB}\left(1 + 0.5 - j0.866\right) \\ &= I_{AB}\sqrt{3}\angle -30^\circ \end{aligned}$$

Since $I_{CA} = I_{AB}\angle -240^\circ,$

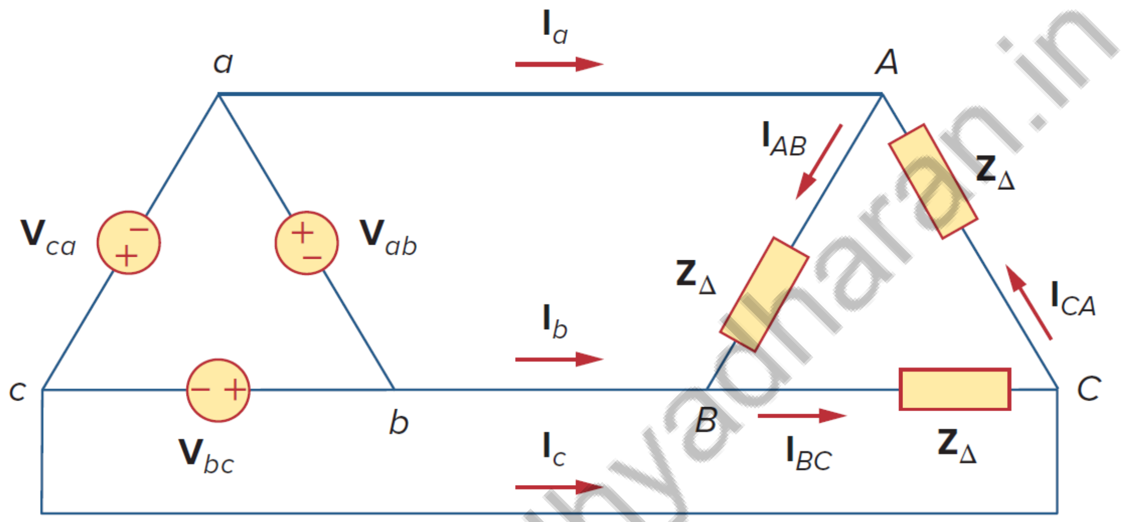
$$I_L = \sqrt{3} I_p$$

Balanced Delta-Delta Connection

A balanced Δ-Δ system is one in which both the balanced source and balanced load are Δ-connected.



Balanced Delta-Delta Connection



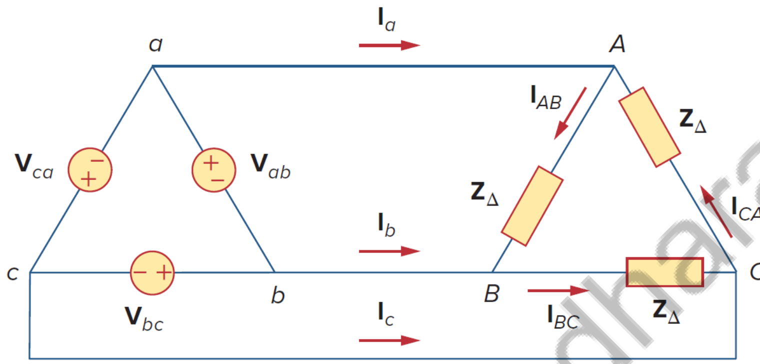
$$V_{ab} = V_p \angle 0 \ , \ V_{bc} = V_p \angle -120 \ , \ V_{ca} = V_p \angle +120$$

Also,

$$V_{ab} = V_{AB}, \ V_{bc} = V_{BC}, \ V_{ca} = V_{CA}$$

In Δ-Δ system, line voltages equal to phase voltages $V_L = V_\varphi$

Balanced Delta-Delta Connection



$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{V_{ab}}{Z_\Delta},\ I_{BC} = \frac{V_{BC}}{Z_\Delta} = \frac{V_{bc}}{Z_\Delta},\ I_{CA} = \frac{V_{CA}}{Z_\Delta} = \frac{V_{ca}}{Z_\Delta}$$

The line currents are also obtained from the phase currents by applying KCL at nodes A, B, and C.

$$I_a = I_{AB} - I_{CA},\ I_b = I_{BC} - I_{AB},\ I_c = I_{CA} - I_{BC}$$

Also, each line current lags the corresponding phase current by 30°. Moreover,

$$I_L = \sqrt{3} I_p$$

$$I_a = I_{AB}\sqrt{3}\angle -30^0$$

$$I_b = I_a\angle -120^0 =$$

$$I_c = I_a\angle +120^0 =$$

Balanced Delta-Delta Connection

Q1. A balanced Δ-connected load with impedance $20 - j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $V_{ab} = 330\angle 0^0$ V. Calculate the phase currents of the load and the line currents.

Ans: The load impedance per phase is $Z_\Delta = 20 - j15 = 25\angle -36.57^0\,\Omega$

Since $V_{AB} = V_{ab}$, the phase currents are,

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{330\angle 0^0}{25\angle -36.87^0} = 13.2\angle 36.87^0 \text{ A}$$

$$I_{BC} = I_{AB}\angle -120^0 = 13.2\angle -83.13^0 \text{ A}$$

$$I_{CA} = I_{AB}\angle +120^0 = 13.2\angle 156.87^0 \text{ A}$$

Balanced Delta-Delta Connection

Q1. A balanced Δ-connected load with impedance $20 - j15\ \Omega$ is connected to a Δ-connected, positive-sequence generator having $V_{ab} = 330\angle 0^0$ V. Calculate the phase currents of the load and the line currents.

For a delta load, the line current always lags the corresponding phase current by 30° and has a magnitude √3 times that of the phase current. Hence, the line currents are,

$$I_a = I_{AB}\sqrt{3}\angle -30^0 = 22.86\angle 6.87^0 \text{ A}$$

$$I_b = I_a\angle -120^0 = 22.86\angle -113.13^0 \text{ A}$$

$$I_c = I_a\angle +120^0 = 22.86\angle 126.87^0 \text{ A}$$

Balanced Delta-Wye Connection

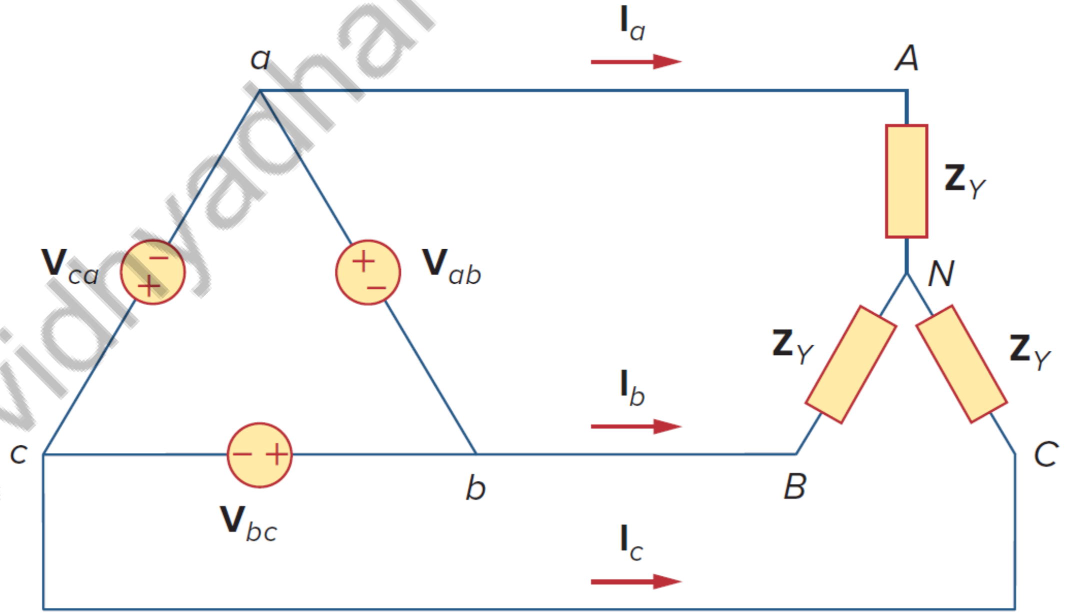
A balanced Δ-Y system consists of a balanced Δ-connected source feeding a balanced Y- connected load.

$$V_{ab} = V_p \angle 0^\circ,$$

$$V_{bc} = V_p \angle -120^\circ,$$

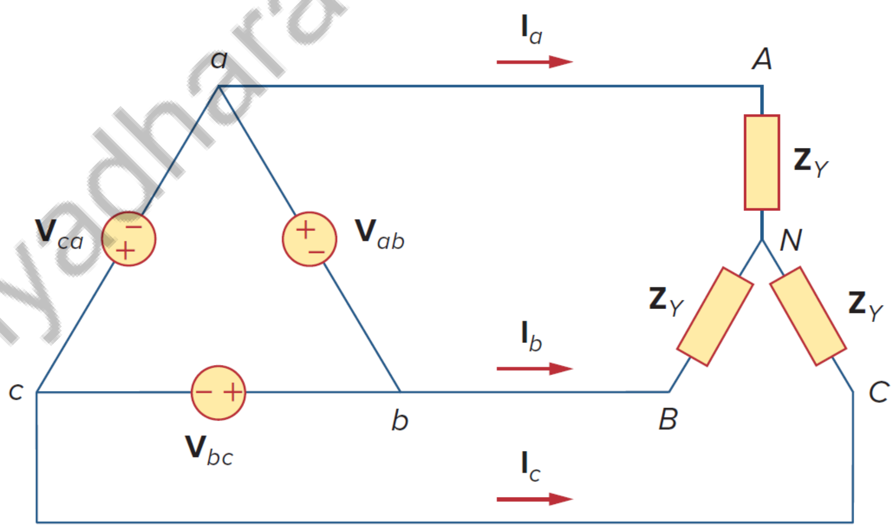
$$V_{ca} = V_p \angle +120^\circ$$

$$V_L = V_\varphi$$



Balanced Delta-Wye Connection

A balanced Δ-Y system consists of a balanced Δ-connected source feeding a balanced Y- connected load.



$$-V_{ab} + Z_Y I_a - Z_Y I_b = 0$$

(or)

$$Z_Y\left(I_a - I_b\right) = V_{ab} = V_p\angle 0^\circ$$

Thus, $\left(I_a - I_b\right) = \dfrac{V_p\angle 0^\circ}{Z_Y}$

Here, $I_b = I_a\angle -120^\circ$

$$I_a - I_b = I_a\left(1 - 1\angle -120\ \right) = I_a\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = I_a\sqrt{3}\angle 30$$

$$\left(I_a - I_b\right) = \frac{V_p\angle 0}{Z_Y} \Rightarrow I_a\sqrt{3}\angle 30^\circ = \frac{V_p\angle 0}{Z_Y} \Rightarrow I_a = \frac{\frac{V_p}{\sqrt{3}}\angle -30^\circ}{Z_Y}$$

$$I_b = I_a\angle -120^\circ,\ I_c = I_a\angle +120^\circ$$

Balanced Delta-Wye Connection

Q1. A balanced Y-connected load with a phase impedance of 40 + j25 Ω is supplied by a balanced, positive sequence Δ-connected source with a line voltage of 210 V. Calculate the phase currents. Use V_{ab} as a reference.

Ans:

The load impedance is

$$Z_Y = 40 + j25 = 47.17\angle 32^0\,\Omega$$

and the source voltage is

$$V_{ab} = 210\angle 0^0 \text{ V}$$

When the Δ-connected source is transformed to a Y-connected source,

$$V_{an} = \frac{V_{ab}}{\sqrt{3}}\angle -30^0 = 121.2\angle -30^0 \text{ V}$$

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Balanced Delta-Wye Connection

The line currents are,

$$I_a = \frac{V_{an}}{Z_Y} = \frac{121.2\angle -30^0}{47.12\angle 32^0} = 2.57\angle -62^0 \text{ A}$$

$$I_b = I_a\angle -120^0 = 2.57\angle -178^0 \text{ A}$$

$$I_c = I_a\angle +120^0 = 2.57\angle 58^0 \text{ A}$$

which are the same as the phase currents.

Power in Three Phase System

Star Connection

$S_{phase} = \frac{E}{\sqrt{3}} \times I$ E is Line Voltage and I the Line Current

This is the power transferred by a single phase. To calculate the power transferred by 3 phase we can multiply this equation by 3.

$$S_{phase} = \frac{E}{\sqrt{3}} \times I \times 3 = \sqrt{3} \times E \times I$$

Delta Connection

$S_{phase} = E \times \frac{I}{\sqrt{3}}$ E is Line Voltage and I the Line Current

Multiply above equation by 3 to calculate power transferred by 3 phase. And you'll get the same result as star connection.

$$S_{phase} = \sqrt{3} \times E \times I$$

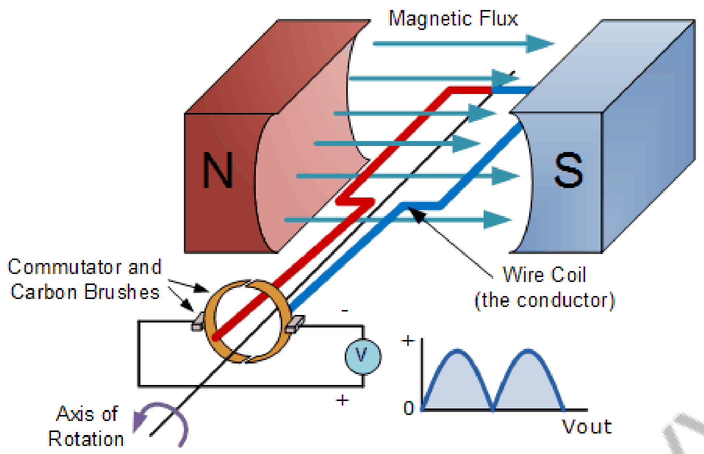
Summary Three Phase System

Sl. No.	Star (Y) Connected System	Delta (Δ) Connected System
1.	In star connected system there is common point known as neutral 'n' or star point. It can be earthed.	There is no neutral point in delta connected system
2.	In star connected system we get 3-phase, three wire system and also 3-phase, 4 wire system is taken out.	Only 3-phase, 3 wire system is possible in delta connected system
3.	Line voltage $V_L = \sqrt{3}\, V_{ph}$ or, $V_{ph} = \frac{1}{\sqrt{3}} V_L$	Line voltage = Phase voltage $V_L = V_{ph}$
4.	Line current = Phase current $I_L = I_{ph}$	Line current $I_L = \sqrt{3}\, I_{ph}$ $I_{ph} = \frac{1}{\sqrt{3}} I_L$
5.	Three phase power $= \sqrt{3}\, V_L\, I_{ph} \cos\phi$ $= 3\, V_{ph}\, I_{ph} \cos\phi$	Three phase power $= \sqrt{3}\, V_L\, I_L \cos\phi$ $= 3\, V_{ph}\, I_{ph} \cos\phi$

Three Phase Alternating Currents

Advantages AC over DC

(i) The generation of AC is cheaper than that of DC.



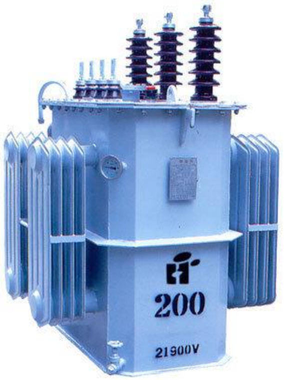
Advantages 3-ϕ AC over Single-ϕ AC

A three-phase system produces three separate waves of power, delivered in sequence. This ensures a continuous uninterrupted flow of power that never drops to zero and makes three-phase generators more powerful than single-phase generators.

Three Phase Alternating Currents

Advantages AC over DC

(ii) When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission. Conversion from high to low voltages and vice-versa is essay.



Advantages 3-ϕ AC over Single-ϕ AC

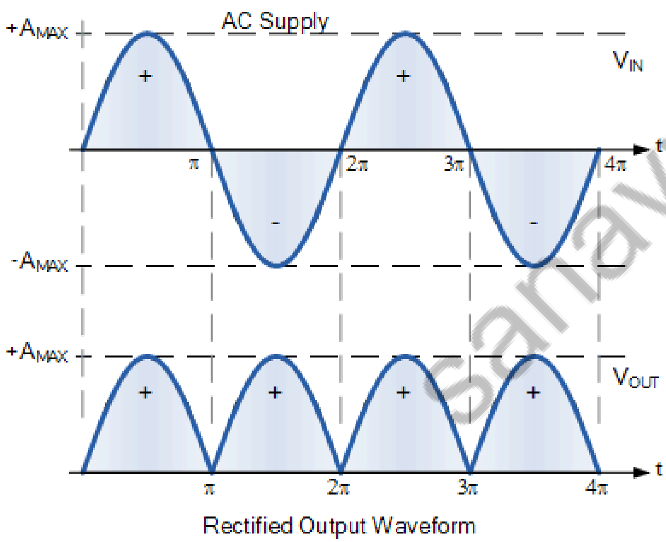
The three-phase transformer performs its functions more efficiently and delivers more power than a single-phase transformer. Lesser total line current than Single-ϕ AC for conveying same power.

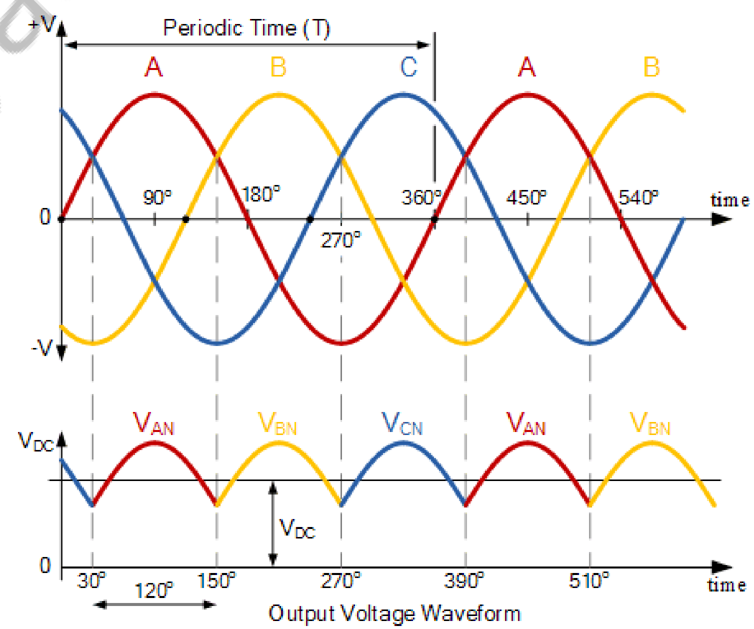
Three Phase Alternating Currents

Advantages AC over DC

(iii) AC can easily be converted into DC with the help of rectifiers.

Advantages 3-ϕ AC over Single-ϕ AC





Power output and therefore rectification efficiency are quite high. Lower Ripple Factor

Thank you