Electrical Science: 2021-22

Lecture 13

AC Response for Pure Resistive, Inductive and Capactive Circuits

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RMS

The RMS voltage/current value can also be defined as the "value of the direct voltage/current that dissipates the same power in a resistor."

$$f_{RMS} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} [f(t)]^2 \, dt},$$

If i is the instantaneous current through the resistance, the average power dissipated is, $I_{RMS}^2 R$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$I^2_{RMS} = \frac{1}{2\pi} \int_0^{2\pi} I^2_m \, \mathrm{Sin}^2 (\theta) d\theta = \frac{1}{2\pi} \int_0^{2\pi} \frac{I^2_m}{2} \, d\theta = \frac{I^2_m}{2}$$

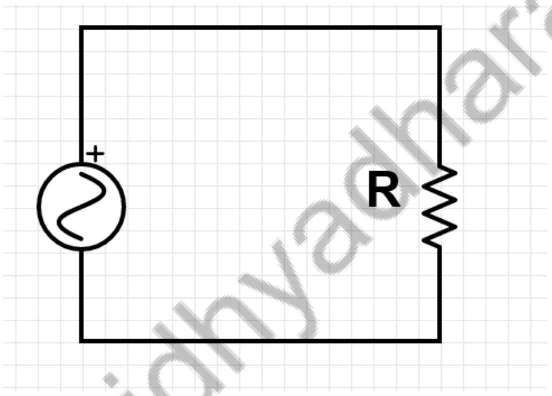
$$\boxed{I_{RMS} = \frac{I_m}{\sqrt{2}} = 0.707 I_m}$$

$V_{RMS} = 0.707 \, V_m$

Power = V_{RMS} * I_{RMS} = I^2_{RMS}* R = $\frac{V^2_{RMS}}{R}$

AC RESPONSE FOR A PURE RESISTIVE CIRCUIT

Consider a circuit having a resistance R ohms connected across the terminals of an AC Source



If the value of the voltage at any instant is v volts, the value of the current at that instant is given by

$$i = \frac{v}{R}$$

When the voltage is zero, the current is also zero and since the current is proportional to the voltage, the waveform of the current is exactly the same as that of the voltage

AC RESPONSE FOR A PURE RESISTIVE CIRCUIT

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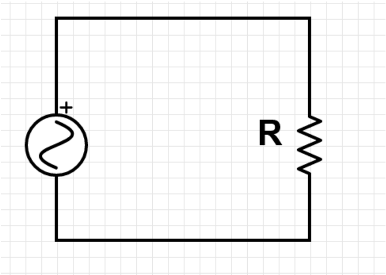

$$i = \frac{v}{R}$$

$$v = V_m \text{ Sin } (\omega t)$$

$$i = I_m \text{ Sin } (\omega t)$$

$$I_m = V_m / R$$

The two quantities are *in phase* as they pass through their zero values at the same instant and attain their maximum values in a given direction at the same instant

Hence the current wave is as shown,

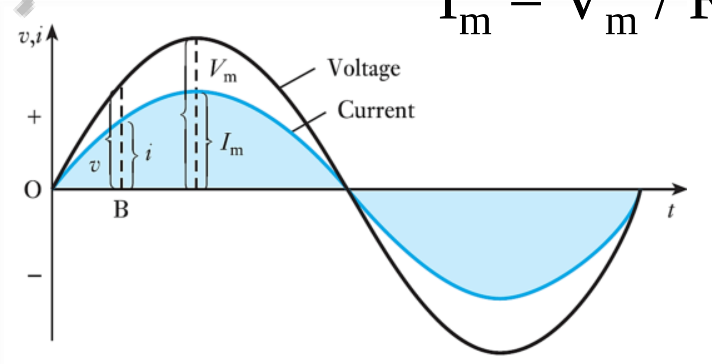

Fig. Voltage and current waveforms for a resistive circuit

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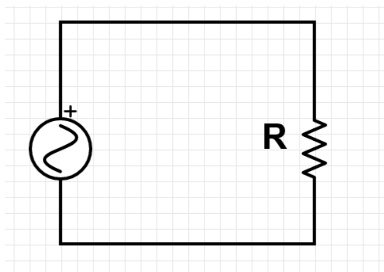
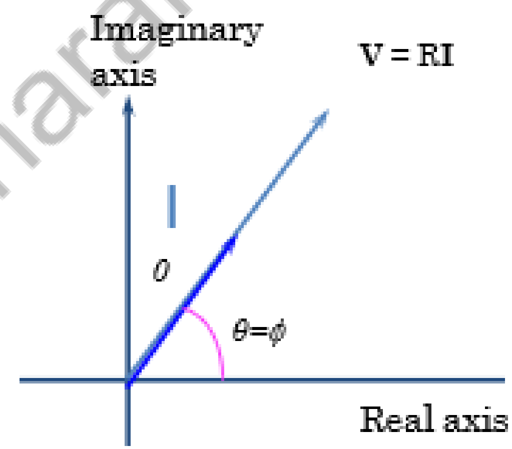


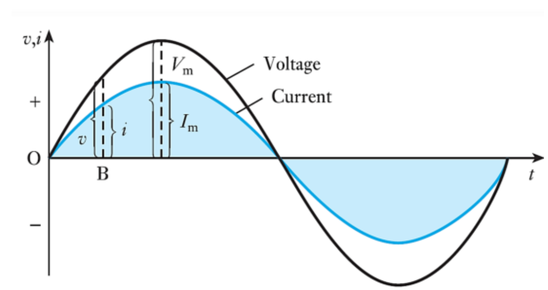
$$i = \frac{v}{R}$$



$$\text{Voltage} \quad v(t) = Ve^{\,j(\omega t+\phi)}$$

$$\text{Current} \quad i(t) = Ie^{\,j(\omega t+\theta)}$$

$$\text{By Ohm's law} \quad V e^{\,j(\omega t+\phi)} = R\,I\,e^{\,j(\omega t+\phi)}$$



Phasor representation :

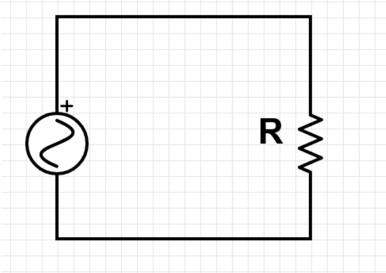
$$V\angle\phi = RI\angle\theta$$

$$\phi = \theta$$

- So current and voltage are in phase for a resistor

AC RESPONSE FOR A PURE RESISTIVE CIRCUIT

Consider a circuit having a resistance R ohms connected across the terminals of an AC Source



$$i = \frac{v}{R}$$

$$\boxed{I_{\mathrm{RMS}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m}$$

$V_{RMS} = 0.707\ V_m$

Power = V_{RMS} * I_{RMS} = $I^2{}_{RMS}$ * R = $\frac{V^2{}_{RMS}}{R}$

AC RESPONSE FOR A PURE RESISTIVE CIRCUIT

Generalised Equation for Power in an AC Circuit

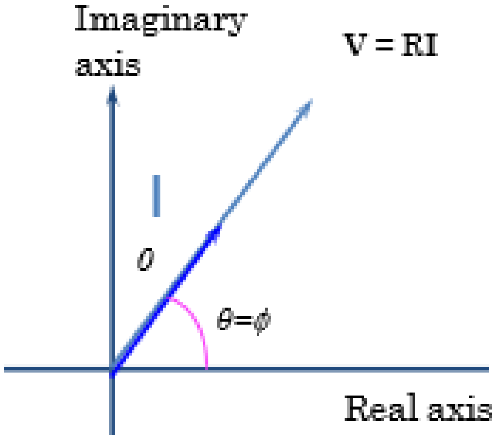
$$P_{avg} = \text{Average of } (V_m \sin\omega t \times I_m \sin(\omega t + \Phi))$$

$$P_{avg} = Average\ of\ \frac{V_m I_m [cos\Phi\ -\ cos(2\omega t + \Phi)]}{2}$$

2 sin A sin B = cos (A- B) - cos (A +B)

$$P_{avg} = \frac{V_m I_m cos\Phi}{2}$$

$$P_{avg} = V_{rms} \times I_{rms} \times cos\Phi$$



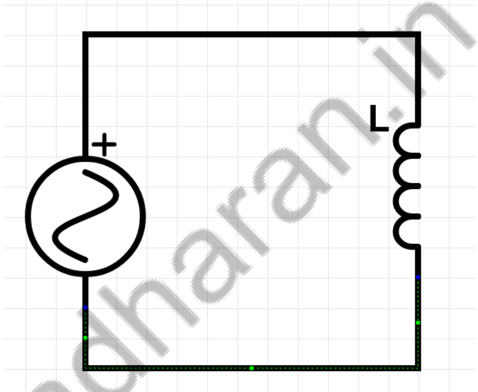
For Pure Resistive Circuit

$\phi = 0$

Power = V_{RMS} * I_{RMS} = I^2_{RMS}* R = $\frac{V^2_{RMS}}{R}$

AC RESPONSE FOR A PURE INDUCTIVE CIRCUIT

$V(t) = V_m \text{ Sin } (\omega t)$



$$V_m \sin \omega t = L \frac{di}{dt}$$

$$di = \frac{V_m}{L} \sin \omega t \; dt$$

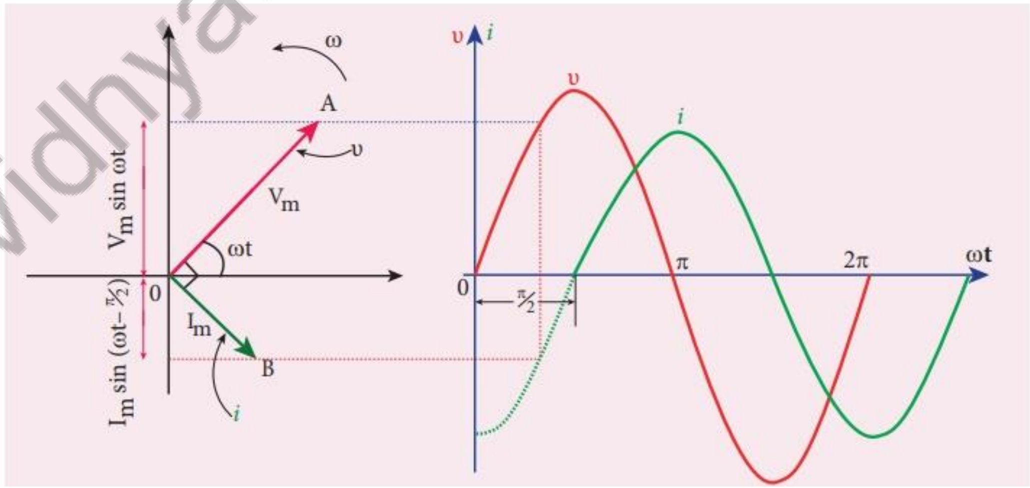
Integrating both sides, we get

$$i = \frac{V_m}{L} \int \sin \omega t \, dt$$

$$i = \frac{V_m}{L\omega} (-\cos \omega t) + \text{constant}$$

$$i = \frac{V_m}{\omega L} \sin\left(\omega t - \pi/2\right)$$

$$\text{or} \quad i = I_m \sin\left(\omega t - \pi/2\right)$$



$$V = j\,L\omega\,I$$

$$V\angle\phi = L\;\omega\,I\angle\theta$$

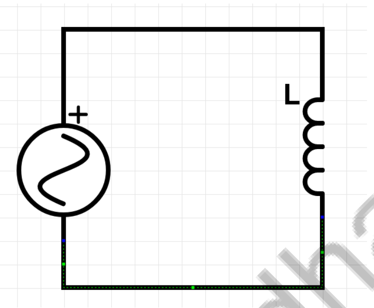
$$\phi = 90^o + \theta$$

- The voltage angle is 90º greater than current angle,
- hence inductor **voltage leads the current by 90º**.

AC RESPONSE FOR A PURE INDUCTIVE CIRCUIT

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \pi/2\right)$$

or $$i = I_m \sin\left(\omega t - \pi/2\right)$$



Inductive reactance X_L

The peak value of current I_m is given by $I_m = V_m / \omega L$. Let us compare this equation with $I_m = V_m / R$ from resistive circuit. The quantity ωL plays the same role as the resistance in resistive circuit. This is the resistance offered by the inductor, called inductive reactance (X_L). It is measured in ohm.

$$X_L = \omega L$$

The inductive reactance (X_L) varies directly as the frequency.

$$X_L = 2\pi f L$$

where f is the frequency of the alternating current. For a steady current, $f = 0$. Therefore, $X_L = 0$. Thus an ideal inductor offers no resistance to steady DC current.

AC RESPONSE FOR A PURE INDUCTIVE CIRCUIT

Example 1:

A 400 mH coil of negligible resistance is connected to an AC circuit in which an effective current of 6 mA is flowing. Find out the voltage across the coil if the frequency is 1000 Hz.

Solution

$L = 400 \times 10^{-3}$ H; $I_{eff} = 6 \times 10^{-3}$ A

$f = 1000$ Hz

Inductive reactance, $X_L = L\omega = L \times 2\pi f$

$= 2 \times 3.14 \times 1000 \times 0.4$

$= 2512\ \Omega$

Voltage across L,

$V = I\,X_L = 6 \times 10^{-3} \times 2512$

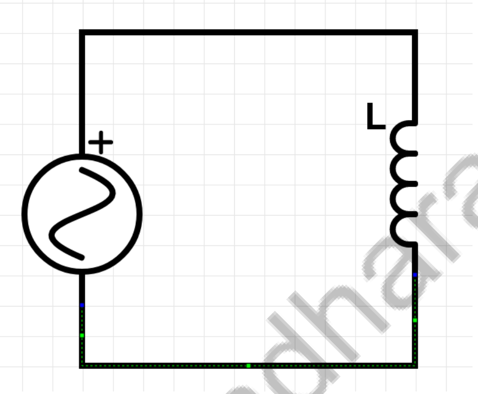
$V = 15.072\,V(RMS)$

AC RESPONSE FOR A PURE INDUCTIVE CIRCUIT

$V(t) = V_m \sin(\omega t)$

$$i = \frac{V_m}{\omega L} \sin\left(\omega t - \pi/2\right)$$

or $$i = I_m \sin\left(\omega t - \pi/2\right)$$



Generalised Equation for Power in an AC Circuit

$$P_{avg} = \frac{V_m I_m cos\Phi}{2}$$

$$P_{avg} = V_{rms} \times I_{rms} \times cos\Phi$$

For Inductor $\phi = -90^0$

$P_{avg} = 0$

AC RESPONSE FOR A PURE CAPACITIVE CIRCUIT

$$V(t) = V_m \sin(\omega t)$$

$$v = \frac{q}{c} \;>>\; q = vc$$

$$i = \frac{dq}{dt} = \frac{d\,C V_m \sin(\omega t)}{dt}$$

$$i = C\omega\, V_m \cos(\omega t)$$

$$i = I_m \cos(\omega t)$$

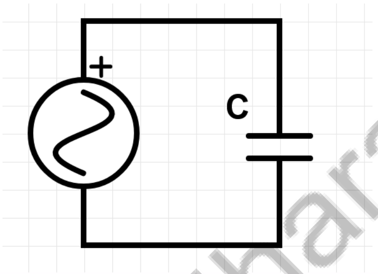
$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

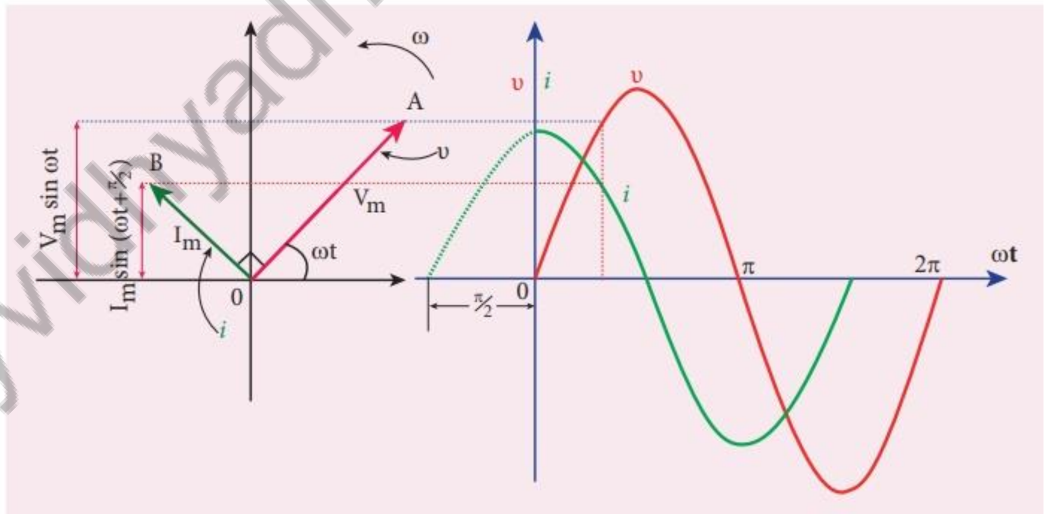
$$I_m = C\omega\, Vm = \frac{V_m}{\frac{1}{C\omega}}$$

$$X_c = \frac{1}{C\omega}$$

$$I\angle\theta = \omega C V\angle\phi \Rightarrow \mathbf{I} = j\omega C\,\mathbf{V}$$

$$\theta = 90^o + \phi$$





- The current angle is 90^o greater than voltage angle,
- hence **capacitor current leads the voltage by 90^o**.
- For DC X_c = Open Circuit

AC RESPONSE FOR A PURE CAPACITIVE CIRCUIT

Example 2:

A capacitor of capacitance $10^2/\pi$ μF is connected across a 220 V, 50 Hz A.C. mains. Calculate the capacitive reactance, RMS value of current and write down the equations of voltage and current.

$$C = \frac{10^2}{\pi} \times 10^{-6} F,\ V_{RMS} = 220\ V;\ f = 50 Hz$$

(i) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2 \times \pi \times 50 \times \frac{10^{-4}}{\pi}} = 100\,\Omega$$

(ii) RMS value of current,

$$I_{RMS} = \frac{V_{RMS}}{X_C} = \frac{220}{100} = 2.2\,A$$

(iii) $V_m = 220 \times \sqrt{2} = 311\ V$

$$I_m = 2.2 \times \sqrt{2} = 3.1 A$$

Therefore,

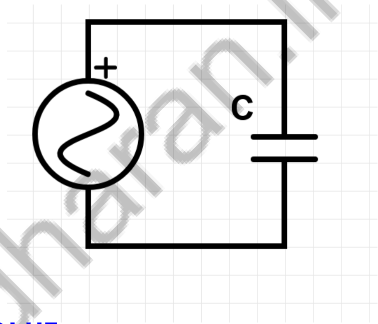
$$\upsilon = 311 \sin 314 t$$

$$i = 3.1 \sin\left(314 t + \pi/2\right)$$

AC RESPONSE FOR A PURE CAPACITIVE CIRCUIT

$V(t) = V_m \, \text{Sin}\,(\omega t)$

$i = I_m \, \text{Cos}\,(\omega t)$



Generalised Equation for Power in an AC Circuit

$$P_{avg} = \frac{V_m I_m cos\Phi}{2}$$

$$P_{avg} = V_{rms} \times I_{rms} \times cos\Phi$$

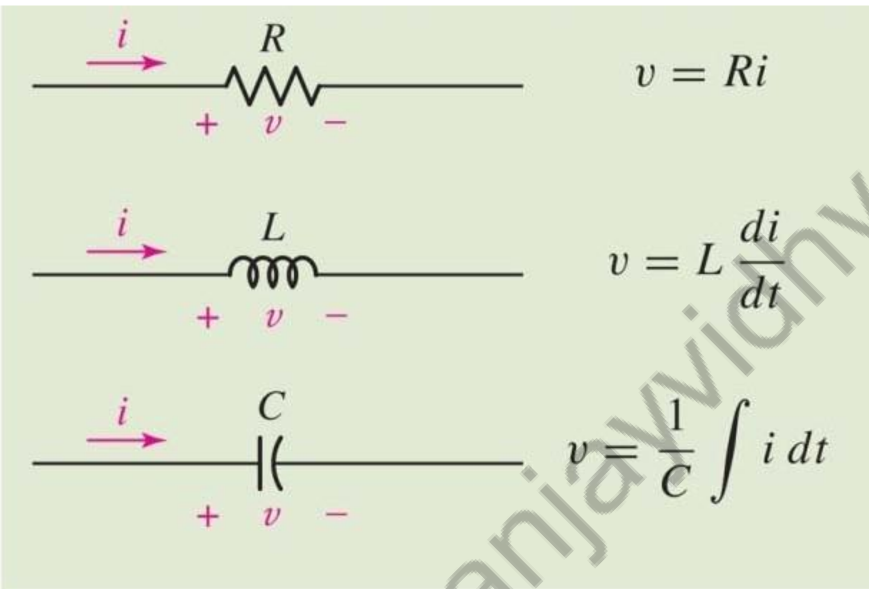
$For\ Capacitor\ \phi = 90^0$

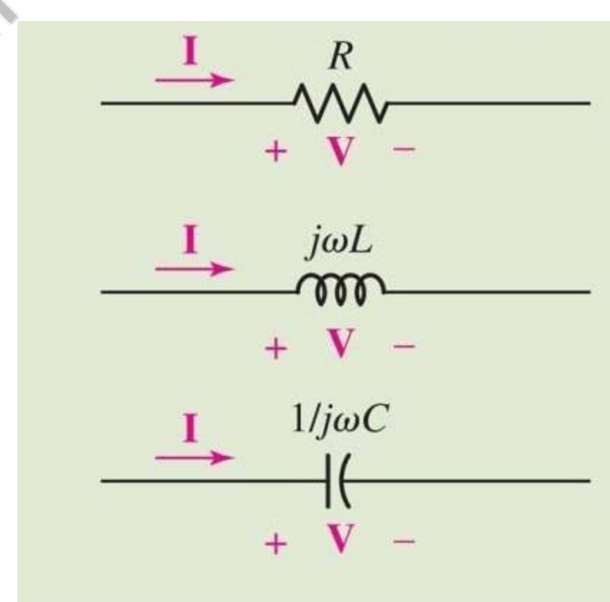
$P_{avg} = 0$

Summary: Phasor Voltage/Current Relationships

Time Domain

Frequency Domain





Calculus (hard but real)

Algebra (easy but complex)

Summary: Phasor Voltage/Current Relationships

General Phasor equation for elements:

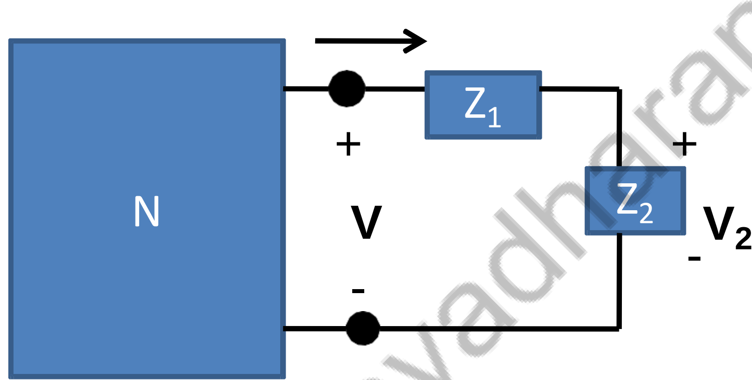
$\mathbf{V} = Z\mathbf{I}$ $\quad Z \rightarrow$ Impedance

$Z_R = R \;;\; Z_L = jwL \;;\; Z_C = 1/jwC$

$\mathbf{I} = Y\mathbf{V}$ $\quad Y \rightarrow$ Admittance

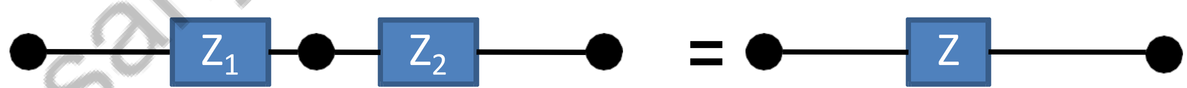
$Y_R = 1/R \;;\; Y_L = 1/jwL \;;\; Y_C = jwC$

Impedances in Series



$$V = V_1 + V_2 = Z_1 I + Z_2 I = (Z_1 + Z_2) I \quad \Rightarrow \quad I = \frac{V}{Z_1 + Z_2}$$

As a result,

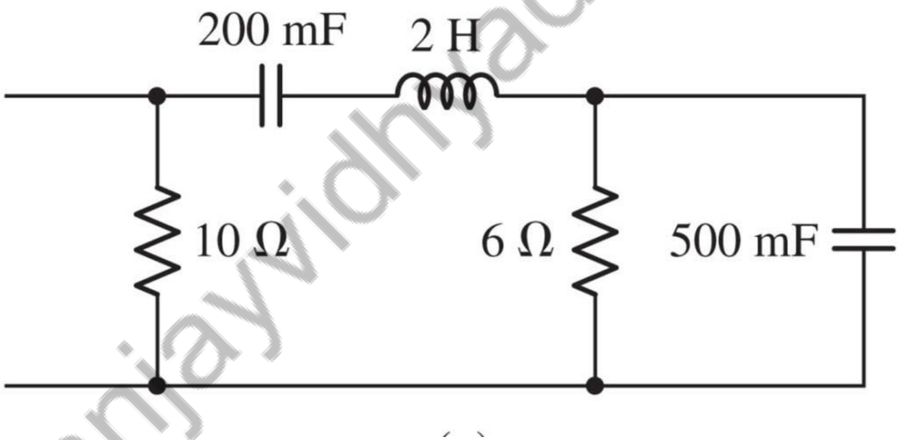


$Z = Z_1 + Z_2$

Impedance of AC circuits

Example 3:

Find the impedance of the network at 5 rad/s.



Impedance of AC circuits

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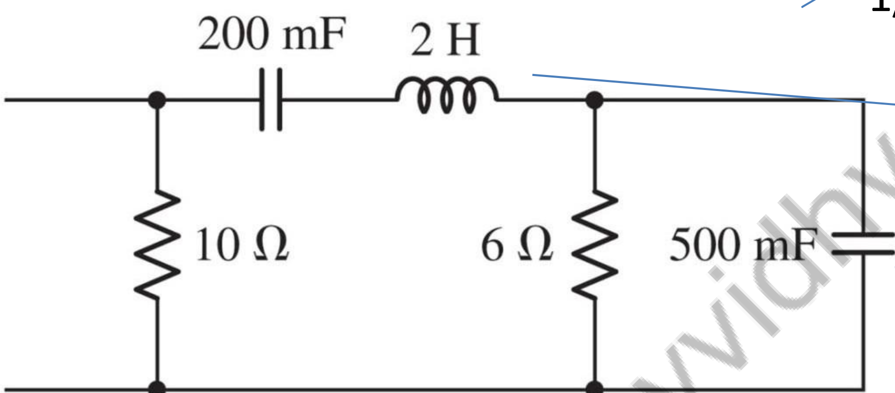
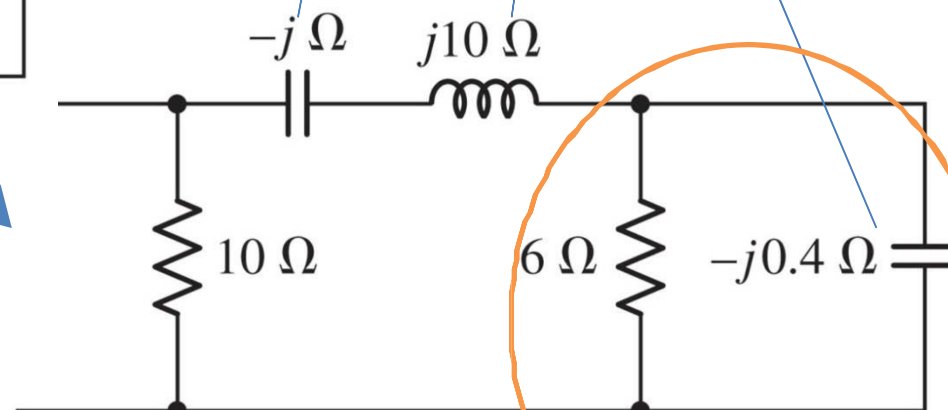


$1/jwC = -j/5 * 200 * 10^{-3} = -j\Omega$

$jwL = j5 * 2 = j10\Omega$

$1/jwC = -j/5 * 500 * 10^{-3} =$

$-j0.4\Omega$



$Z = Z_1 * Z_2/(Z_1+Z_2)$

$6*-j0.4/(6-j0.4)$

Answer: 4.255 + j4.929 Ω

Thank you