Electrical Science: 2021-22

Lecture 12

Introduction to AC Circuits

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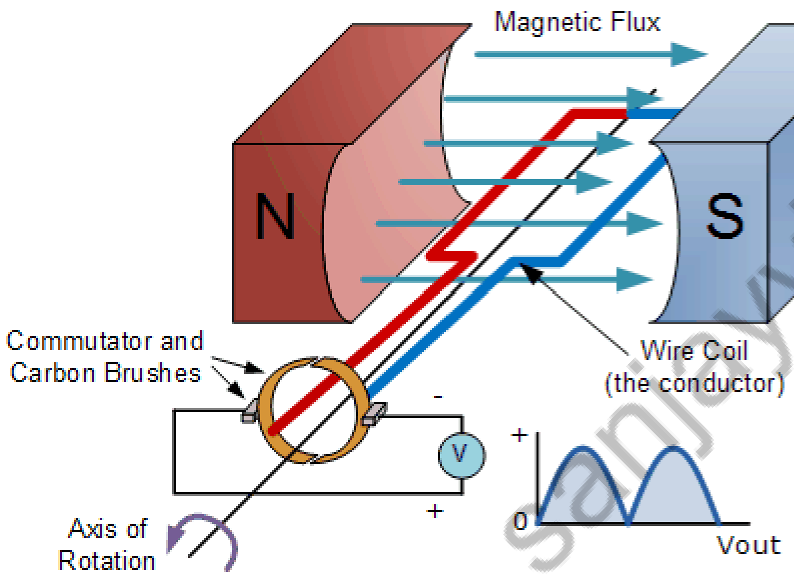
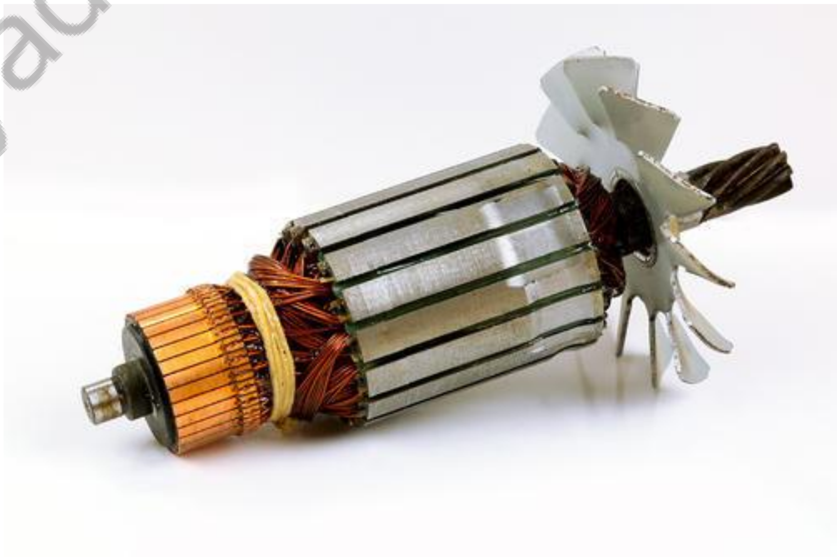
Alternating Currents

Advantages AC over DC

(i) The generation of AC is cheaper than that of DC.





Alternating Currents

Advantages AC over DC

(ii) When AC is supplied at higher voltages, the transmission losses are small compared to DC transmission. Conversion from high to low voltages and vice-versa is essay.

Alternating Currents

Advantages AC over DC

(iii) AC can easily be converted into DC with the help of rectifiers.



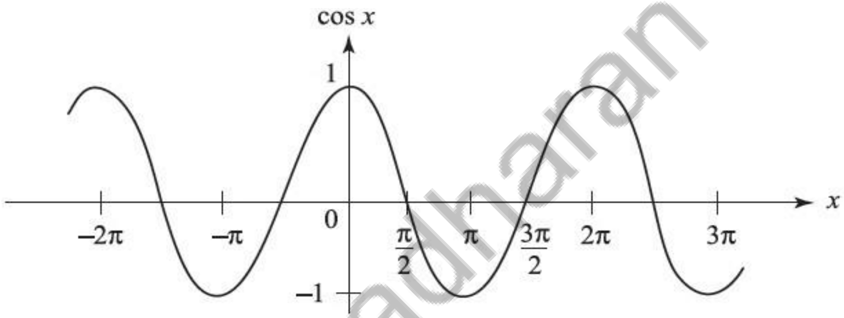
Alternating Currents

Disadvantages of AC w.r.t DC

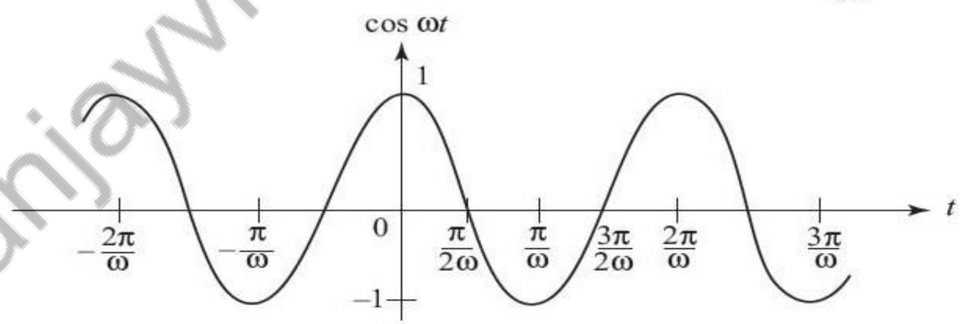
(i) Design of AC circuits more complex than DC
(ii) Paralleling AC more complex than DC sources
(iii) Losses due to radiation

Alternating Currents

cos x vs x
x → rad



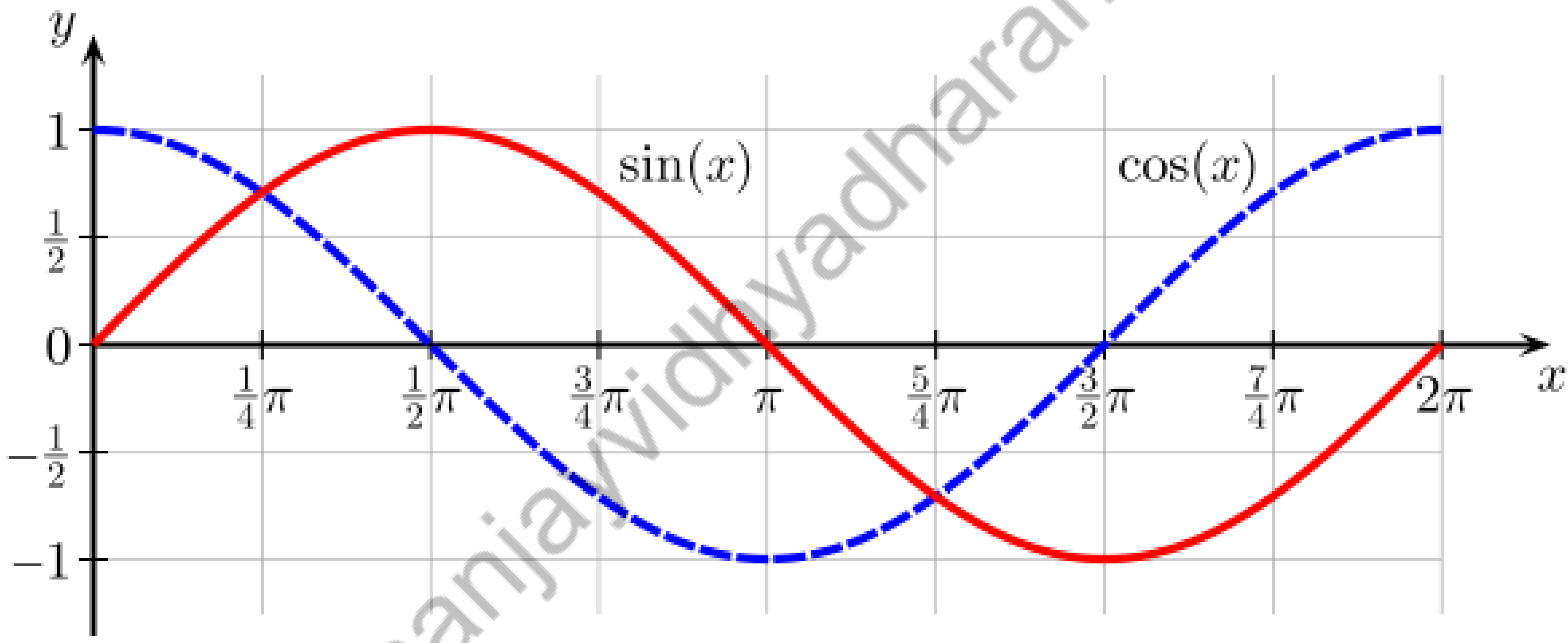
cos ω t vs t
t → s



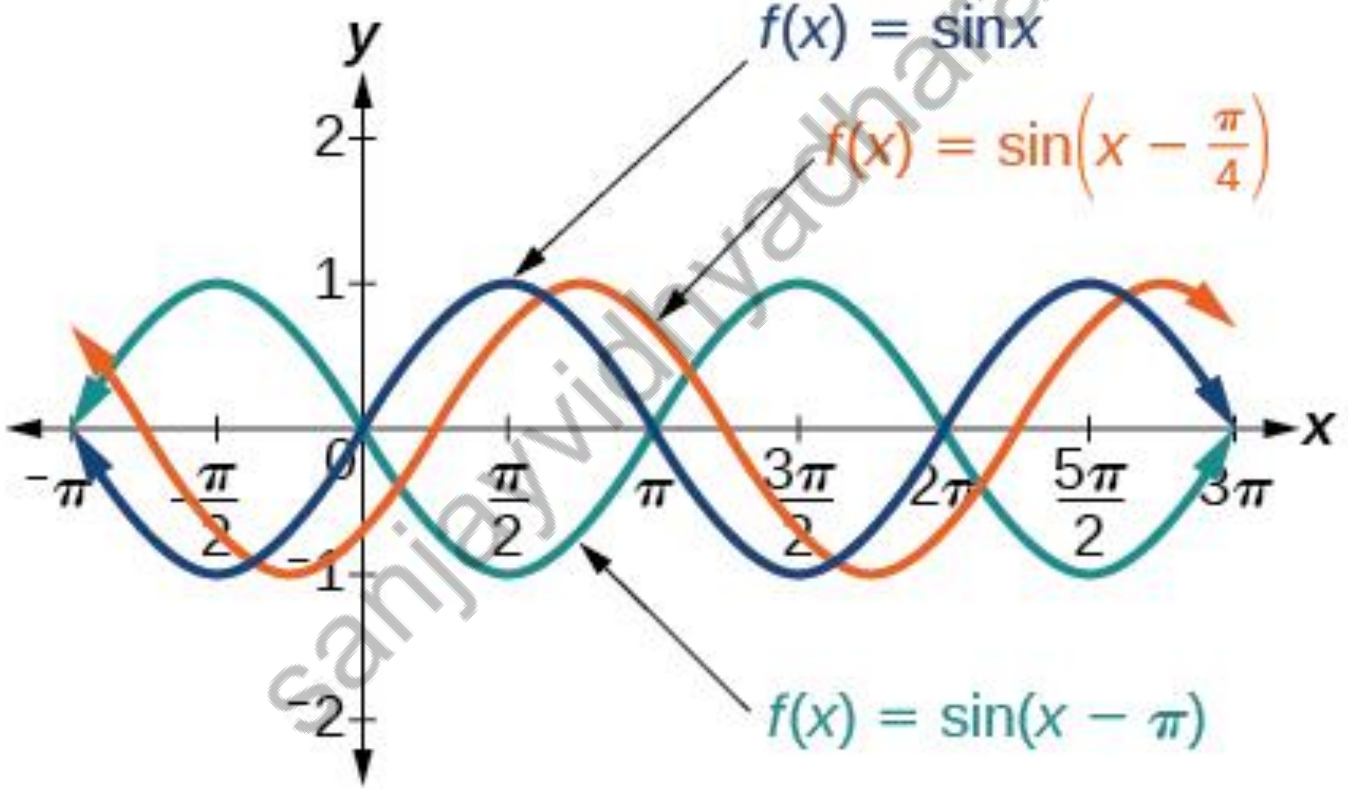
Time period $T = \frac{2\pi}{\omega}$

Frequency $f = \frac{\omega}{2\pi}$

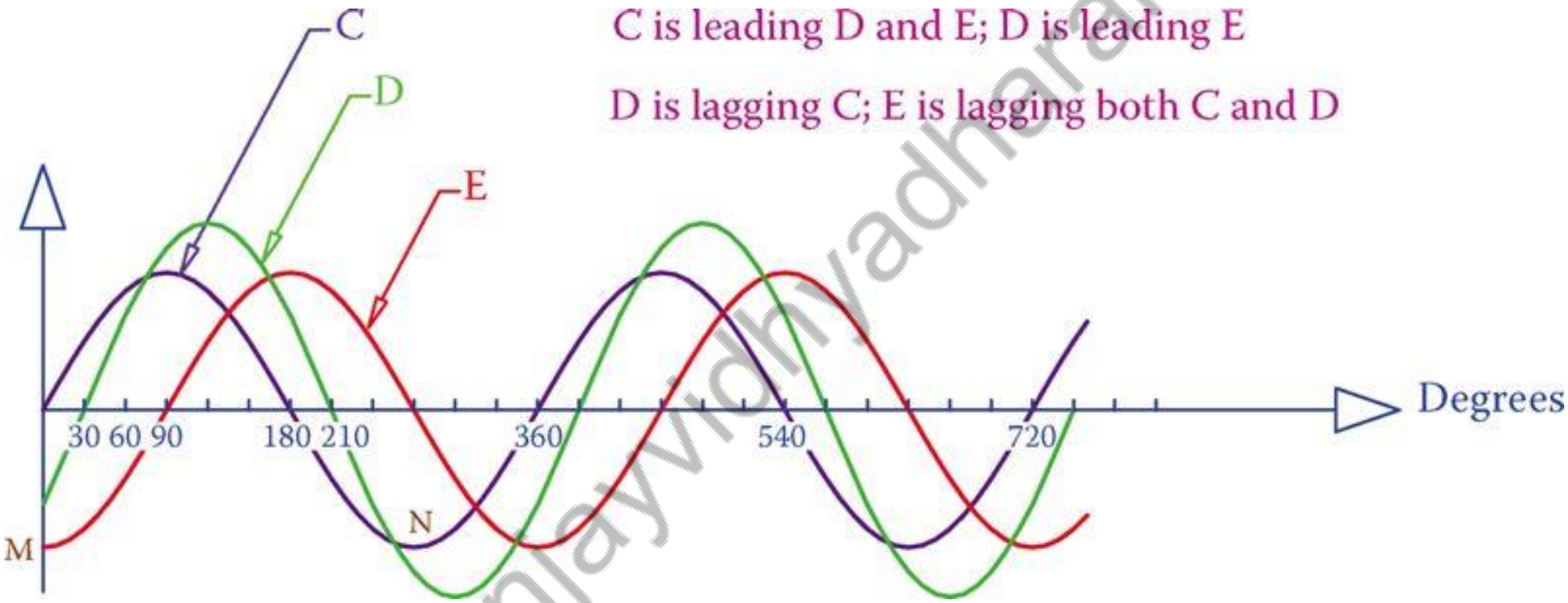
Alternating Currents



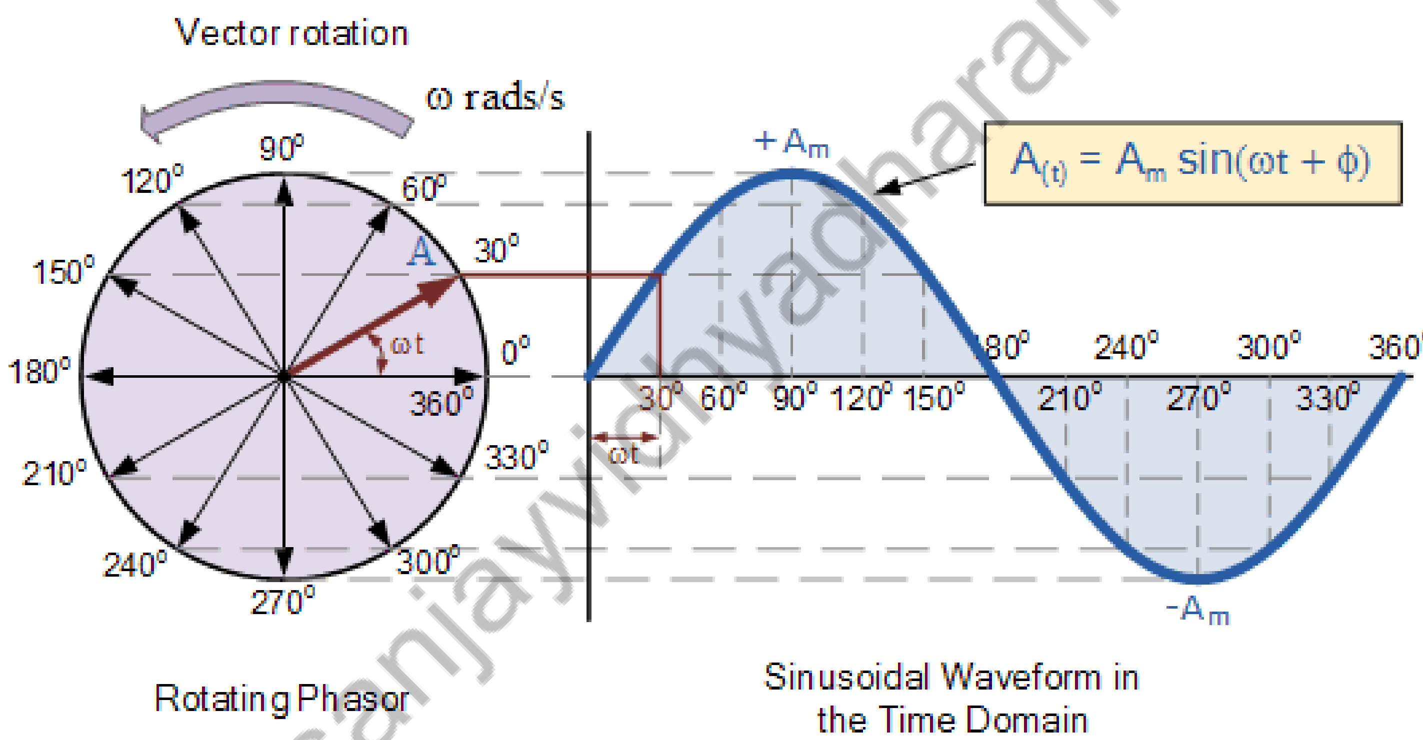
Alternating Currents



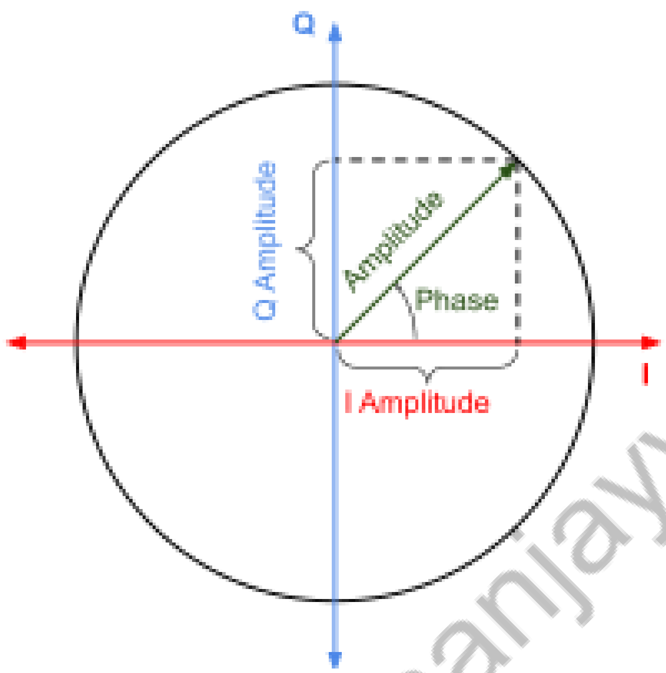
Alternating Currents

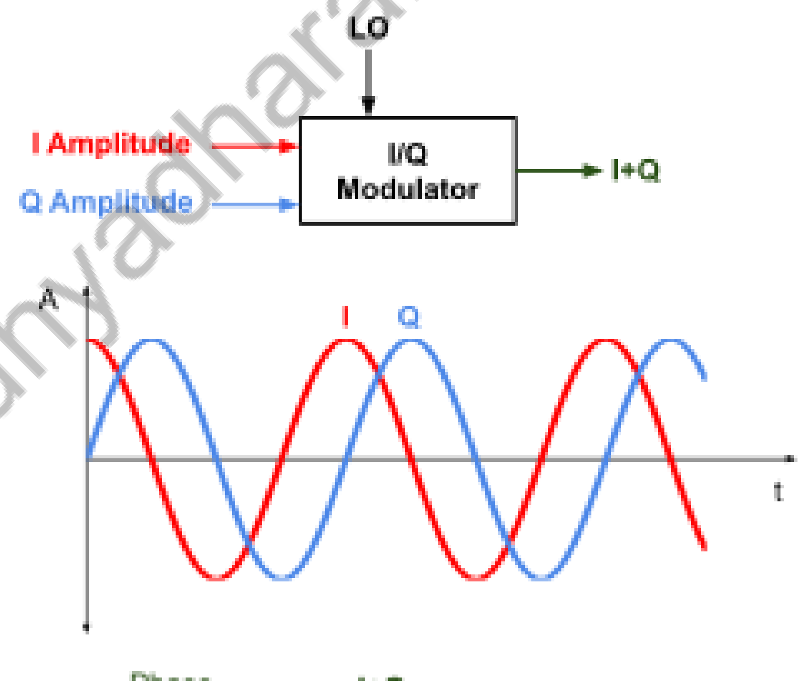


Alternating Currents



Alternating Currents





Review of Complex Numbers

Complex Numbers

- $i = \sqrt{-1}$
- $i^2 = (\sqrt{-1})^2 = -1$
- $i^3 = i^2 \times i = -1 \times i = -i$

$i^4 = (i^2)^2 = (-1)^2 = 1$

- Complex number $A = a + ib$
- Complex conjugate $A^* = a - jb$

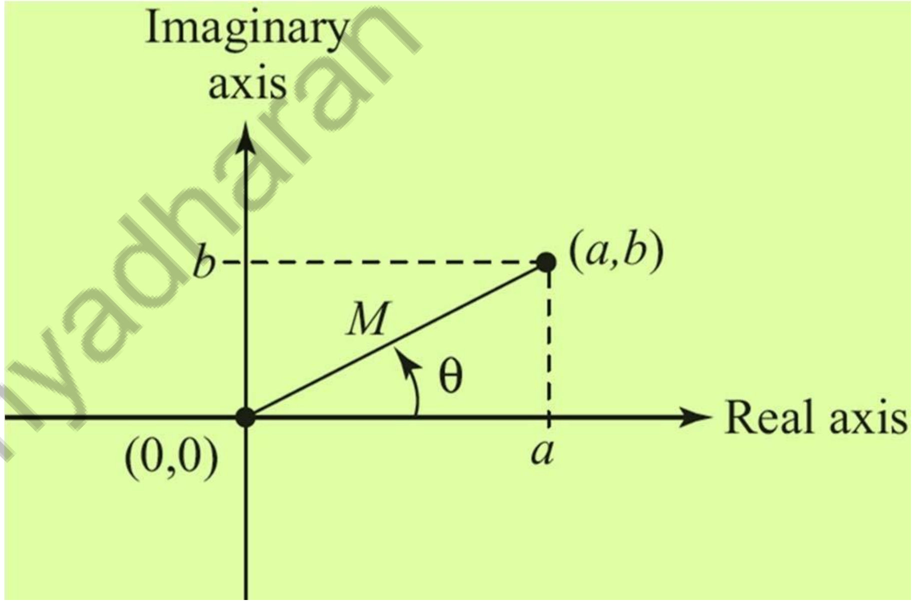
Review of Complex Numbers

Euler's Formula

Exponential Form

$\cos\theta + i\sin\theta = e^{i\theta}$

$\cos\theta - i\sin\theta = e^{-i\theta}$



Rectangular form

$A = a + ib$

Can we write?

$A = a+ib = M e^{i\theta} = M\cos\theta + i M\sin\theta$

$\Rightarrow a = M\cos\theta;\ b = M\sin\theta$

$\Rightarrow \tan\theta = b/a$

$\Rightarrow M = \sqrt{(a^2+b^2)}$

Phasors

Euler's Formula

$$e^{i\phi} = \cos\phi + i\sin\phi$$

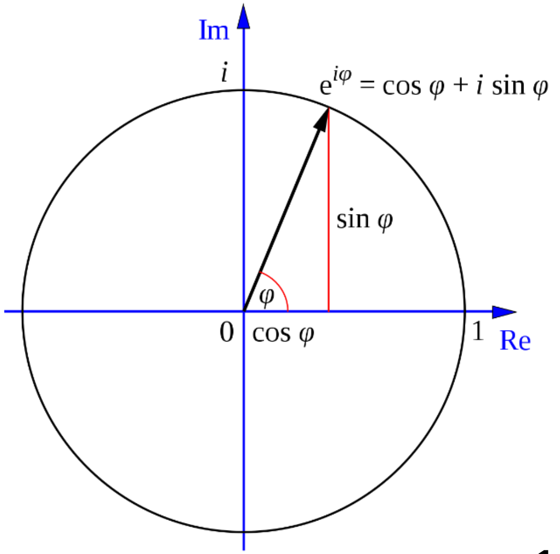
- By considering cos ϕ and sin ϕ as the real and imaginary parts of $e^{j\phi}$,

$$\cos\phi = \text{Re}(e^{j\phi}), \sin\phi = \text{Im}(e^{j\phi})$$

- Given a sinusoid $v(t) = V_m \cos(\omega t + \phi)$,

$$v(t) = V_m \cos(\omega t + \phi) = \text{Re}(V_m e^{j(\omega t + \phi)})$$

$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow V = V_m \angle\phi$$



Phasor Representation

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90°$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90°$

Phasor Conversion Examples

Numerical 1: Transform these sinusoids to phasors,

(a) $i = 6\cos(50t - 40^\circ)$ A
(b) $v = -4\sin(30t + 50^\circ)$ V

Solution:

(a) $i = 6\cos(50t - 40^\circ)$A has the phasor, $I = 6\angle -40^\circ A$

(b) Since, $-\sin A = \cos(A + 90^\circ)$,

$$v = -4\sin(30t + 50^\circ) = 4\cos(30t + 50^\circ + 90^\circ) = 4\cos(30t + 140^\circ) \text{ V}$$

The phasor form of v is $V = 4\angle 140^o V$

Phasor Operations

Phasor Operations

addition: $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$

subtraction: $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$

multiplication: $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$

division: $\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$

inverse: $\frac{1}{z} = \frac{1}{r} \angle -\phi$

square root: $\sqrt{z} = \sqrt{r} \angle \phi/2$

complex conjugate: $z^* = x - jy$
$= r \angle -\phi = r\, e^{-j\phi}$

$z = x + jy$

$z = r \angle \phi$

$z = z e^{j\phi}$

$r = \sqrt{x^2 + y^2}$

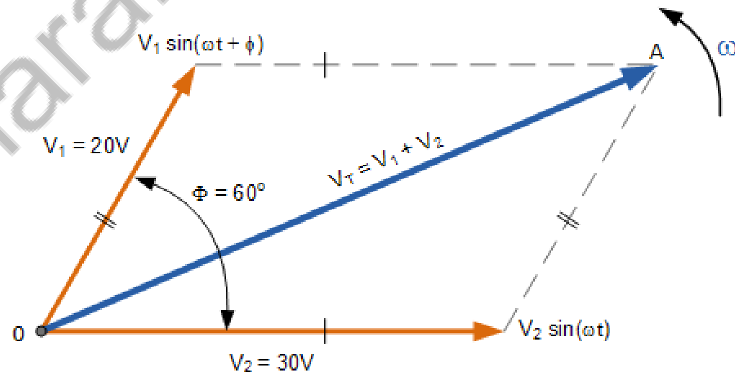
$\phi = \tan^{-1}\left(\frac{x}{y}\right)$

$y = r\sin\phi$

$e^{j\phi} = \cos\phi + j\sin\phi$

$\cos\phi = \mathrm{Re}(e^{j\phi})$

$\sin\phi = \mathrm{Im}(e^{j\phi})$



$V_1 = 50 \angle 60^\circ \quad V_2 = 30 \angle -50^\circ$

$V_1 + V_2 = ?$

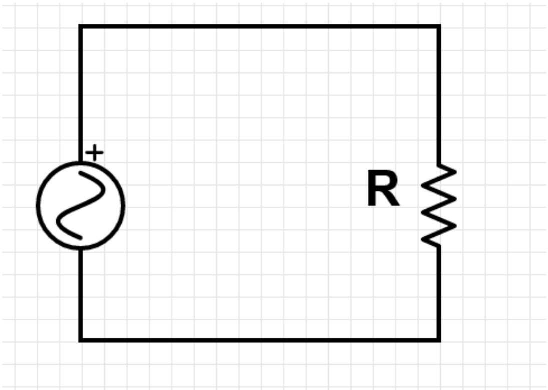
$V_1 = 50\,[\cos 60^\circ + j\sin 60^\circ] = 25 + j\,43.3$

$V_2 = 30\,[\cos(-50^\circ) + j\sin(-50^\circ)] = 19.3 - j\,23.0$

$V_1 + V_2 = 44.3 + j\,20.3 \qquad V_M = \sqrt{V_1^2 + V_2^2} = 48.7$

$V_1 + V_2 = 48.7 \angle 24.6^\circ \qquad \phi = \tan^{-1}\left(\frac{V_1}{V_2}\right) = 24.6^\circ$

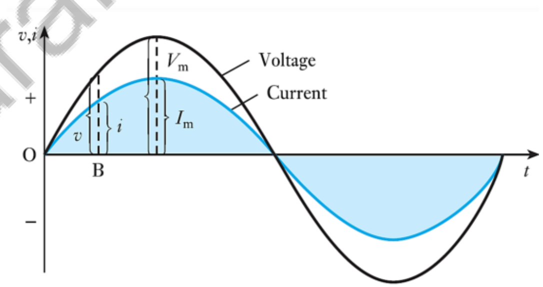
Average and RMS



$$v = V_m \text{ Sin } (\omega t)$$

$$i = I_m \text{ Sin } (\omega t)$$

$$I_m = V_m / R$$



$$V_{avg} = \int_0^{2\pi} V_m \text{ Sin } (\omega t) dt = 0$$

The RMS voltage/current value can also be defined as the "value of the direct voltage/current that dissipates the same power in a resistor."

$$f_{RMS} = \sqrt{\frac{1}{T_2 - T_1} \int_{T_1}^{T_2} [f(t)]^2 \, dt},$$

Average and RMS

The RMS voltage/current value can also be defined as the "value of the direct voltage/current that dissipates the same power in a resistor."

$$f_{\mathrm{RMS}} = \sqrt{\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} [f(t)]^2\, dt},$$

If i is the instantaneous current through the resistance, the average power dissipated is, $I_{RMS}^2 R$

$$I^2_{RMS} = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{I}^2_{\mathrm{m}}\ \mathrm{Sin}^2\,(\theta)d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{\mathrm{I}^2_{\mathrm{m}}}{2}\, d\theta = \frac{\mathrm{I}^2_{\mathrm{m}}}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$I_{\mathrm{RMS}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

$V_{RMS} = 0.707\ V_m$

Power = V_{RMS} * I_{RMS} = I^2_{RMS} * R = $\frac{V^2_{RMS}}{R}$

Average and RMS

The Average of a Sinusoidal Signal

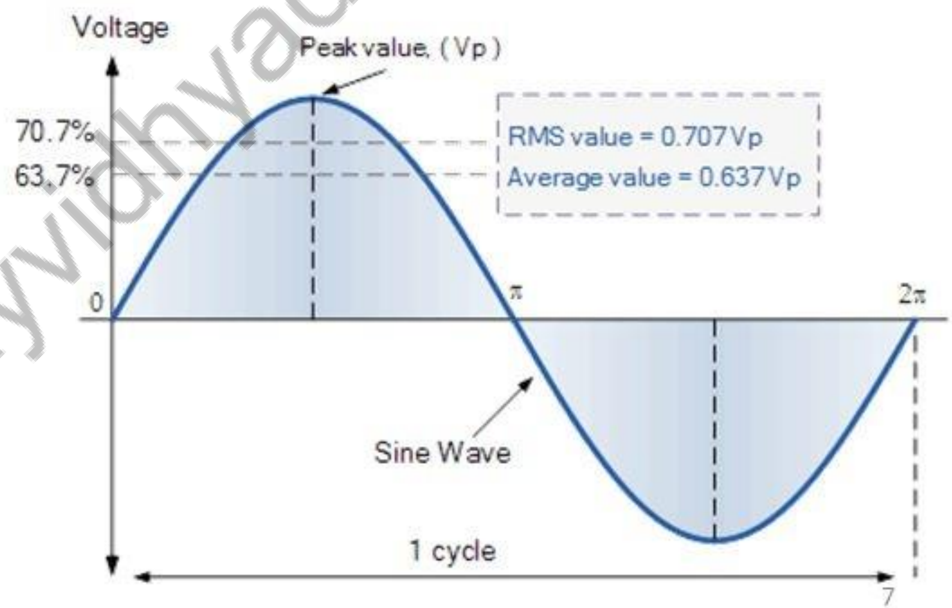
- Then the average value is obtained by adding the instantaneous values of voltage over one half cycle only.

$$V_{av} = \frac{1}{\pi}\int_{0}^{\pi} V_p \sin\theta \; d\theta$$

$$V_{av} = \frac{V_p}{\pi}[-\cos\theta]_0^{\pi}$$

$$V_{av} = \frac{2V_p}{\pi} = 0.637V_p$$

$$\boldsymbol{I_{av} = 0.637 I_m}$$



Average and RMS

- RMS value of a sinusoidal current or voltage is,

$$\boxed{I = 0.707 I_m}$$

Peak factor

– for ***any waveform*** the peak factor is defined as

$$\text{Peak factor} = \frac{\text{peak value}}{\text{r.m.s. value}}$$

– for a ***sine wave*** this gives

$$\text{Peak factor} = \frac{V_p}{0.707 V_p} = 1.414$$

Average and RMS

- RMS value of a sinusoidal current or voltage is,

$$I = 0.707 I_m$$

Form factor

– for ***any waveform*** the form factor is defined as

$$\text{Form factor} = \frac{\text{r.m.s. value}}{\text{average value}}$$

– for a ***sine wave*** this gives

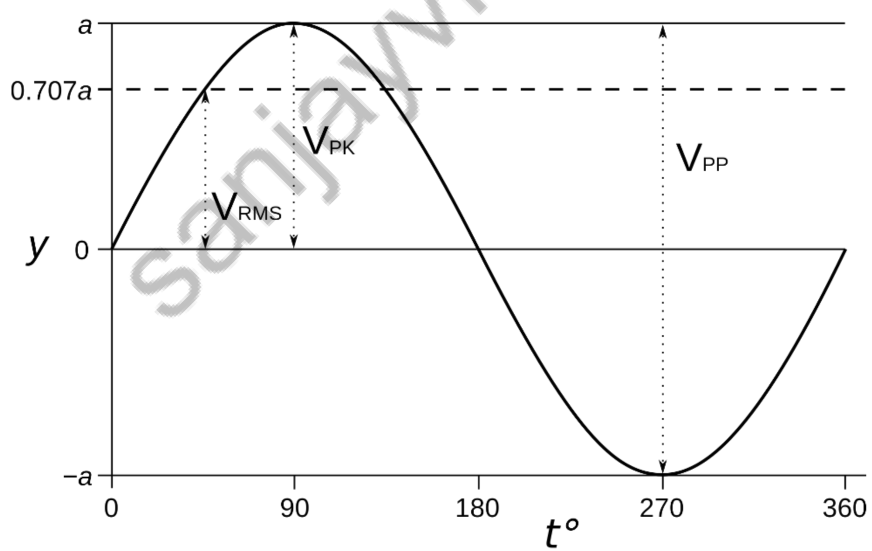
$$\text{Form factor} = \frac{0.707\,V_p}{0.637\,V_p} = 1.11$$

Average and RMS

Numerical 1: An alternating current of sinusoidal waveform has an RMS value of 10.0 A. What is the peak-to-peak value of this current?

Solution: $I_m = \dfrac{I}{0.707} = \dfrac{10}{0.707} = 14.14\,A$

The peak-to-peak value is therefore **14.14** − **(-14.14) = 28.28 A**



Average and RMS

Numerical 2: An alternating voltage has the equation $v = 141.4 \sin 377t$, what are the values of (a) RMS voltage (b) frequency (c) the instantaneous voltage when $t = 3$ ms?

Solution: The relation is of the form $v = V_m \sin\omega t$ and by comparison,

$(a) V_m = 141.4V = \sqrt{2}V$ Hence, $V = \frac{141.4}{\sqrt{2}} = 100V$

(b) Also by comparison,

$$\omega = 377 rad/s = 2\pi f, f = \frac{377}{2\pi} = 60 Hz$$

(c) Finally, $v = 141.4 \sin 377t$

when $t = 3 \times 10^{-3}$ sec,

$$v = 141.4 \sin(377 \times 3 \times 10^{-3}) = 141.4 \sin 1.131$$

$$= 141.4 \times 0.904 = 127.8V$$

Average and RMS

The peak voltage of 220 Volt AC mains (in Volt) is :

A 155.6

B 220.0

C 311

D 440.0

Solution

Correct option is C 311

$$\text{Peak voltage} = V_{Rms} \times \sqrt{2}$$

$$= 220 \times \sqrt{2}$$

$$= 311\text{V}.$$

Thank you