Electrical Science: 2021-22
Lecture 10
First Order Circuits

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First Order Circuit

- Any circuit with a single energy storage element, an arbitrary number of sources, and an arbitrary number of resistors is a circuit of order 1.

- Any voltage or current in such a circuit is the solution to a 1st order differential equation.

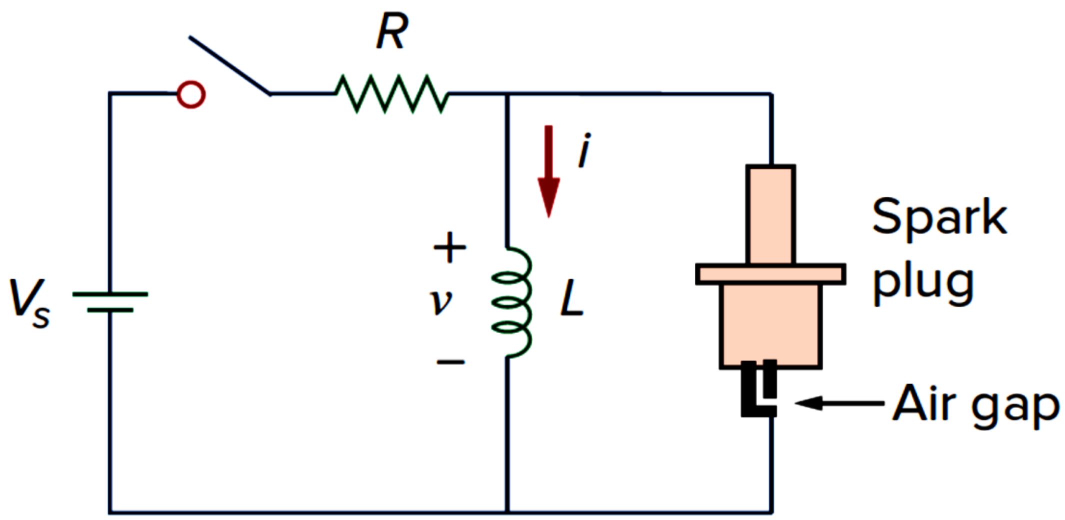
First Order Response

Natural Response : Source free response results from energies stored in dynamic circuit elements and is characterized by the nature of circuit itself

The forced response is **what the circuit does** with the sources turned on, but with the initial conditions set to zero. The natural response is what the circuit does including the initial conditions, but with the input suppressed. The total response is the sum of the forced response plus the natural response.

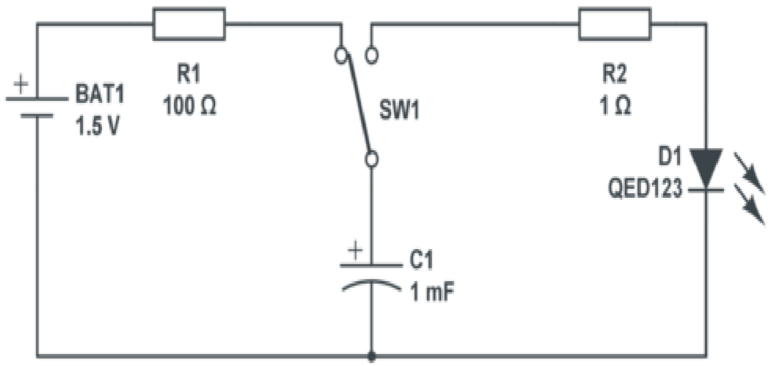
Applications of First Order Circuits

Automobile Ignition Circuit



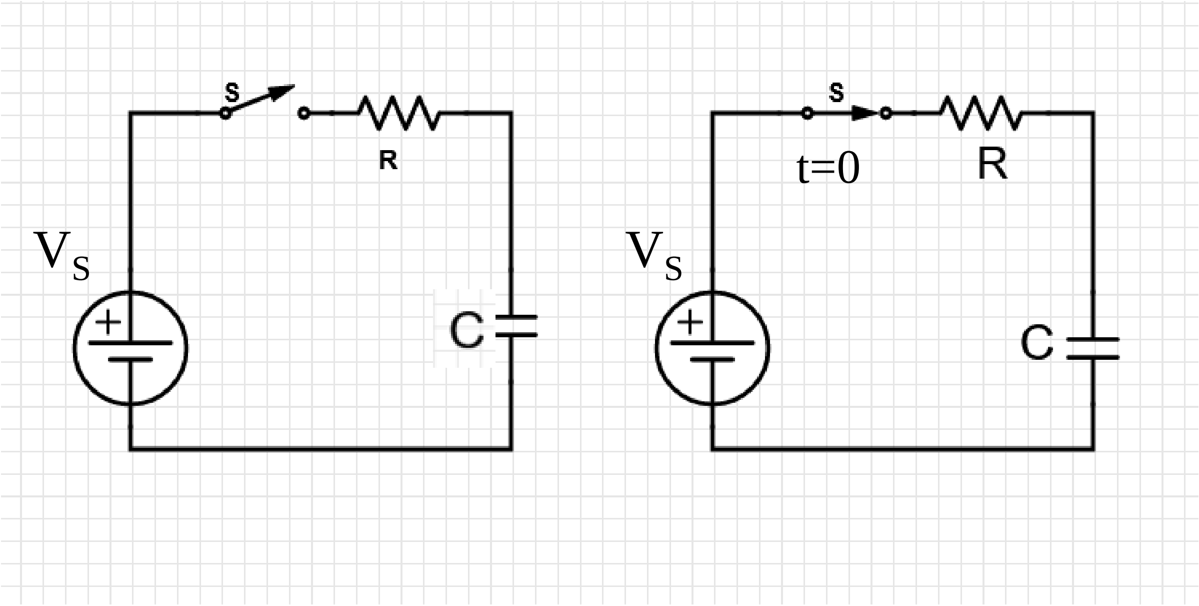
Applications of First Order Circuits

Camera Flash



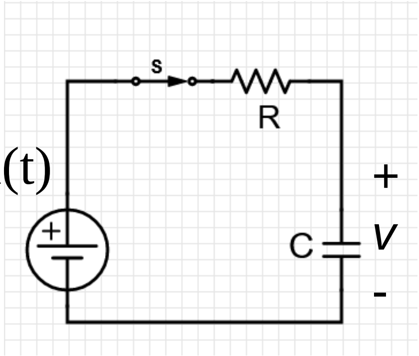
Step Response of a RC Circuit

When a DC Voltage or Current source is suddenly turned on, the source can be modelled as a step function.



u(t): Unit-step function
= 0 for t < 0
= 1 for t > 0

Step Response of a RC Circuit



$$C = \frac{q}{v} \gg q = cv \gg i = C\frac{dv}{dt}$$

$$C\frac{dv}{dt} = \frac{V_S\,u(t) - v}{R}$$

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{V_s}{RC}$$

$$\frac{dv}{dt} = -\frac{v - V_s}{RC}$$

$$\frac{dv}{v - V_s} = -\frac{dt}{RC}$$

$$\ln(v - V_s)\Big|_{V_0}^{v(t)} = -\frac{t}{RC}\Big|_0^t$$

$$\ln(v(t) - V_s) - \ln(V_0 - V_s) = -\frac{t}{RC} + 0;\ \ln\frac{v - V_s}{V_0 - V_s} = -\frac{t}{RC}$$

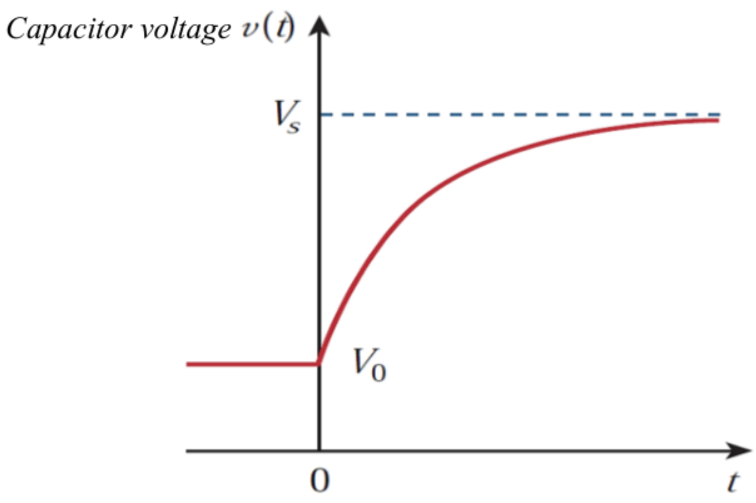
Taking exponential on both sides

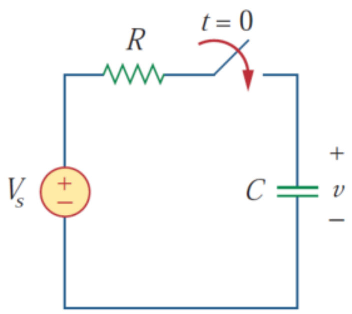
$$\frac{v - V_s}{V_0 - V_s} = e^{-\frac{t}{\tau}}, \tau = RC;\ v(t) = V_s + (V_0 - V_s)e^{-\frac{t}{\tau}}, t > 0$$

$$v(t) = \begin{cases} V_0 & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{\tau}} & t > 0 \end{cases}$$

Step Response of a RC Circuit

$$v(t)=\begin{cases} V_0 & t<0 \\ V_s+(V_0-V_s)e^{-\frac{t}{\tau}} & t>0 \end{cases}$$





Step Response of a RC Circuit

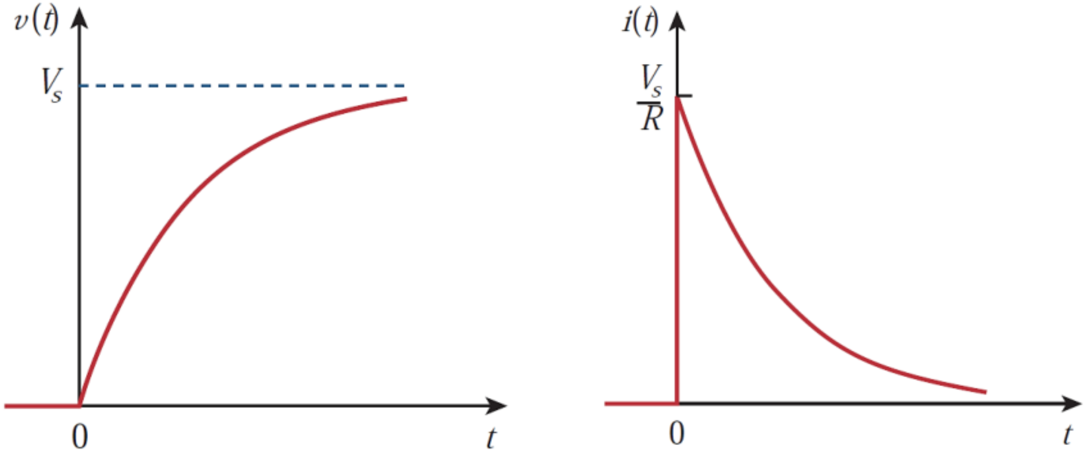
If Initial charge on capacitor is zero

$$v(t) = \begin{cases} V_0 & t < 0 \\ V_s + (V_0 - V_s)e^{-\frac{t}{\tau}} & t > 0 \end{cases}$$

$$v(t) = V_s(1 - e^{-\frac{t}{\tau}})u(t)$$

$$i(t) = C\frac{dv}{dt} = \frac{C}{\tau}V_s e^{-\frac{t}{\tau}}; \ \tau = RC; \ t > 0$$

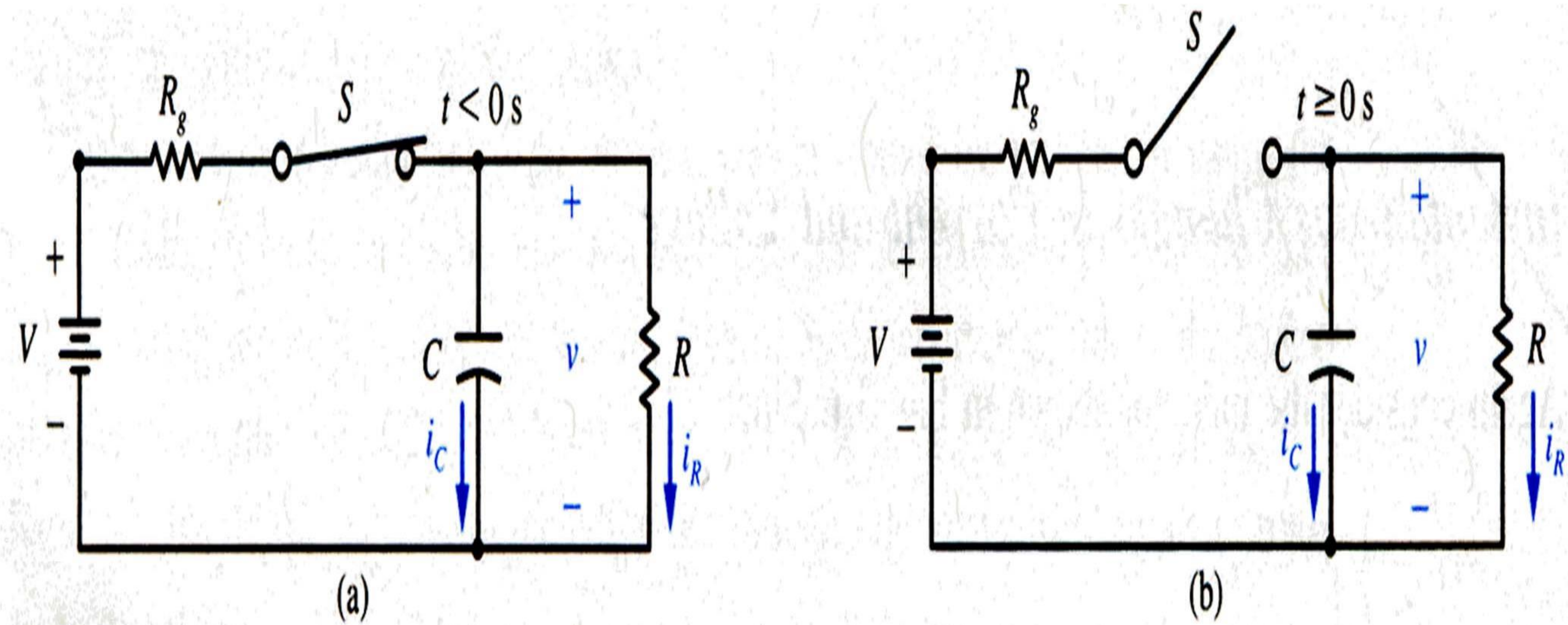
$$i(t) = \frac{V_s}{R}e^{-\frac{t}{\tau}}u(t)$$



Step Response of a RC Circuit

Switch is

a. **Closed for t < 0 s**
b. **Open for t > 0 s**



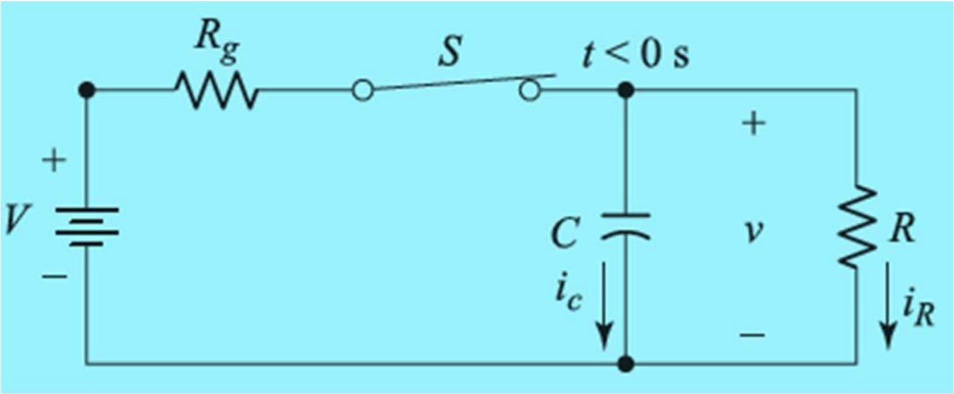
RC circuit (*a*) with a closed switch, and (*b*) with the switch opened.

Step Response of a RC Circuit

For t < 0, the circuit is a DC circuit therefore, the capacitor behaves as Open Circuit.

By Voltage division ,

$$v = \frac{V * R}{R_g + R}$$



Step Response of a RC Circuit

For t $\geq$ 0,

$$v(0) = \frac{V * R}{R_g + R}$$

$$i_C + i_R = 0$$

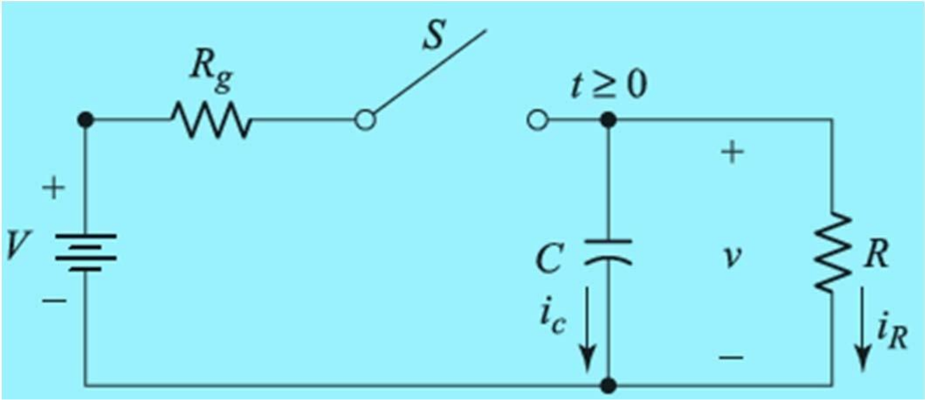
$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

First Order Differential Equation

$$\frac{dv}{dt} = -\frac{1}{RC}v$$

$$\int \frac{dv}{v} = \int -\frac{1}{RC}dt$$

$$\Rightarrow \ln v(t) = -\frac{t}{RC} + K$$



$$v(t) = e^{-\frac{t}{RC}+K} = e^{-\frac{t}{RC}}e^{K}$$

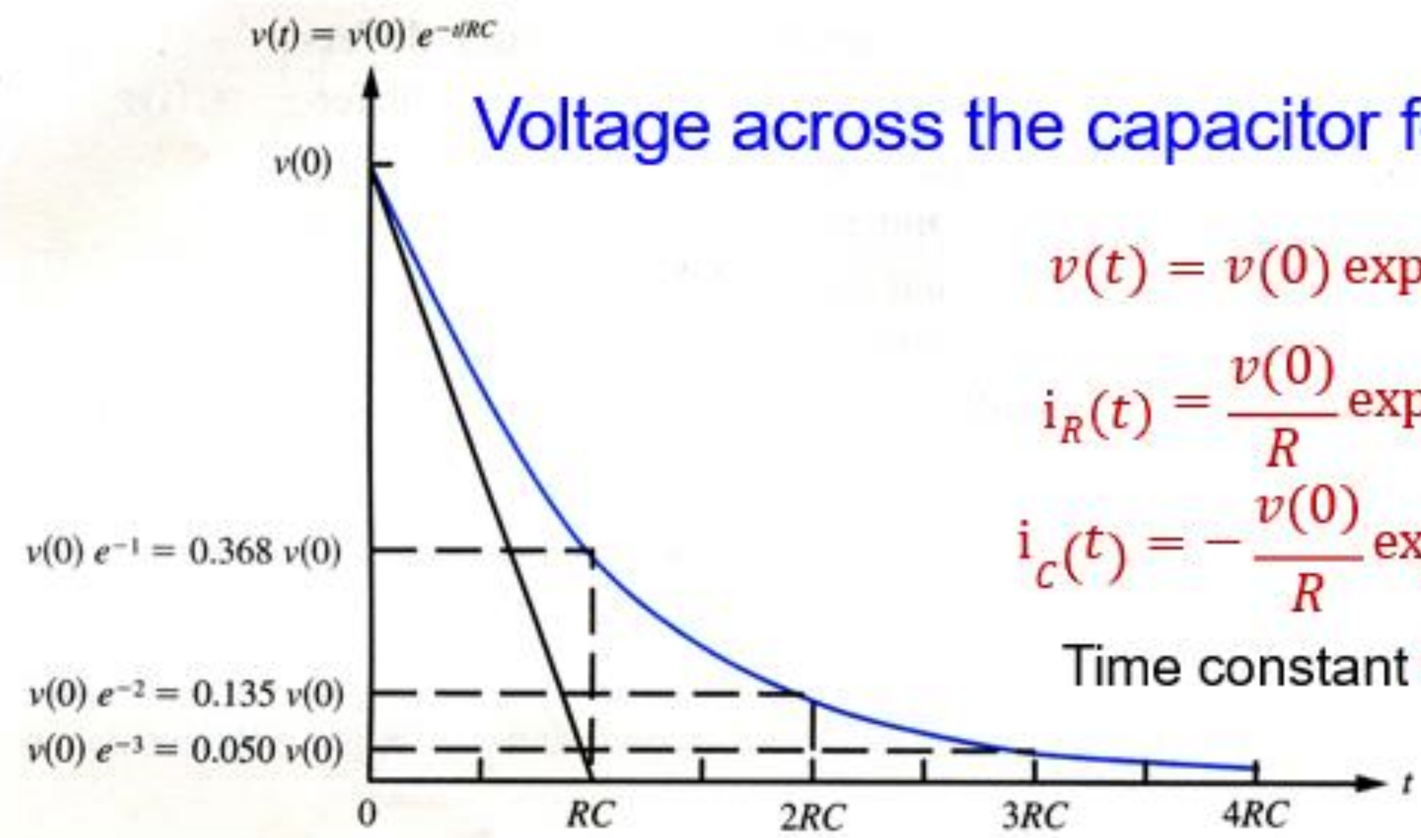
$$\text{At } t = 0, v(0) = e^{K}$$

$$\text{For } t \geq 0\,\text{s}, v(t) = v(0)e^{-\frac{t}{RC}}$$

$$i_R(t) = \frac{v(t)}{R} = \frac{v(0)}{R}e^{-\frac{t}{RC}}$$

RC is a time constant.

Step Response of a RC Circuit



Voltage across the capacitor for t ≥0s

$$v(t) = v(0)\exp\left(-\frac{t}{RC}\right)$$

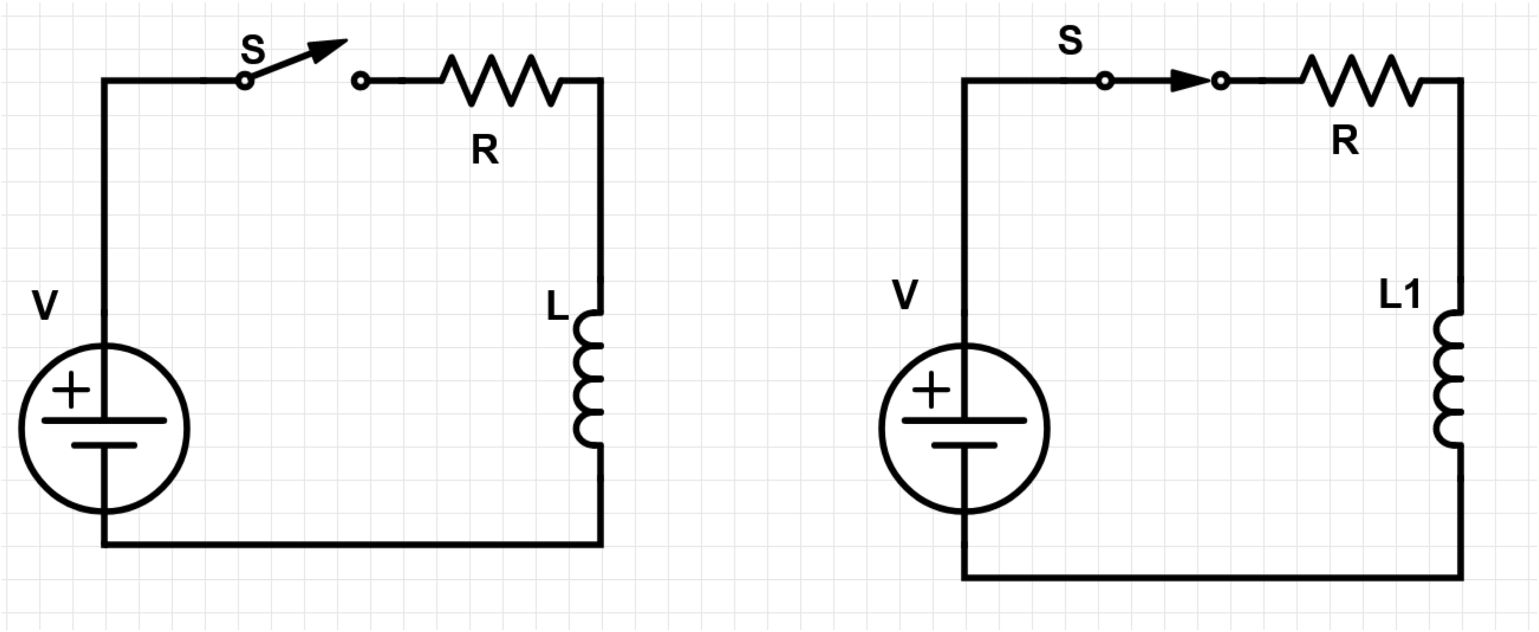
$$i_R(t) = \frac{v(0)}{R}\exp\left(-\frac{t}{RC}\right)$$

$$i_c(t) = -\frac{v(0)}{R}\exp\left(-\frac{t}{RC}\right)$$

Time constant τ = RC

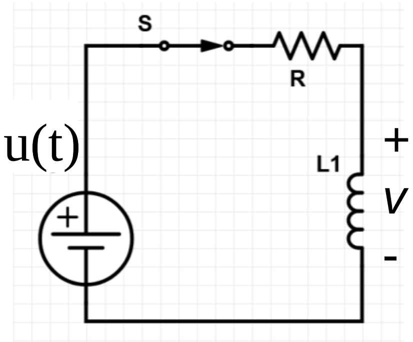
Step Response of a RL Circuit

When a DC Voltage or Current source is suddenly turned on, the source can be modelled as a step function.



u(t): Unit-step function
= 0 for t < 0
= 1 for t > 0

Step Response of a RL Circuit



$V_S\,u(t)$

Transient response is always a decaying exponential

$$i_t = Ae^{-\frac{t}{\tau}}; \ \ \tau = \frac{L}{R}$$

$$i_{ss} = \frac{V_s}{R};$$

$$i = Ae^{-\frac{t}{\tau}} + \frac{V_s}{R}$$

Let I_0 be the initial current through the inductor

$$\text{At } t = 0, \ I_0 = A + \frac{V_s}{R}$$

$$A = I_0 - \frac{V_s}{R}$$

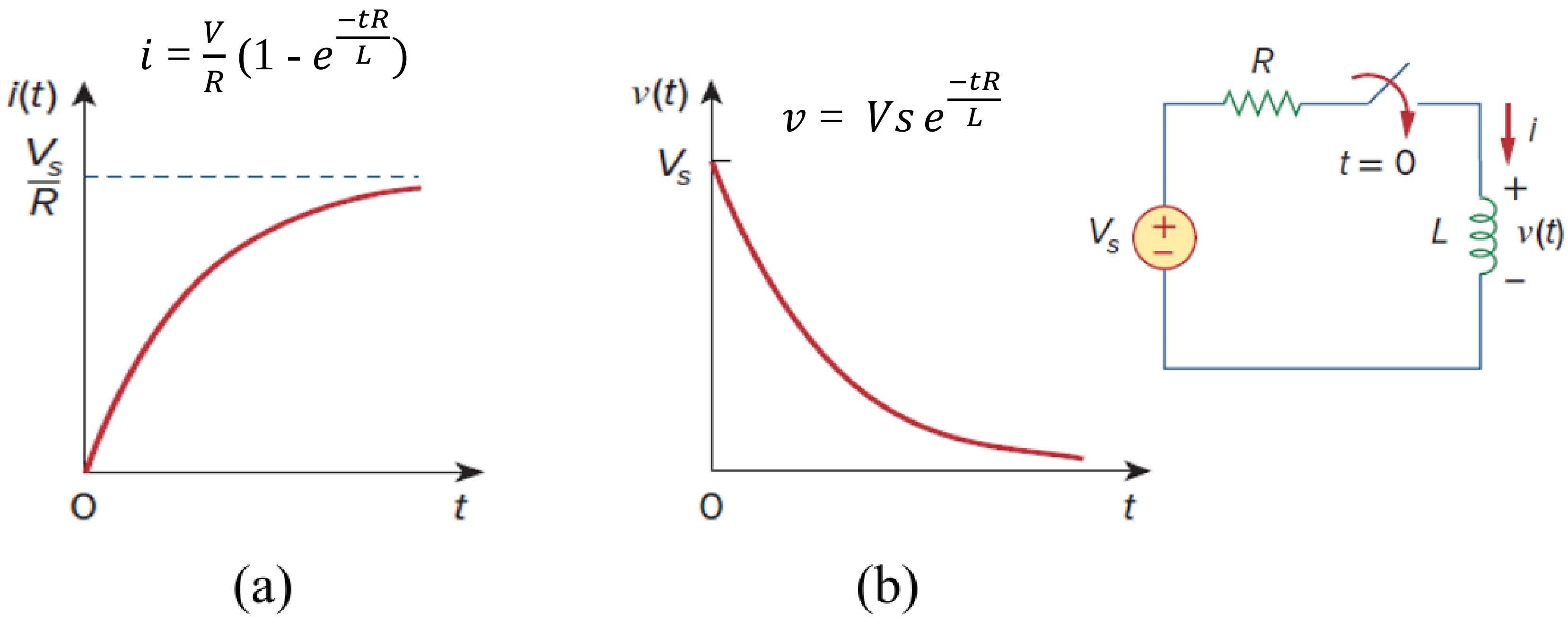
$$\boxed{i(t) = \frac{V_s}{R} + (I_0 - \frac{V_s}{R})e^{-\frac{t}{\tau}}}$$

If $I_0=0$,

$$i(t) = \begin{cases} 0 & t < 0 \\ \frac{V_s}{R}(1 - e^{-\frac{t}{\tau}}) & t > 0 \end{cases}$$

$$\boxed{i(t) = \frac{V_s}{R}(1 - e^{-\frac{t}{\tau}})u(t)}$$

Step Response of a RL Circuit

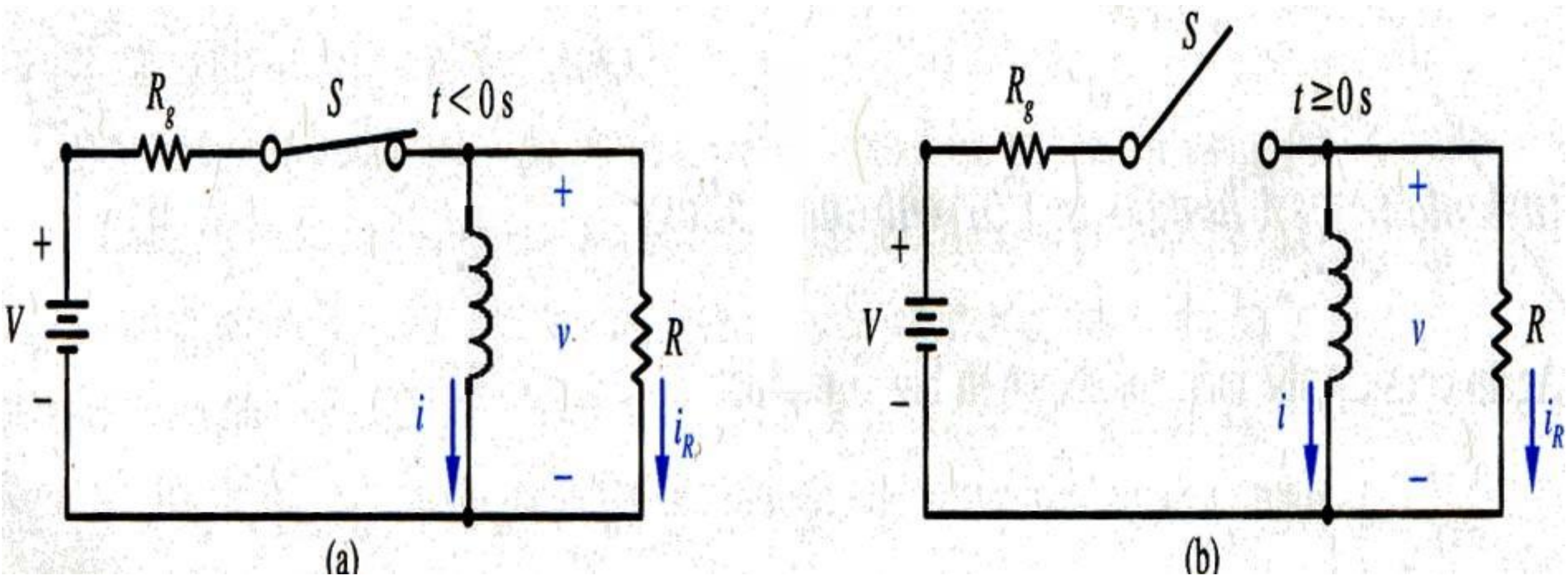


Step responses of an RL circuit with no initial inductor current

Step Response of a RL Circuit

Switch is

a. **Closed for t<0**
b. **Open for t>0**



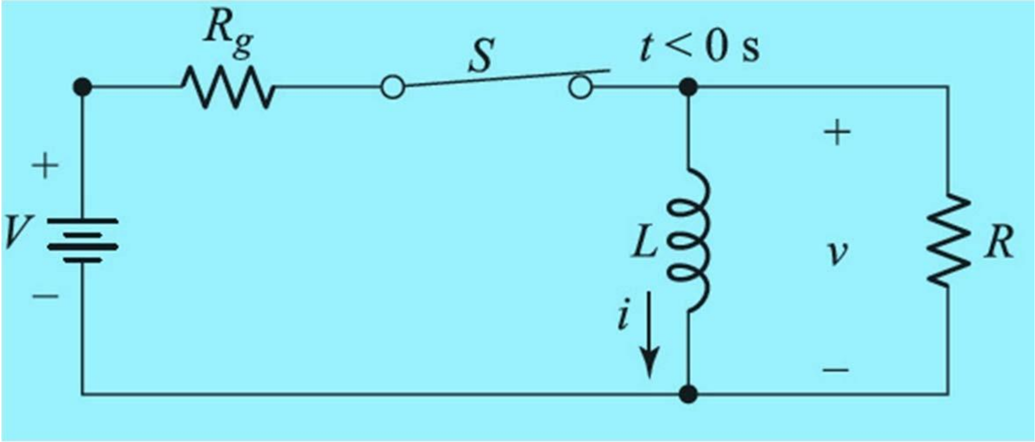
Step Response of a RL Circuit

- Switch is closed at $t < 0s$.
- Inductor acts as short circuit for dc, voltage

$$v_L = v_R = v = 0$$

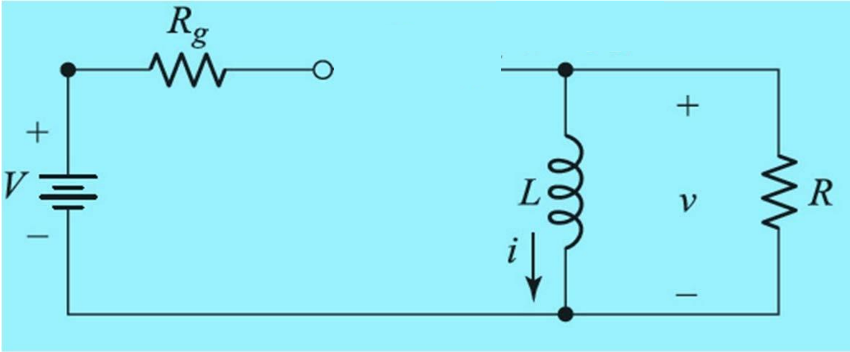
$$\Rightarrow i_R = 0,$$

$$i(t) = \frac{V}{R_g}$$



Step Response of a RL Circuit

- Switch opens at t = 0 sec and remains open for t > 0.
- Current through inductor can not change instantaneously



$$i(0) = \frac{V}{R_g}$$

$$L\frac{di}{dt} + Ri = 0,$$

$$\frac{di}{dt} + \frac{R}{L}i = 0$$

$$\frac{di}{dt} = -\frac{R}{L}i$$

$$\frac{1}{i}\frac{di}{dt} = -\frac{R}{L} \Rightarrow \int \frac{1}{i}\frac{di}{dt}dt = -\int \frac{R}{L}dt$$

$$\int \frac{1}{i}\frac{di}{dt}dt = \int \frac{di}{i} = \int -\frac{R}{L}dt$$

$$\Rightarrow \ln i(t) = -\frac{Rt}{L} + K$$

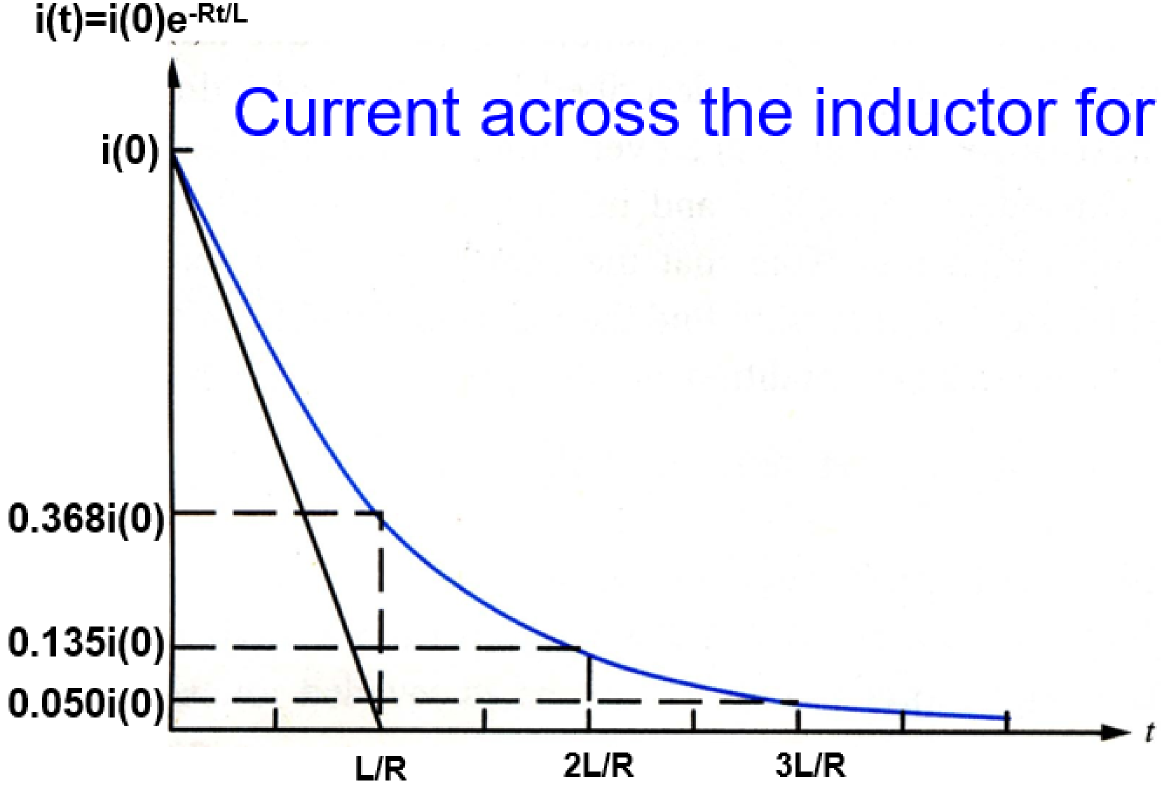
$$i(t) = e^{\frac{-Rt}{L}}e^{K}$$

At $t = 0$, $i(0) = e^{K}$

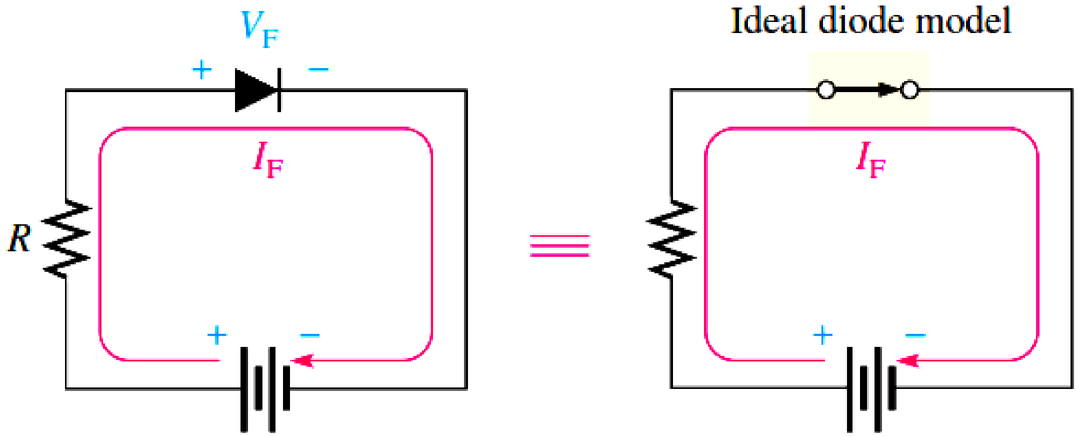
For $t \geq 0\,\mathrm{s}, i(t) = i(0)e^{\frac{-Rt}{L}}$

Step Response of a RL Circuit

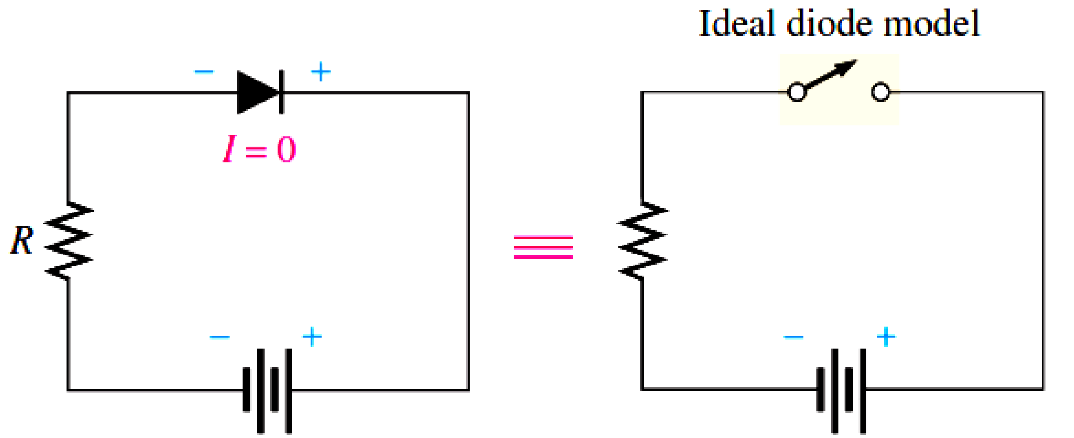
For $t \geq 0\,\text{s}, i(t) = i(0)e^{\frac{-Rt}{L}}$



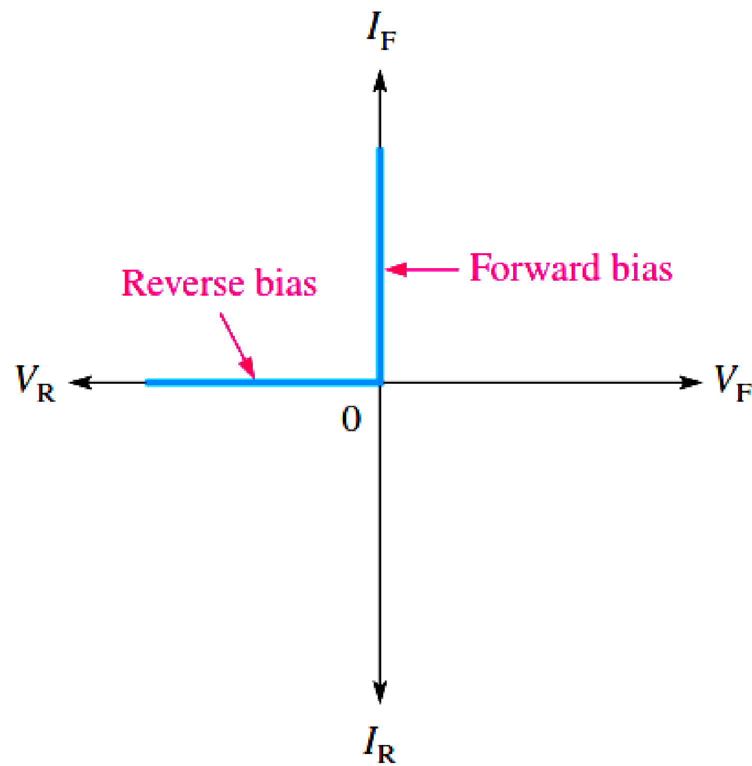
Ideal Diode Model



(a) Forward bias



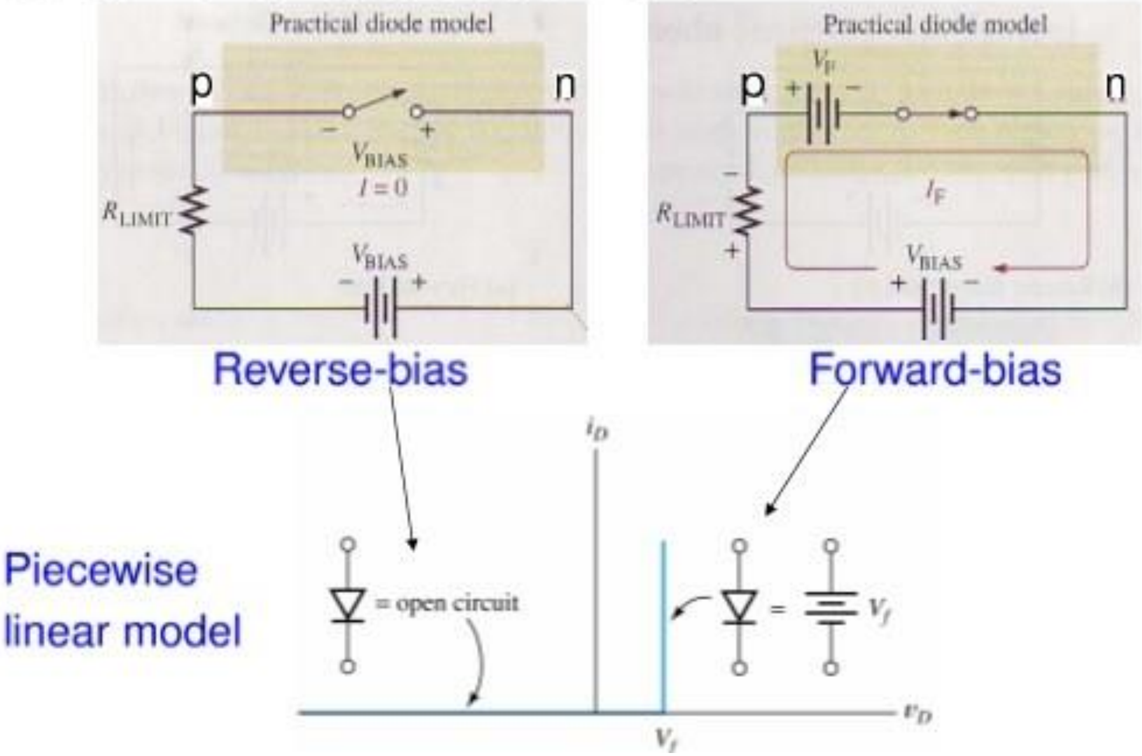
(b) Reverse bias



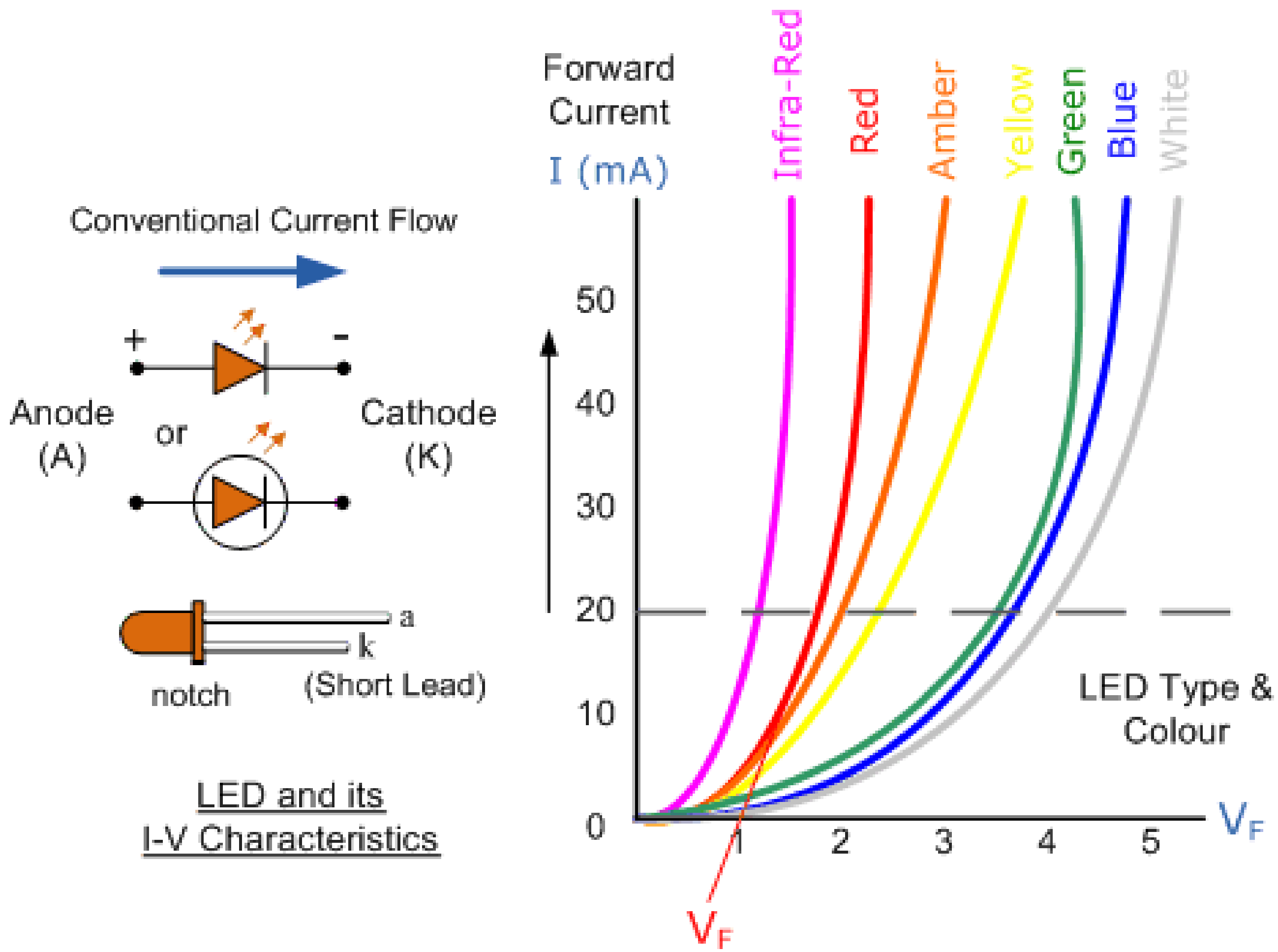
(c) Ideal *V-I* characteristic curve (blue)

Practical Diode Model

- The practical diode model adds a 0.7V voltage source in series with an ideal diode model.



LED Model



Conventional Current Flow
+
-
Anode
(A)
or
Cathode
(K)
a
k
notch
(Short Lead)
LED and its
I-V Characteristics
Forward
Current
I (mA)
Infra-Red
Red
Amber
Yellow
Green
Blue
White
50
40
30
20
10
0
1
2
3
4
5
V_F
LED Type &
Colour
V_F

Thank you